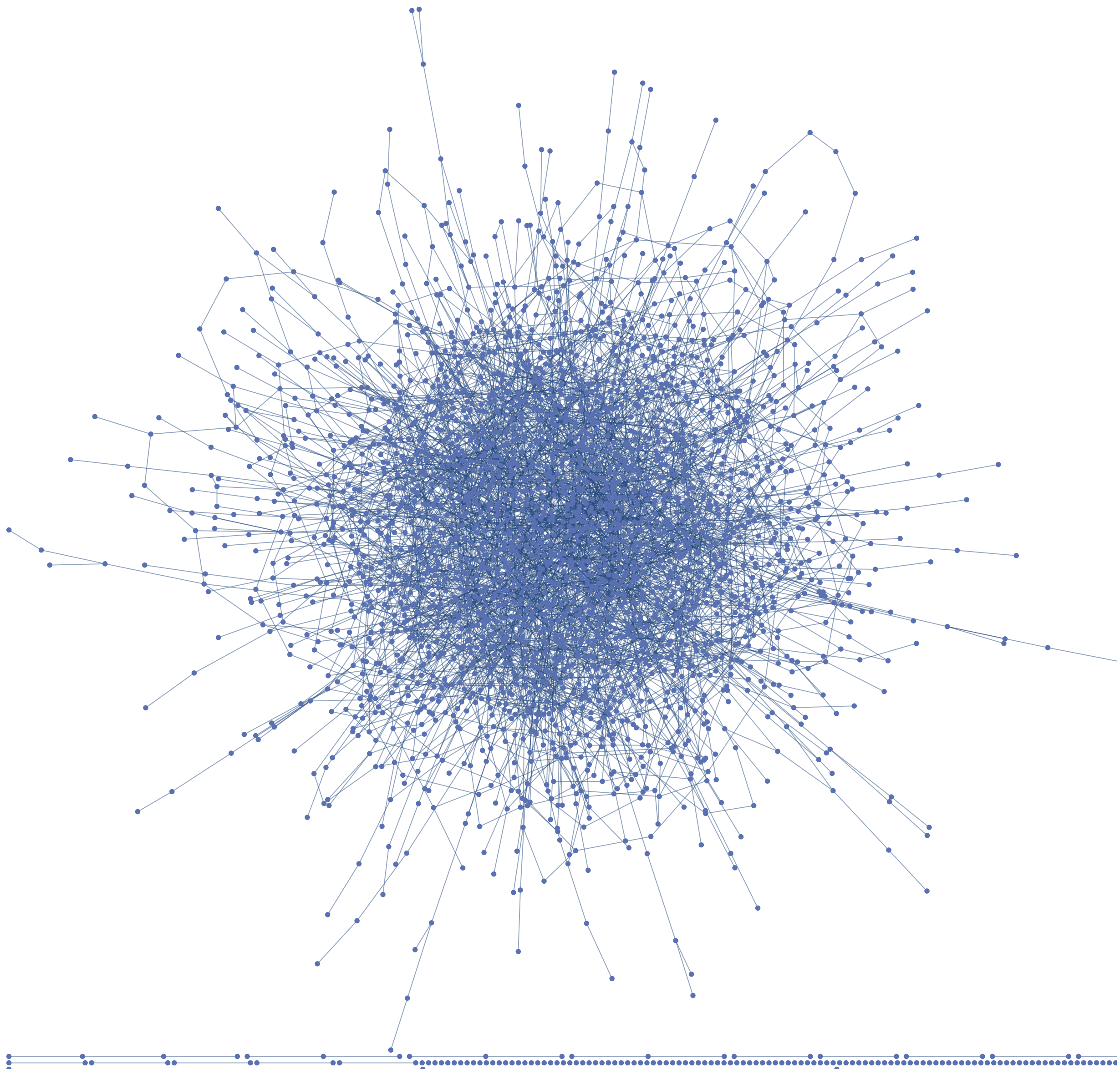
```
(*Problem 1. Erdos-Renyi network.*)
n = 3000;
p = 10^-3;
g = RandomGraph[BernoulliGraphDistribution[n, p]]
```

Out[152]=



In[17]:=

```
(*1: <L>*)
EdgeCount[g]
n^2/2 * p
```

Out[17]= 4528

Out[18]= 4500

```
(*2: <k>*)
N@Mean[VertexDegree[g]]
n * p
```

Out[21]= 3.01867

Out[22]= 3

```
(*3: regime*)
```

It's in supercritical regime.

```
In[23]:= (*4: critical linking probability*)
pc = 1. / 3000
```

Out[23]= 0.000333333

```
In[34]:= (*5: average path length*)
d = Log[n] / Log[n * p] // N
```

Out[34]= 7.28771

```
In[35]:= (*6: what is n if <k>=3 and d=100*)
Exp[100 * Log[3]]
```

Out[35]= 515 377 520 732 011 331 036 461 129 765 621 272 702 107 522 001

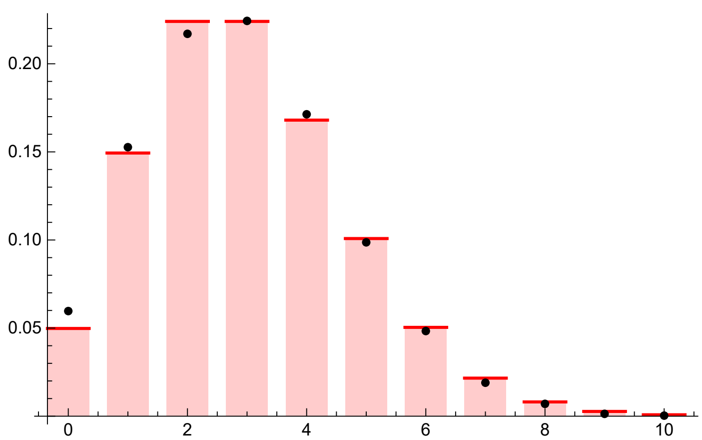
```
In[179]:= (*7: Poisson distribution of k*)
GraphPropertyDistribution[VertexDegree[g, 1], g ≈ BernoulliGraphDistribution[n, p]]
dist = PoissonDistribution[n * p]
```

Out[179]= BinomialDistribution[-1 + n, p]

Out[180]= PoissonDistribution[n p]

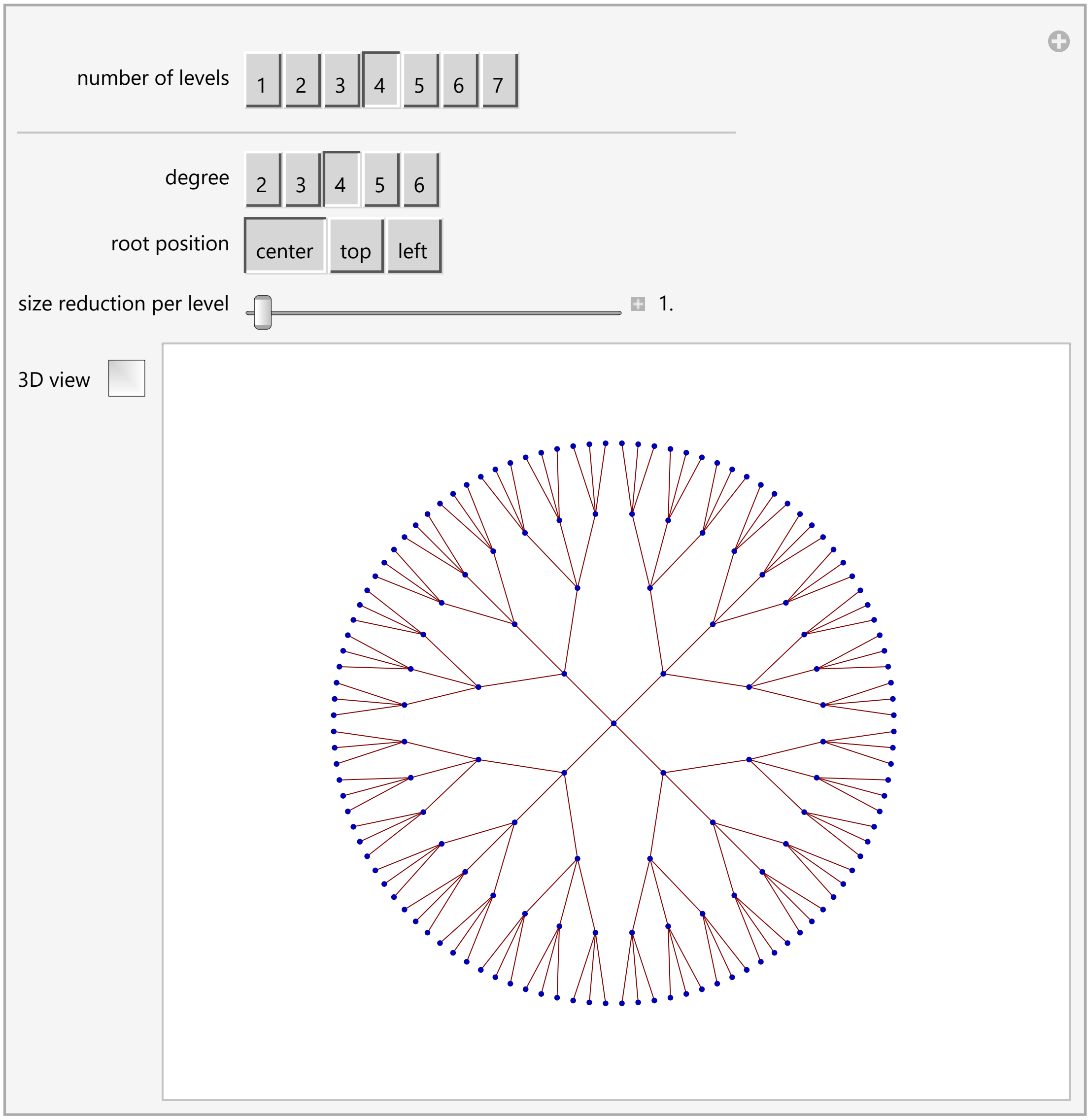
```
In[149]:= Show[DiscretePlot[PDF[dist, k], {k, 0, 10}, PlotStyle → Red, ExtentSize → 0.7],
  ListPlot[({#[[1]], #[[2]] / n}) & /@ Tally[VertexDegree[g]], PlotStyle → Black]]
```

Out[149]=




```
In[210]:= (*Problem 2. Cayley Tree.*)
ClearAll["Global`*"]
```

Out[234]=



(*1*)

There should be $1+k+k(k-1)+k(k-1)^2+...+k(k-1)^{n-1}$ nodes reachable in n steps, or,

In[182]:=
```
Piecewise[
 {{1 + Sum[k (k - 1) ^i, {i, 0, n - 1}], n ≤ r}, {1 + Sum[k (k - 1) ^i, {i, 0, r - 1}], n > r}}]
```

Out[182]=

$$\begin{cases} 1+\frac{\left(-1+(-1+k)^{n}\right)k}{-2+k} & n\leq r \\ 1+\frac{\left(-1+(-1+k)^{r}\right)k}{-2+k} & n>r \\ 0 & \text{True} \end{cases}$$

(*2*)

There are $n_{outer}=k(k-1)^{r-1}$ nodes in the outermost layer. The average path length between leaves is

$$2 * \frac{k-2}{n_{outer}-1} + 4 * \frac{(k-1)(k-2)}{n_{outer}-1} + 6 * \frac{(k-1)(k-1)(k-2)}{n_{outer}-1} + ... +$$
$$2(r-1) * \frac{(k-1)^{r-2}(k-2)}{n_{outer}-1} + 2r * \frac{(k-1)^{r}}{n_{outer}-1}, \text{ or,}$$

In[178]:= $\text{Sum}\left[2(i+1)\frac{(k-1)^{i}(k-2)}{k(k-1)^{r-1}-1}, \{i, 0, r-2\}\right] + 2r\frac{(k-1)^{r}}{k(k-1)^{r-1}-1}$ // FullSimplify

Out[178]= $$\frac{2\left(-1+k+(-1+k)^{r}\left(1-k+(-2+k)\,k\,r\right)\right)}{(-2+k)\left(1+\left(-1+(-1+k)^{r}\right)k\right)}$$

(*3*)

The size of a Cayley tree is to the order of k^r, while the diameter or the average path length between leaves is only to the order of r.