Herd behavior in a complex adaptive system

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In order to survive, self-serving agents in various kinds of complex adaptive systems (CASs) must compete against others for sharing limited resources with biased or unbiased distribution by conducting strategic behaviors. This competition can globally result in the balance of resource allocation. As a result, most of the agents and species can survive well. However, it is a common belief that the formation of a herd in a CAS will cause excess volatility, which can ruin the balance of resource allocation in the CAS. Here this belief is challenged with the results obtained from a modeled resource-allocation system. Based on this system, we designed and conducted a series of computer-aided human experiments including herd behavior. We also performed agent-based simulations and theoretical analyses, in order to confirm the experimental observations and reveal the underlying mechanism. We report that, as long as the ratio of the two resources for allocation is biased enough, the formation of a typically sized herd can help the system to reach the balanced state. This resource ratio also serves as the critical point for a class of phase transition identified herein, which can be used to discover the role change of herd behavior, from a ruinous one to a helpful one. This work is also of value to some fields, ranging from management and social science, to ecology and evolution, and to physics.

experimental econophysics | computational econophysics | market-directed resource-allocation game | minority game | agent-based model

Most of the social, ecological, and biological systems that involve a large number of interacting agents can be seen as complex adaptive systems (CASs), because they are characterized by a high degree of adaptive capacities to the changing environment. CAS dynamics and collective behaviors have attracted much attention among physical scientists (1–3). In order to survive, self-serving agents in these CASs must compete against others for limited resources with biased or unbiased distribution by conducting strategic behaviors. This competition can globally result in balanced or unbalanced resource allocation. Examples of such phenomena involve many species like human beings. For instance, drivers select different traffic routes, people bet on horse racing with odds, and so on. In general, the allocation of the resources in a CAS could reach a balanced state due to the preferences and decision making ability of agents, as revealed by investigating a resource-allocation problem (4). In practice, however, it will sometimes fail to reach the balanced state. For this, one important reason is due to the formation of a herd. In fact, herding extensively exists in collective behaviors of many species in CASs, including human beings. Though human decisions are basically made according to individual thinking, people tend to pay heed to what others are doing, emulate successful persons, or those of higher status, and thus follow the current trend. For example, young girls often copy the clothing style of some famous stars named as trendsetters in the fashion world. Similarly, researchers would rather choose to work on a topic that is currently hot in the scientific society. As a result, large numbers of people may act in concert, and this unplanned formation of crowds is called herd behavior (5). Locally speaking, either the irrationality (6, 7) or rationality (8–10), of an individual agent can be the cause of herd behavior. The global view of herding often implies the ruin of balance by causing excessive volatility in the resource allocation system. Accordingly, herd behavior is commonly seen as a tailor-made cause for explaining bubbles and crashes in a CAS with the existence of extremely high volatility. But is this "common sense" always right? Based on results of this study, we argue that herd behavior should not be labeled like the killer of balance and stability all the time. Here we focus on the effect of herding on the whole CAS for resource allocation, because it is most important for as many agents (involving human beings) as possible to survive in various kinds of CASs like social, ecological or biological systems. Therefore, we shall not study or consider the details on how to reach a herd through contagion and/or imitating. In fact, our results are not dependent on the process of herding formation.

Experiment

We design and conduct a series of computer-aided human experiments, on the basis of the resource-allocation system (4, 11–13), in order to study the necessary conditions for a CAS to reach the ideal balanced state. Using this kind of experimental settings will allow us to investigate the herd behavior in a well regulated abstract system for resource allocation, which reflects the fundamental characteristics of many CASs (14–17). Human participants of the resource-allocation experiment are students recruited from several departments of Fudan University. Before the start of experiments, a leaflet (as shown in *SI Text: Part I*) was provided which explains configurations of the experiment and actions of the participants. There are two rooms (Room 1 and Room 2) and the amounts of resource in these two rooms are M_1 and M_2 ($\leq M_1$), respectively. As the experiment evolves, M_1 and M_2 are kept fixed and unknown to all the participants. For each experiment round, each participant has to choose one of the two rooms to enter. Those who go into the same room should share alike the virtual resource (M_1 or M_2) in it. Apart from human participants, there are also imitating agents joining the experiment. All the imitating agents are generated by a computer program, because their decisions are simply made by mimicking human participants' behaviors. In particular, each imitating agent will randomly select a group (of size five) of human participants at every experiment round, and then follow the choice of the best participant (who has the highest score) in the group for the next round. In each round of the experiment, the number of human participants and imitating agents in Room 1 is denoted as N_1 and the number in Room 2 as N_2. Therefore the total number of human participants and imitating agents can be counted as $N = N_1 + N_2$. The human participants or imitating agents who earned more than the global average $(M_1 + M_2)/N$ are regarded as winners of the round, and the room which the winners had entered as the winning room. The total number of human parti-

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cipants or imitating agents can also be expressed as $N = N_n + N_m$. Here N_n is the total number of human participants who make decisions by their own, and N_m is the total number of imitating agents who do not have their own ideas. The ratio between imitating agents and human participants is defined as $\beta = N_m/N_n$. More details about the experiment can be found at the end of the main text.

The resource-allocation experiments are conducted repeatedly with different values of M_1/M_2 and β. The modeled system is designed as an open system in which the number of human participants N_n is fixed while the number of imitating agents N_m is increased in an implicit manner. As shown in the previous study (4), the heterogeneity of preferences is an indispensable factor for the whole system to reach the balanced state. Hence the preferences of human participants need to be checked under the influence of imitating agents. For a human participant in the experiment, his/her preference is evaluated as the average rate that he/she chooses to enter Room 1. Preferences of the 44 participants are plotted in Fig. 1 with different M_1/M_2's and/or β's. Fig. 1*A* shows the preferences of human participants when $M_1/M_2 = 1$ and the imitating agents are absent. Distinctions among the preferences of human participants can be easily identified. For example, the fourth participant is strongly partial to entering Room 2 while the sixth participant prefers Room 1 much more. It can be found in Fig. 1 *B* and *C*, that the human participants still have diverse preferences even when M_1 becomes much larger than M_2. In addition, the heterogeneity of preferences remains even for the cases in which $N_m = N_n/2$ imitating agents are involved; see Fig. 1 *G–I*. Despite of this heterogeneity, the average of participants' preferences changes along with M_1/M_2. In other words, human participants have the ability to adapt themselves to fit the environment.

Comparisons of the distributions of human participants' preferences, as the resource distribution M_1/M_2 is varied and/or the imitating agents are involved, are shown in Fig. 1 *D–F*, *J–L*. From Fig. 1 *D* and *E*, one can find that when M_1/M_2 is not so biased, human participants alone can do the analysis of the system so well that they can make the whole system reach the balanced state. Note that the preference distribution has a peak at 0.5 in Fig. 1*D* and the participants' preferences are mainly distributed around 0.75 in Fig. 1*E*. Both of the two observations can be deduced from the resource distribution, $M_1/M_2 = 1$ and $M_1/M_2 = 3$. When the imitating agents are involved, however, the two preference distributions have some changes in Fig. 1 *J* and *K*. In particular, the

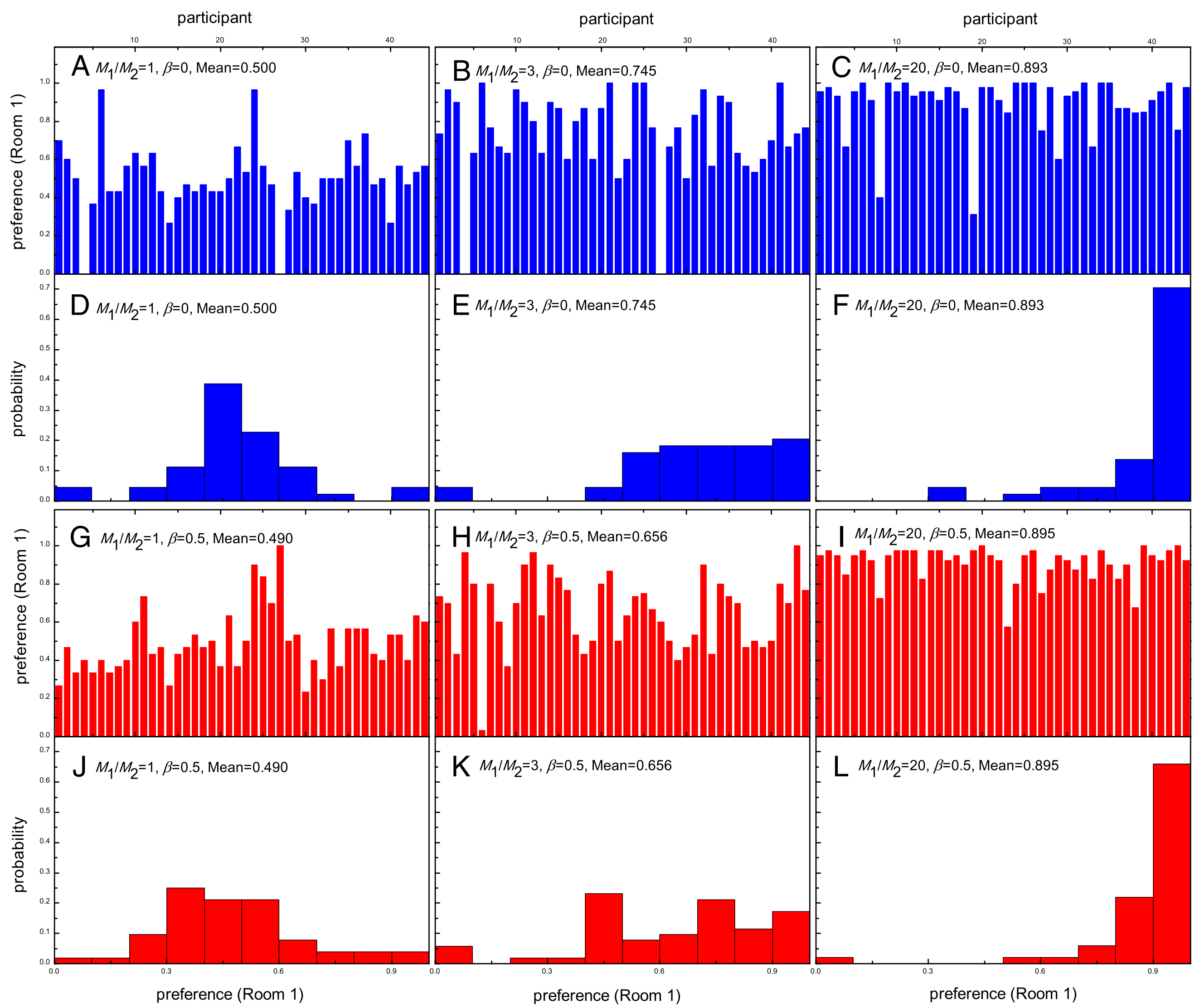

Fig. 1. Data obtained from the human experiment. (*A–C*,*G–I*) Preferences of the 44 participants in sequence to Room 1 for the cases (*A–C*) without and (*G–I*) with imitating agents, $\beta = (A\text{-}C)$ 0 and (*G–I*) 0.5, for the resource distributions $M_1/M_2 = (A,G)$ 1, (*B*,*H*) 3, and (*C*,*I*) 20. Here, "Mean" denotes the average value of the preferences of the 44 participants. (*D–F*,*J–L*) Distribution of the 44 participants' preferences.

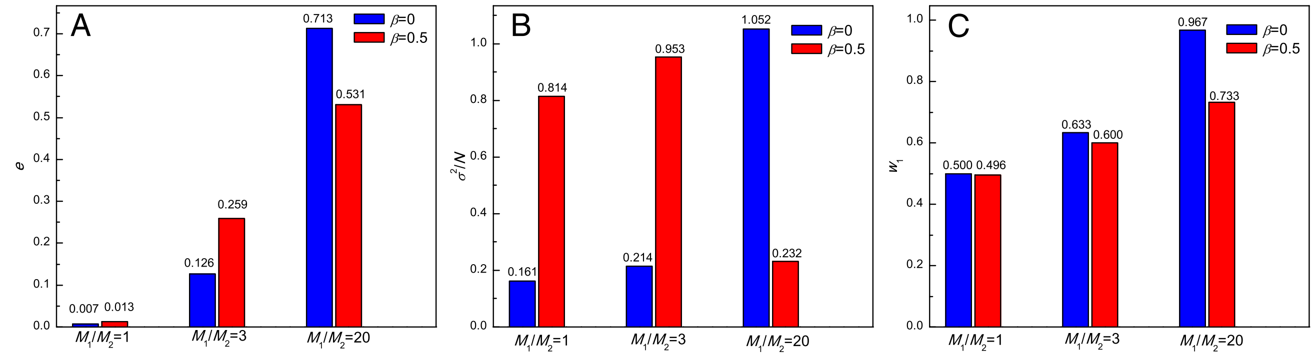

Fig. 2. Experimental results for (*A*) efficiency *e*, (*B*) stability σ^2/N, and (*C*) predictability w_1 of the modeled resource-allocation system, with human participants $N_n = 50$. $\beta = 0$ and 0.5 correspond to imitating agents $N_m = 0$ and 25, respectively. Each experiment lasts for 30 rounds.

peak almost disappears in Fig. 1*J* and the mean value of participants' preference deviates from the resource distribution bias in Fig. 1*K*. A possible reason for these changes can be inferred as that human participants may get confused by the behavior of imitating agents. Hence in this case the herd (which is formed by imitating agents) indeed disturbs the system and weakens the analyzing ability of human participants. Things are different if M_1/M_2 gets even larger, as shown in Fig. 1 *F* and *L*. Here the involvement of imitating agents does not bring much change to the preference distribution of human participants. One may say that, in this case, herd behavior has no harmful effect on the analyzing ability of the human participants. Finally, it is interesting to note from the same figure, that a minority of human participants with preference to Room 2 can stay alive even in a highly biased system ($M_1/M_2 \gg 1$) when the imitating agents exist.

To evaluate the performance of the whole system, we have calculated efficiency (which, herein, only describes the degree of balance of resource allocation), stability, and predictability of the resource-allocation system. The efficiency of the whole system can be defined as $e = |\langle N_1\rangle/\langle N_2\rangle - M_1/M_2|/(M_1/M_2)$. A smaller e means a higher efficiency in the allocation of resources. The stability of the resource-allocation system can be described as $\sigma^2/N \equiv \frac{1}{2N}\sum_{i=1}^{2}\langle(N_i - \tilde{N}_i)^2\rangle$, where $\langle \mathrm{A}\rangle$ denotes the average of time series A. This definition describes the fluctuation (volatility) in the room population away from the balanced state, where the optimal room populations $\tilde{N}_i = M_i N/\sum M_i$ can be realized. The predictability of the system is measured by the "uniformity" of the winning rates in different rooms. The winning rate in Room 1 is denoted as w_1. It is obvious that if w_1 is close to 0.5, choices of the two rooms are symmetrical and the system is unpredictable. If the winning rates were too biased, smart participants should be able to predict the next winning room in the experiment. As shown in Fig. 2, when M_1/M_2 is small ($M_1/M_2 = 1$ or 3), adding some imitating agents will lower the efficiency and cause large fluctuations. On the other hand, when M_1/M_2 get even larger ($M_1/M_2 = 20$), the formation of herd can improve the efficiency, the stability, and the unpredictability of the resource-allocation system.

Agent-Based Modeling

An agent-based model is developed in order to fully understand the preceding experimental results. Consider a situation where N agents repeatedly join a resource-allocation system. Among these agents, there are N_n normal agents (which correspond to human participants in the preceding experiments) and N_m imitating agents, so that the total number of agents can be calculated as $N = N_n + N_m$. To play in the resource-allocation system, each normal agent will take S strategies from the full strategy space and compose a strategy book. A strategy for the resource-allocation experiment is typically a choice table which consists of two columns. The left column is for the P possible situations, and the right column is filled with bits of 0 or 1. Bit 1 is linked to the choice for the entrance of Room 1, while bit 0 to that of Room 2. In the strategy book of a normal agent, strategies differ from each other in the preference, which is defined as an integer L $(0 \leq L \leq P)$. To model the heterogeneity of preference, let the normal agent pick up a preference number L first. Then each element of the strategy's right column is filled in by 1 with the probability L/P, and by 0 with the probability $(P - L)/P$ (more detailed explanations can be found in *SI Text: Part II*). The process will be repeated S times, each time with a randomly chosen L for each normal agent to complete the construction of its strategy book. From the start of the resource-allocation experiment, each normal agent will score all the strategies in its strategy book so as to evaluate how successful they are to predict the winning room. Following the hitherto best performing strategy in their strategy books, normal agents are enabled to make decisions to enter one of the two rooms, once the current situation is randomly given*. Imitating agents in the model behave in a different way during the process of decision making. Before each round of the play starts, each imitating agent will randomly select a group of k $(1 \leq k \leq N_n)$ normal agents †. Within this group, the imitating agent will find the normal agent who has the best performance so far and imitate its behavior in the following experiment round. It is assumed that the imitating agents know neither the historical record of the winning room nor the details of strategy books of other group members. The only information for them to access is the performance of the normal agents, that is, the virtual money that these normal agents have earned from the beginning of the experiment. If the number of imitating agents N_m kept increasing, there would be more and more positive correlations among agents' decisions, which would trigger the formation of a herd in the system.

Simulation Results of the Agent-Based Modeling

Agent -based simulations are carried out in an open system condition, in reference to the experiments. (Please refer to *SI Text: Part III* to see the results for a closed system.) Following the analysis of experimental results, we first investigate the simulation results for the preferences of normal agents. Clearly, Fig. 3 shows distributions of the preferences similar to those shown in Fig. 1. The qualitative agreement indicates that our agent-based modeling has taken into account the heterogeneity of preferences with a reasonable modeling of the decision making process for the human participants. (We had also investigated the preferences

*Here the situation is not the history of winning rooms. Broadly speaking, it can be explained as a mixture of endogenous and exogenous system information. Results obtained with the real history bit-strings have no essential difference with the current study, though the use of random information makes the theoretical analysis easier.

†This process corresponds to the case of primary imitators. In fact, in the real system, there might exist multilevel imitations where some imitators can copy other imitators' behavior. Similar conclusions could be achieved.

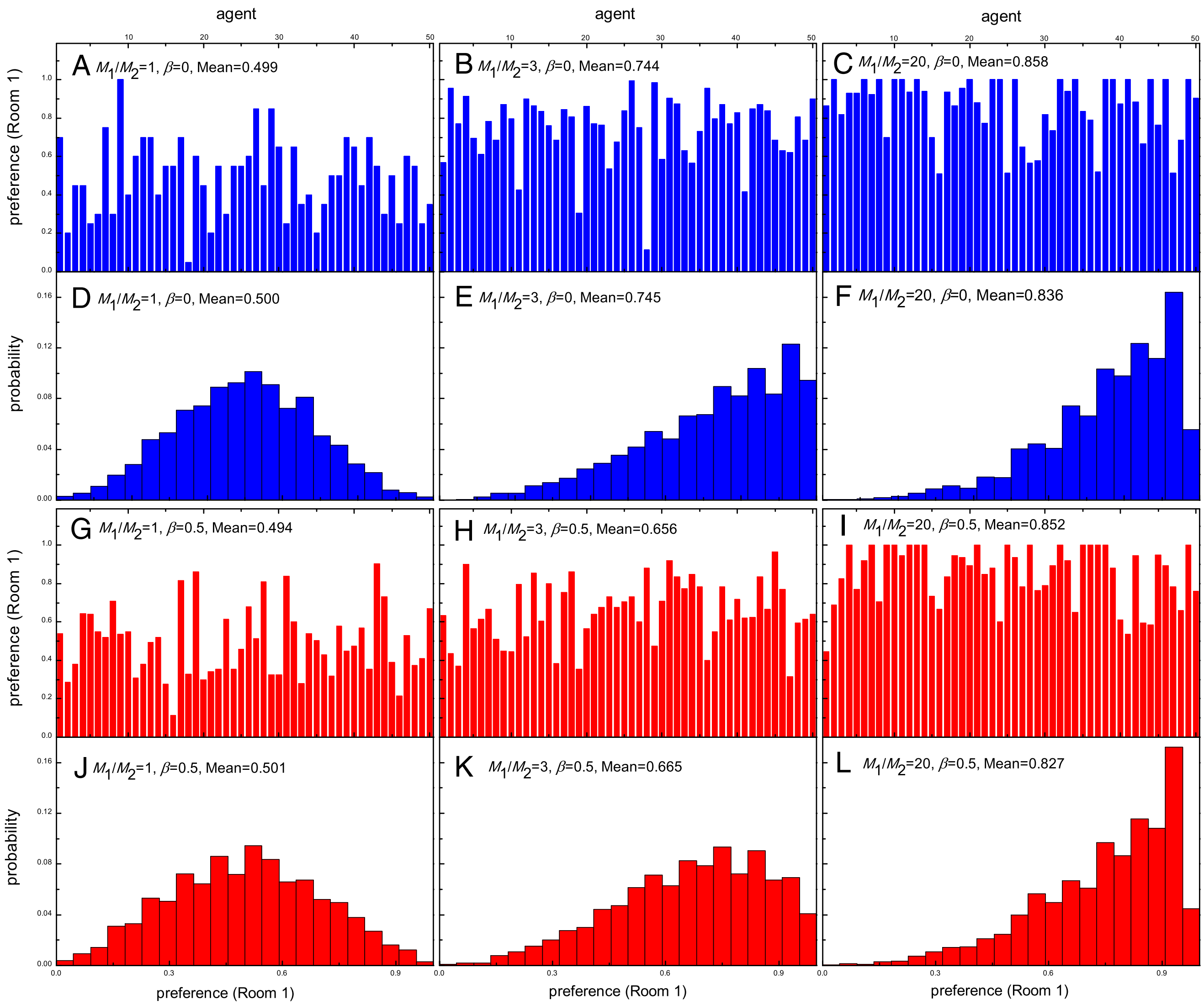

Fig. 3. Simulation results obtained from the agent-based simulations. (*A–C,G–I*) Preferences of the 50 normal agents to Room 1 for the cases (*A–C*) without ($\beta = 0$) and (*G–I*) with ($\beta = 0.5$) imitating agents, and for the resource distributions $M_1/M_2 =$ (*A,G*) 1, (*B,H*) 3, and (*C,I*) 20. We have run the simulations for 200 times, each over 400 time steps (first half for equilibration, the remaining half for statistics). (*A–C,G–I*) are typical results of one of the 200 runs. In (*A–C,G–I*), "Mean" denotes the mean value of the preferences of the 50 normal agents. (*D–F,J–L*) Distribution of the 50 normal agents' preferences. Note that (*D–F,J–L*) are obtained from the average over the 200 runs, and also the "Mean" in (*D–F,J–L*) denotes this average. Simulation parameters: $S = 4$, $P = 16$, and $N_n = 50$.

of normal agents in an alternative way by analyzing the Shannon information entropy; see *SI Text: Part IV*) Next, efficiency, stability, and predictability of the whole modeled system are calculated according to the definitions made in the experimental study. The change of system behavior along with the variation of the resource ratio M_1/M_2 is shown in Fig. 4. Differently colored symbols in the figure represent results obtained under different values of β. As shown in Fig. 4*A*, when the resource distribution is comparable ($M_1/M_2 \approx 1$), the averaged population ratio $\langle N_1\rangle/\langle N_2\rangle$ can always be in concert with M_1/M_2 no matter imitating agents are involved or not. On the other hand, as the resource distribution gets more and more biased (M_1/M_2 increases), surprisingly the whole system tends to reach the balanced state only if more imitating agents (larger β) join the system. Fig. 4*B* shows the change of efficiency of the resource-allocation system. The tendency is that when the resource ratio gets more biased, a larger size of herd is needed to realize a higher efficiency of the resource distribution. From both the subfigures, the so-called "M_1/M_2 phase transition" (4), where M_1/M_2 plays the role of control parameter, can also be identified. As shown in Fig. 4*C*, the increase of the number of imitating agents will cause larger fluctuations in the low M_1/M_2 region. However, as M_1/M_2 increases, more imitating agents can yield higher stability of the resource-allocation systems. Comparing system behaviors for the cases of $\beta = 0$ and $\beta \neq 0$, the M_1/M_2 phase transition also indicates the change of role for the herd behavior, namely, from a ruinous herd into a helpful herd. It is clear that the critical point of the M_1/M_2 phase transitions get larger when the number of imitating agents increases. Denoted as $(M_1/M_2)_c$ hereafter, the critical point refers to the M_1/M_2 value where the minimum of σ^2/N is achieved. This definition together with the mechanism for the increase of $(M_1/M_2)_c$ will be further discussed in the theoretical analysis of the model. Finally, the effect of herd behavior on the predictability of the resource-allocation system is shown in Fig. 4*D*. When more imitating agents are introduced to the system for large M_1/M_2, the prediction of the next winning room becomes more difficult as winning rates for the two rooms are more symmetric. Notice that the system behavior under various conditions found herein by the agent-based simulations echoes with the observations in the experiment.

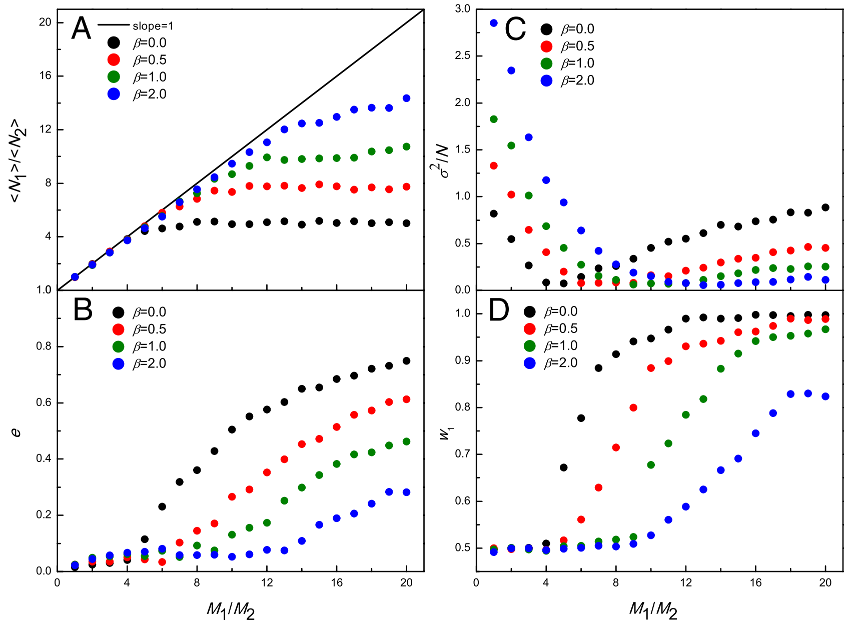

Fig. 4. (*A*) $\langle N_1 \rangle / \langle N_2 \rangle$, (*B*) *e*, (*C*) σ^2/N, and (*D*) w_1 as a function of M_1/M_2, for an open system in the agent-based simulations. Parameters: $N_n = 50$, $S = 4$, $P = 16$, $k = 5$, and $\beta = 0$, 0.5, 1.0, and 2.0. For each parameter set, simulations are run for 200 times, each over 400 time steps (first half for equilibration, the remaining half for statistics). In (*A*), "slope = 1" denotes the straight line with slope being 1.

We summarize the simulation results here and make some more comments to emphasize the significance of findings in our study. The performance of the resource-allocation system consisting of normal agents or human participants with the full decision making ability is, in some cases, inferior to those including imitating agents (who form the herd). This argument might seem questionable at first sight. In particular, it may be argued that the failure to reach the balanced resource allocation for large M_1/M_2 when $\beta = 0$ is only due to the relatively small population of the normal agents. However, it has been proved in the theoretical analysis [see the equation for the population in the next section or Eq. S6 in *SI Text: Part II*] and the agent-based simulation of resource-allocation systems (4) that the total number of agents is indeed not a key factor. When the resource distribution is not biased so much, the normal agents can play pretty well so that the resource-allocation system behaves in a healthy manner (efficient or balanced, stable and unpredictable). In such kind of situations, adding imitating agents will only bring about a "crowded system" in which larger fluctuations (volatility) turn up. In this respect, our study shares some common features with the Binary-Agent-Resource model (18, 19). In particular, the "crowd effect" has been observed in these models and the inclusion of imitating agents in our model can be explained as a special kind of networking effects. Only if the resource distribution becomes so biased that most of the normal agents cannot completely solve the decision making problem by referencing their strategy books, adding the imitating agents could become a helpful factor in consuming the remained arbitrage opportunities in the system. The discussion above explains the reason why the herd behavior in the resource-allocation system can effectively help the system to realize the balanced state and reduce instability and predictability in the mean time.

Theoretical Analysis of the Agent-Based Modeling

To further understand the underlying mechanism for these phenomena, we also conduct a theoretical analysis by deriving the critical points $(M_1/M_2)_c$ for the M_1/M_2 phase transition identified in the agent-based simulations. (For the details of derivation, please refer to *SI Text: Part II*.) As a result of the theoretical analysis, the maximum of population ratio in Room 1 $\langle R_1 (= N_1/N) \rangle_{\max}$ can be obtained under the condition $M_1 \geq M_2$. Its formula reads as the following (the meaning of the symbols can also be found in *SI Text: Part II*),

$$\langle R_1 \rangle_{\max} = 1 - \frac{1}{(\beta+1)P} \sum_{\tilde{L}=1}^{P} \left[\left(\frac{\tilde{L}}{P+1} \right)^{s} + \beta \left(\frac{\tilde{L}}{P+1} \right)^{ks} \right],$$

where $\tilde{L}$ stands for the preference of a normal agent's strategy. If $\langle R_1 \rangle_{\max}$ is not less than $M_1/(M_1 + M_2)$, the system can fluctuate around the balanced state. Otherwise, the system can never reach the balanced state. Then some insightful comments can be added:

- The state of the resource-allocation system depends only on M_1/M_2, β, k, P, and S. This state has no concern with N_n or N_m.
- An optimized value of β may be calculated by setting $\langle R_1 \rangle_{\max} = M_1/(M_1 + M_2)$, which could make the system most stable. After substituting this expression into the equation for $\langle R_1 \rangle_{\max}$, we can obtain numerical solutions for the critical

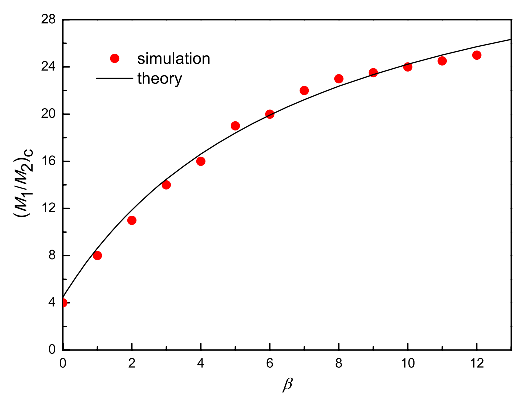

Fig. 5. Critical points of the M_1/M_2 phase transition, $(M_1/M_2)_c$, varying with different population ratios β: simulation results (symbols) vs. theoretical results (line). The simulation results are obtained from the data in Fig. 4 *A* and *C*.

points $(M_1/M_2)_c$ of the phase transitions. Fig. 5 shows a good agreement between the simulation results and those of theoretical derivation for the critical points.

- It is easy to prove that $\partial\langle R_1\rangle_{\max}/\partial\beta > 0$, which means that β and $\langle R_1\rangle_{\max}$ are positively related. When $\beta \to \infty$, the population ratio will converge to $\langle R_1\rangle_{\max} \to 1 - \frac{1}{P}\sum_{\tilde{L}=1}^{P}(\frac{\tilde{L}}{P+1})^{ks}$. At this limit, the model suggested here will be equivalent to the original resource-allocation model without the imitating agents (4), except that in this case, each agent would occupy kS (instead of S) strategies.

Discussion and Conclusions

We have revealed that, if the bias between the two resources M_1/M_2 were large and unknown to the participants, a herd of a typical size could help the overall system to reach the optimal state, namely, the state with a minimal fluctuation, a high efficiency, and a relatively low predictability. The corresponding ratio between the two resources also works as the critical point of a class of M_1/M_2 phase transition. The phase transition can be used to discover the role change of herd behavior, namely from a ruinous herd to a helpful herd as the resources distribution gets more and more biased. The main reason for this generalization could be understood as follows. When a large bias exists in the distribution of resource, the richer room will offer more arbitrage opportunities so that it deserves to be chosen without too much deliberation. Because imitating agents learn from the local best human participant or normal agent, the herd formed by these agents will certainly be more oriented to the richer room. To balance a highly biased resource distribution, in fact, it correspondingly needs a suitable number of participants who have a highly biased orientation in their choices. But every coin has two sides. Normal agents will be confused if too many imitating agents are involved. Because in that case, normal agents have to estimate not only the unknown system but also the behavior of the herd. The effect of herd behavior would become negative again under these situations. We emphasize that these arguments are quite general. In particular such arguments are independent of the process of herding. In *SI Text: Part V*, results of a different agent-based model, in which imitating agents follow the majority of the linked group, rather than the best normal agent, are shown. Similar results are achieved indeed.

This work is also expected to be important to some fields, ranging from management and social science, to ecology and evolution, and to physics. In management and social science, administrators should not only conduct risk management after the formation of herd, but also need to consider system environment and timing to see whether the herd is globally helpful or not. In ecology and evolution, it is not only necessary to study the mechanism of herd formation as usual, but also to pay more attention to the effect of herding on the whole ecological system and/or evolution groups. For physics, this work not only presents the existence of phase transition in such a complex adaptive system, but also proposes a new equilibrium theory. Namely, in the presence of symmetry breaking, a complex adaptive system is likely to reach the equilibrium state only through the performance of typically sized clusters.

About the Computer-Aided Human Experiment

All the experiments are carried out in an online manner. Human participants can get the necessary information only from their computer terminals. The desktop designs of the experiment-control computer program are shown in Fig. S1. The control panel for the experiment coordinator is configurated as panel (A), and that for human participants as panel (B). At the beginning of the experiment, the coordinator input the value of M_1/M_2 and β, and set the time length (60 s) for the human participants to make their decisions. When all the human participants have logged in, the coordinator can click the "start" button to start the experiment. After all the participants have made their choices, the coordinator clicks the "reset" button to end the current round and set anew. On panel (B), buttons with numbers of 1 and 2 are used to choose Room 1 and Room 2. The left of the panel displays the current score (a) of the participant and the current experiment round (t). To keep every participant conducting the experiment independently, procedures and rules of the experiment are designed carefully so that possible direct or indirect communications can be shut off. For example, participants can only make their own choices by clicking the button instead of raising their hands. This limitation could make sure that participants cannot get information from sounds, expressions, or gestures of the others. There is also no need for the experiment coordinator to announce the result of winning room. Participants can only deduce the winning room from the change of their scores on the desktop panels. In addition, no human participants had been kicked off (please refer to *SI Text: Part I*) during the experiments. For all the experiments with $M_1/M_2 = 1$, 3, and 20, the total number of human participants was kept to be 50. Among those, 44 human participants played through all the three experiment sessions. On the other hand, we had member-changes for the remained six participants.

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Supporting Information

Zhao et al. 10.1073/pnas.1105239108

SI Text

Part I: Leaflet to the Human Experiments. There are totally 50 participants doing the experiments together. The experiment situation is the same for everyone. Once the experiments begin, any kind of communication is not allowed.

Together with other participants, you shall engage in a resource-allocation experiment. For the experiment, there are two virtual rooms (Room 1 and Room 2), and the amounts of virtual money in the two rooms are M_1 and M_2, respectively. The value of M_1/M_2 is fixed in one experiment, but is not announced. In each round, you have to choose to enter one of the two rooms, in order to share alike the virtual money inside the room. After everyone has made decision, those who earned more than the global average are regarded as winners of the round, and the room which the winners had entered as the winning room.

After you log in, you will see the choosing panel on the computer screen (as shown in Fig. S1*B*), buttons with numbers of 1 and 2 are used to choose Room 1 and Room 2. The left displays your current score (*a*) and the current experiment round (*t*). During the experiment, 60 s were given for making choice. If you could not decide your choice within 60 s, the experiment-control computer program would assign you a random choice with probability 50%. Nevertheless, the participant who borrowed the computer's choice twice would be automatically kicked out of the experiment. In each round of the experiment, the experiment-control computer program will update the score for each participant after all the participants have made their choices. If your score is added 1 point, it means that the room you have chosen happened to be the winning room. If the score keeps unchanged, it may have two possible interpretations: either the other room won or neither of the rooms won (i.e., the experiment ended in a draw).

The initial capital of each participant is 0 point and the total payoff of a participant is the accumulated scores (points) of all the experiment rounds. At the end of the experiments, as a premium, this payoff (points) will be converted to the monetary payoff in Renminbi with a fixed exchange rate 1 : 1 (namely, one point equals to one Chinese Yuan). Try to win more points, and then you can get more premium.

Part II: The Open Complex Adaptive System (CAS)—Theoretical Analysis of the Agent-Based Modeling. Besides the simulations performed in the main text here we present some theoretical analysis for the same open system. It is reasonable to assume that, if P is not too small, the right column of a strategy filled in by 1 with probability L/P is equal to the one filled in 1 with the number of L. Hence strategies with the same preference number L can be regarded as the same. It is worth noting that if the situations vary in a random manner, the probability is L/P for a normal agent to choose Room 1 using a strategy with preference number L. Next, we assume that the preference number of the best strategy held by normal agent i at time T, is L_i. Denote the choice of room as x_i so that $x_i = 1$ if Room 1 is chosen and $x_i = 0$ otherwise. At the same time, let imitating agent j choose to follow the normal agent μ, the best agent (who has the highest score) in the group of size k ($1 \le k \le N_n$). For the imitating agent, its choice of room is $y_j = x_\mu$, and its preference number becomes $L_j = L_\mu$. With these definitions, the total number of agents in Room 1 at time T can be written as

$$N_1 = \sum_{i=1}^{N_n} x_i + \sum_{j=1}^{N_m} y_j. \quad \text{[S1]}$$

It is obvious that $\langle x_i \rangle = L_i/P$, which can be used to derive the expectation and the variance of the population in Room 1 as

$$\langle N_1 \rangle = \frac{1}{P}\left(\sum_{i=1}^{N_n} L_i + \sum_{j=1}^{N_m} L_j\right), \quad \text{[S2]}$$

$$\sigma_{N_1}^2 = \sum_{i=1}^{N_n} \sigma_{x_i}^2 + \sum_{j=1}^{N_m} \sigma_{y_j}^2 + \sum_{i=1}^{N_n}\sum_{j=1}^{N_m}(\langle x_i y_j \rangle - \langle x_i \rangle \langle y_j \rangle) + \sum_{p,q=1,p\neq q}^{N_m}(\langle y_p y_q \rangle - \langle y_p \rangle \langle y_q \rangle). \quad \text{[S3]}$$

Owing to the specific method for the construction of strategies in the resource-allocation model, the covariance between the choices of different normal agents can be neglected. On the right-hand side of Eq. **S3**, the third term is the correlation between choices of the normal agents and those of the imitating agents who followed them. The fourth item is the correlation between the choices of different imitating agents who followed the same normal agent. Both terms should always be positive, which means that adding the imitating agents could cause large fluctuations (volatility) in the resource-allocation system. It should be emphasized here that the stability defined in the main text is different from the traditional definition of variance. The former characterizes both the deviation and the fluctuation to the idealized room population in the balanced state, while the latter only represents the fluctuation to the mean value of the time series. When the resource distribution is comparable ($M_1/M_2 \approx 1$), because normal agents are able to produce the idealized population or $\langle N_1 \rangle / \langle N_2 \rangle \approx M_1/M_2$, these two kinds of definitions are approximately equal. This condition explains why the stability can be destroyed when imitating agents are involved in situations with a nearly unbiased resource distribution. However, when the system environment becomes difficult for the normal agents to adapt to, the difference between the "variance" and the "stability" cannot be neglected. If no imitating agents are involved, the normal agents alone cannot make the system reach the balanced state. In that case, even if the fluctuation of N_1/N_2 to its average value could be made small, the deviation to the idealized population ratio can still be very large. This situation would make the system suffer from a higher dissipation. If an appropriate portion of imitating agents is added, the deviation of N_1/N_2 to the idealized room population diminishes, leaving only some fluctuations around M_1/M_2, which could result in a reduction of waste in the resource allocation.

Then, we study the performance of different strategies (namely, strategies with different preference numbers). We also assume the condition of $M_1/M_2 \ge 1$, as used in the main text. Assume that at time T, the winning rate of Room 1 is $\alpha(T)$. The expectation of the increment of score for the strategy with the preference number L should be $1 - \frac{L}{P} + (\frac{2L}{P} - 1)\alpha(T)$. Then the expectation of the cumulative score for this strategy from $t = 1$ to $t = T$ can be expressed as

$$f(L,T) = \left(1 - \frac{L}{P}\right)T + \left(\frac{2L}{P} - 1\right)\sum_{t=1}^{T}\alpha(t). \qquad \textbf{[S4]}$$

From this expression, we can calculate the dependence of the cumulative score on the preference number as

$$\frac{\Delta f}{\Delta L} = \frac{2}{P}\sum_{t=1}^{T}[\alpha(t) - 0.5]. \qquad \textbf{[S5]}$$

It is easy to find from Eq. **S3** that if $\sum_{t=1}^{T}[\alpha(t) - 0.5] > 0, f$ should be a monotonically increasing function with L. Now we assume that $[\alpha(T) - 0.5]$ is always positive, which is not a too stringent condition as long as M_1 is large enough. As the experiment evolves under this assumption, the gap of scores among different strategies of different preference numbers will become larger and larger. Eventually, the best performed strategy owned by a normal agent would be the one with the largest L in its strategy book. As a consequence, imitating agents will choose to follow those who own the strategy with the largest preference number $L_{\max}$. From Eq. **S2**, it is obvious that $\langle N_1 \rangle$ will also reach its maximum value $\langle N_1 \rangle_{\max}$, when both L_i and L_j reach their maximum values. With this maximum value of the expected population in Room 1, we can propose the following two conditions:

- If $\langle N_1 \rangle_{\max} < \frac{M_1}{M_1+M_2}N$, the system can never reach the balanced state.
- If $\langle N_1 \rangle_{\max} > \frac{M_1}{M_1+M_2}N$, the system can fluctuate around the balanced state.

Denoting the population ratio $\langle R_1 \rangle = \langle N_1 \rangle / N$, we need to calculate $\langle R_1 \rangle_{\max} = \langle N_1 \rangle_{\max} / N$, to evaluate the conditions above. As the normal agents construct their strategies in a random way, a strategy with an arbitrary preference number may be picked up with a uniform probability $1/(P+1)$. Thus, among the S strategies of a normal agent, the probability to have $L_{\max} = \tilde{L}$ is $p(\tilde{L}) = (\frac{\tilde{L}+1}{P+1})^S - (\frac{\tilde{L}}{P+1})^S$. Because an imitating agent would choose the best normal agent among the k group members, the probability to have $(L_{\max})_{ks} = \tilde{L}$ should be $p'(\tilde{L}) = (\frac{\tilde{L}+1}{P+1})^{kS} - (\frac{\tilde{L}}{P+1})^{kS}$. With these probabilities, we obtain the population ratio as

$$\begin{aligned}\langle R_1 \rangle &= \frac{1}{NP}\left(\sum_{i=1}^{N_n} L_i + \sum_{j=1}^{N_m} L_j\right) \\ &= \frac{1}{NP}\left[N_n \sum_{\tilde{L}=1}^{P} \tilde{L}\, p(\tilde{L}) + N_m \sum_{\tilde{L}=1}^{P} \tilde{L}\, p'(\tilde{L})\right] \\ &= 1 - \frac{1}{(\beta+1)P}\sum_{\tilde{L}=1}^{P}\left[\left(\frac{\tilde{L}}{P+1}\right)^{s} + \beta\left(\frac{\tilde{L}}{P+1}\right)^{ks}\right]. \qquad \textbf{[S6]}\end{aligned}$$

Part III: A Closed CAS—Simulations Based on the Agent-Based Modeling. For the open system discussed in the main text, if there are too many imitating agents in the resource-allocation system, it may still become a disturbing factor to the system. For the completeness of the study, here we consider a closed system in which the number of normal and imitating agents is fixed at $N = 150$ with the parameter β being varied. As shown in Fig. S2, in the larger M_1/M_2 region, situations with the imitating agents ($\beta = 2.0$ and 4.0) are generally better than those without the imitating agents (β =0), similar to cases of the open system. Meanwhile, there clearly exists an optimized β (=4.0 in the current case) with which the best state of the closed system can be realized in the aspects of the efficiency (which, herein, only describes the degree of balance of resource allocation in the model system) and the stability. When $\beta = 9.0$, the system seems to be disturbed by the imitating agents and the performance (except the system unpredictability) becomes even worse than the case of $\beta = 2.0$. The reason for this phenomenon may be explained as follows. If too many imitating agents join the system, even the best normal agents may be confused. Typically the best normal agents might have wrong estimations about the system situation and then make incorrect decisions. When the best normal agents' decisions are learnt by the imitating agents, the herd will overconsume the arbitraging opportunities in the system as a result of the distribution of biased resources, thus yielding a less efficient (or equivalently less balanced) and less stable but still unpredictable state.

Part IV: An Alternative Approach to Analyzing Preferences of Normal Agents and Imitating Agents in the Agent-Based Modeling: Analysis of the Shannon Information Entropy. In order to study the agents' preferences and their estimation of the system, the Shannon information entropy (S1) may be introduced to our agent-based modeling. The information entropy S_I of a discrete random variable X with possible values $\{x_1, \ldots x_n\}$ is defined as $S_I(X) = -\sum_{i=1}^{n} P(x_i)\ln P(x_i)$, in which $P(x_i)$ denotes the probability mass function of x_i. In the agent-based model, the information entropy for a normal agent is $S_{Ii} = -\frac{L_i}{P}\ln\frac{L_i}{P} - \frac{P-L_i}{p}\ln\frac{P-L_i}{P}$, where L_i stands for the preference of the current strategy. If the normal agent would choose two rooms with an equal probability, this information entropy could reach the maximum value of $\ln 2$. On the other hand, the information entropy S_{Ij} for imitating agent j will be the same as that of the normal agent he/she follows in the local group. Thus the averaged information entropy of all the agents (i.e., normal agents and imitating agents) can be calculated as

$$S_I = \frac{1}{N}\left(\sum_{i=1}^{N_n} S_{Ii} + \sum_{j=1}^{N_m} S_{Ij}\right), \qquad \textbf{[S7]}$$

and the results are shown in Fig. S3*A*. As the averaged information entropy decreases as M_1/M_2 becomes larger, a clear-cut average preference of agents emerges as the distribution of resources gets more biased. This observation agrees with the analysis of participants' preferences in the human experiments; see Fig. 1 in the main text. Furthermore, the information content of agent i can be defined as $I_i = (\ln 2 - S_{Ii})/\ln 2$. Note that a larger I_i indicates that the agent has more confidence in a certain room. The averaged information content for all the normal agents (I_n) and imitating agents (I_m) are shown in Fig. S3*B*. In this figure, I_n decreases with the increase of the population of imitating agents when M_1/M_2 is small. This observation means that normal agents can be confused by the actions of imitating agents in a rather uniform distribution of the resource. When M_1/M_2 gets larger, I_n is nearly a constant implying that imitating agents will no longer affect the estimation of the normal agents. All of these arguments go well with the analysis of the experimental results in Fig. 1. The averaged information content of imitating agents has a rather drastic change as the environment varies. When $M_1/M_2 = 1$, I_m is pretty low, even lower than that of the normal agents, a fact indicating that imitating agents have almost unbiased preferences when the resource distribution is uniform. As M_1/M_2 increases, imitating agents are apt to flood into a specific room and thus form the herd in the modeled system.

Part V: A Different Agent-Based Modeling in Which Imitating Agents Follow the Majority, Rather than the Best Agent: an Open CAS vs. a Closed One. To make our work more general, a different modeling of the formation of herd is studied. Following the most successful person is often seen in daily life, and there is another common case following the majority. For example, people often decide on which store or restaurant to patronize on the basis of how popular they seem to be. In this sense, we make some changes to our agent-based modeling adopted in the main text. Models' main structures are similar, but the difference between them is that the "imitating agent" now follows the majority decision of his/her group (namely, the current imitating agents act as local majority-followers). This way of imitation means if more than half of the group members choose Room 1, he/she will enter Room 1 accordingly.

As shown in Fig. S4 and Fig. S5, the system behaviors are similar as those of "imitating the best." The main difference between Fig. 4 and Fig. S4 lies in Fig. S4*C*. When M_1/M_2 is small, adding imitating agents (local majority-followers) will cause relatively larger fluctuations. From Fig. S5 we can find that in the closed system of "following the majority," there also exists a proper β for one certain system environment M_1/M_2. Compared with Fig. S2, Fig. S5 shows that following the majority works not that well as following the best and this is easy to understand. The best normal agent owns the best strategy and is usually much more sensitive than common ones, while the majority reflects the average level of all the normal agents in the group. Although quantitatively different, the main conclusions arising from the two kinds of "following" are similar. Thus, the mechanism on how to form the herd is not an essential problem, and both imitating the best and following the majority can lead to similar conclusions.

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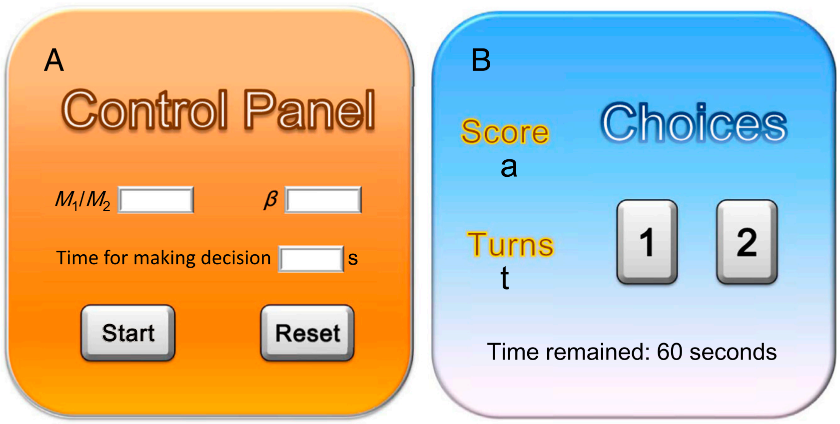

Fig. S1. The desktops of the experiment-control computer program used in the computer-aided human experiments: The control panel (*A*) for the coordinator and (*B*) for the human participants.

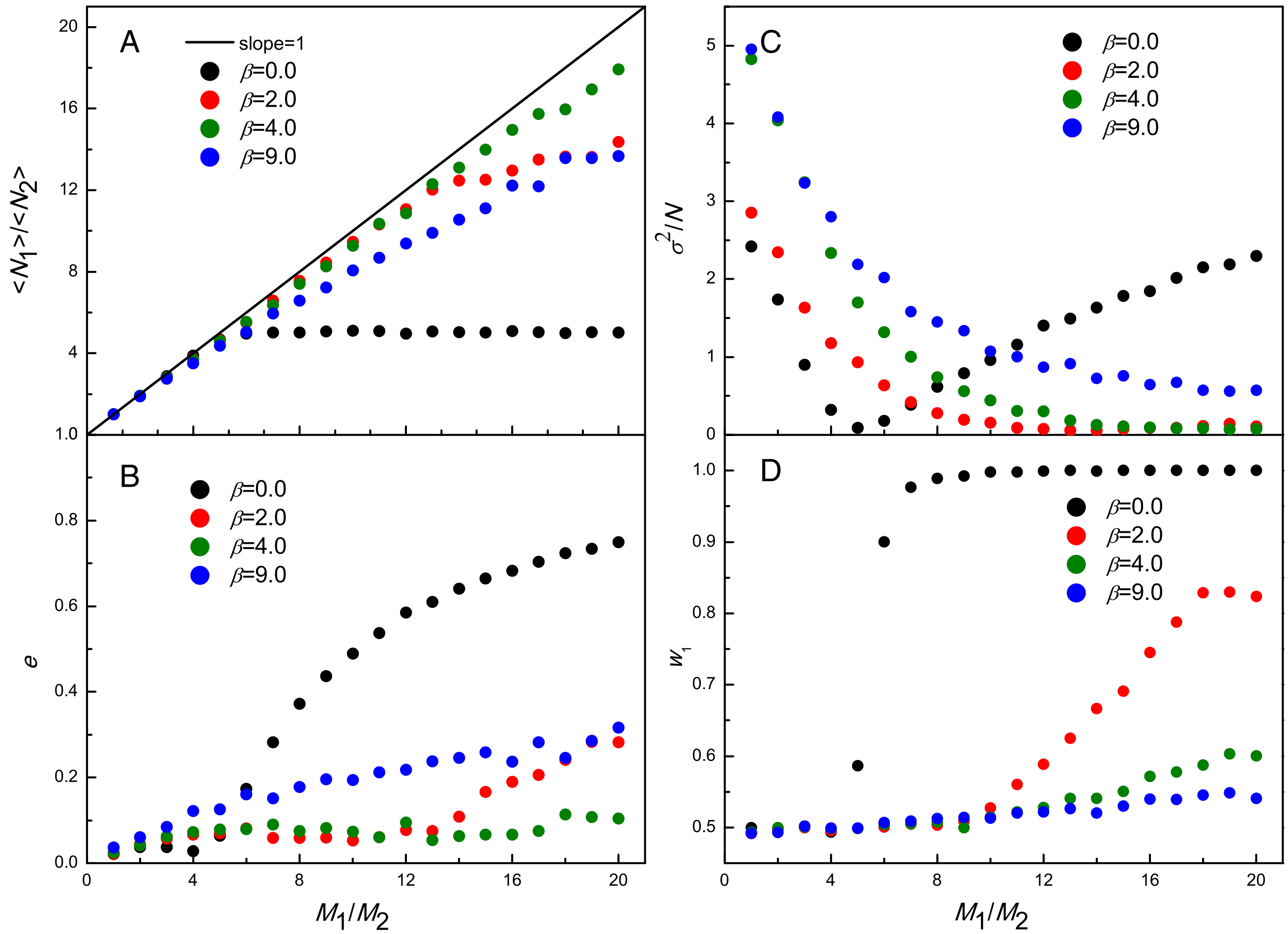

Fig. S2. (*A*) $\langle N_1 \rangle / \langle N_2 \rangle$, (*B*)*e*, (*C*) σ^2/N, and (*D*) w_1 as a function of M_1/M_2, for a closed system. Parameters: $N = 150$, $S = 4$, $P = 16$, $k = 5$, and $\beta = 0,2.0,4.0$, and 9.0. Simulations are run for 200 times, each over 400 time steps (first half for equilibration, the remaining half for statistics). In (*A*), "slope = 1" denotes the straight line with slope being 1.

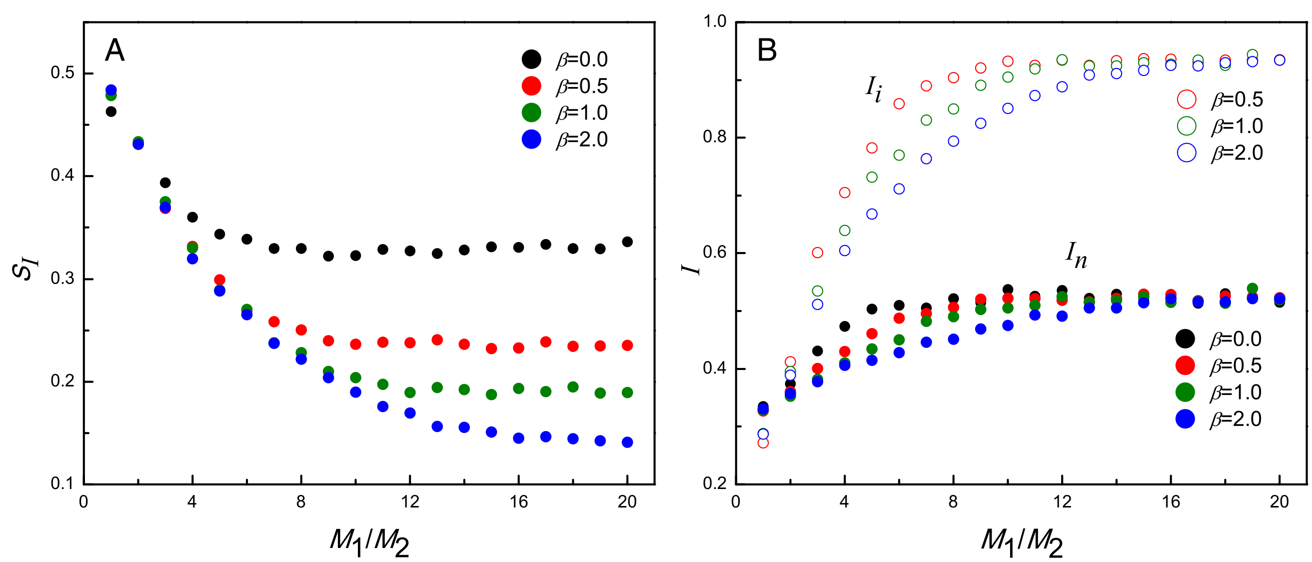

Fig. S3. The change of (*A*) the averaged information entropy (S_I) for all the agents including the normal agents and imitating agents and (*B*) the averaged information content for the normal agents (I_n) and imitating agents (I_m), respectively. Simulations are run for 200 times, each over 400 time steps (first half for equilibration, the remaining half for statistics). Parameters: $N_n = 50$, $S = 4$, $P = 16$, and $k = 5$.

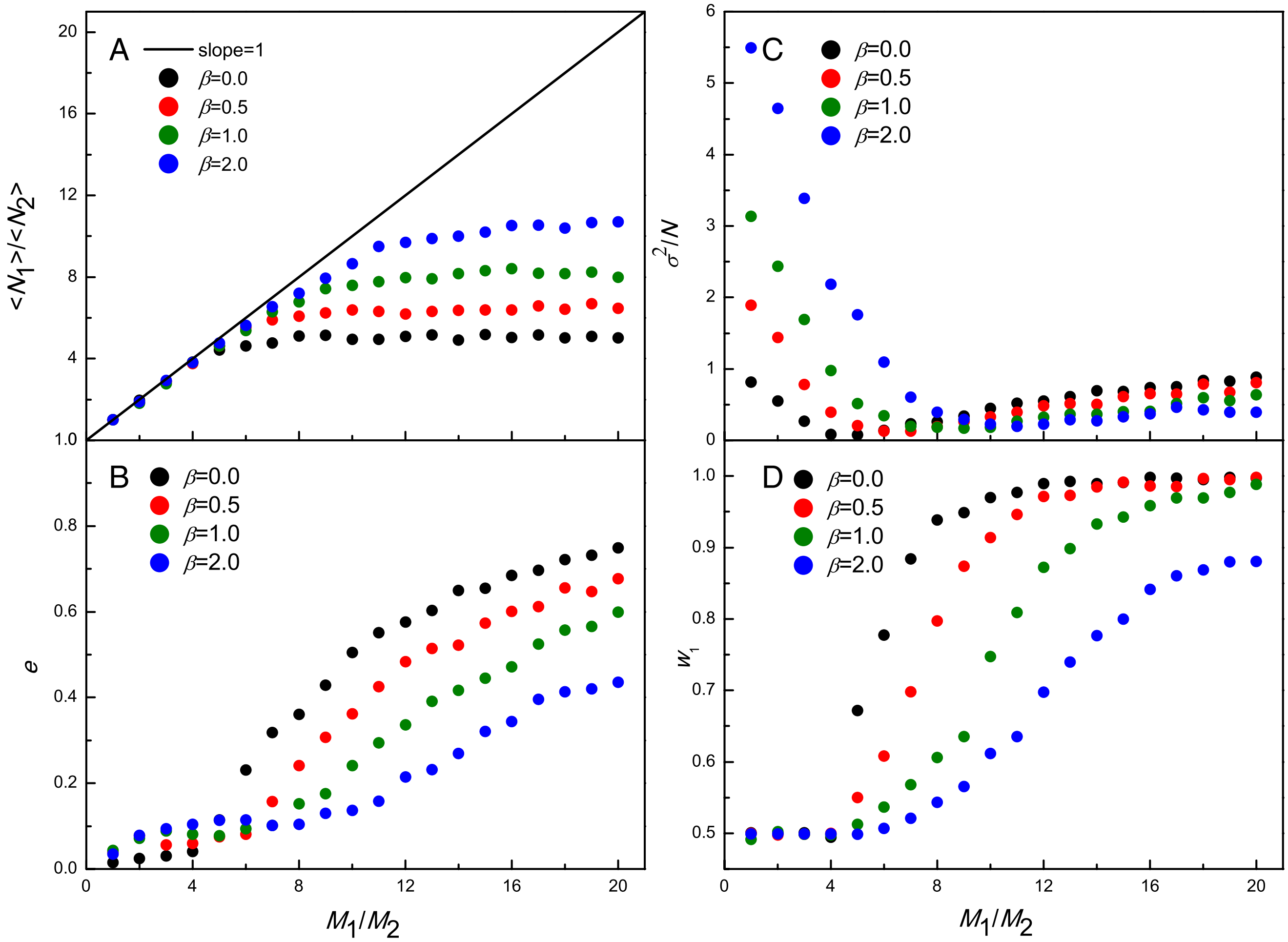

Fig. S4. (*A*) $\langle N_1\rangle/\langle N_2\rangle$, (*B*)e, (*C*) σ^2/N, and (*D*) w_1 as a function of M_1/M_2, for an open system. Parameters: $N_n = 50$, $P = 16$, $S = 4$, $k = 5$, and $\beta = 0, 0.5, 1.0$, and 2.0. The imitating agents follow the majority of their local groups. Simulations are run for 200 times, each over 400 time steps (first half for equilibration, the remaining half for statistics). In (*A*), "slope = 1" denotes the straight line with slope being 1.

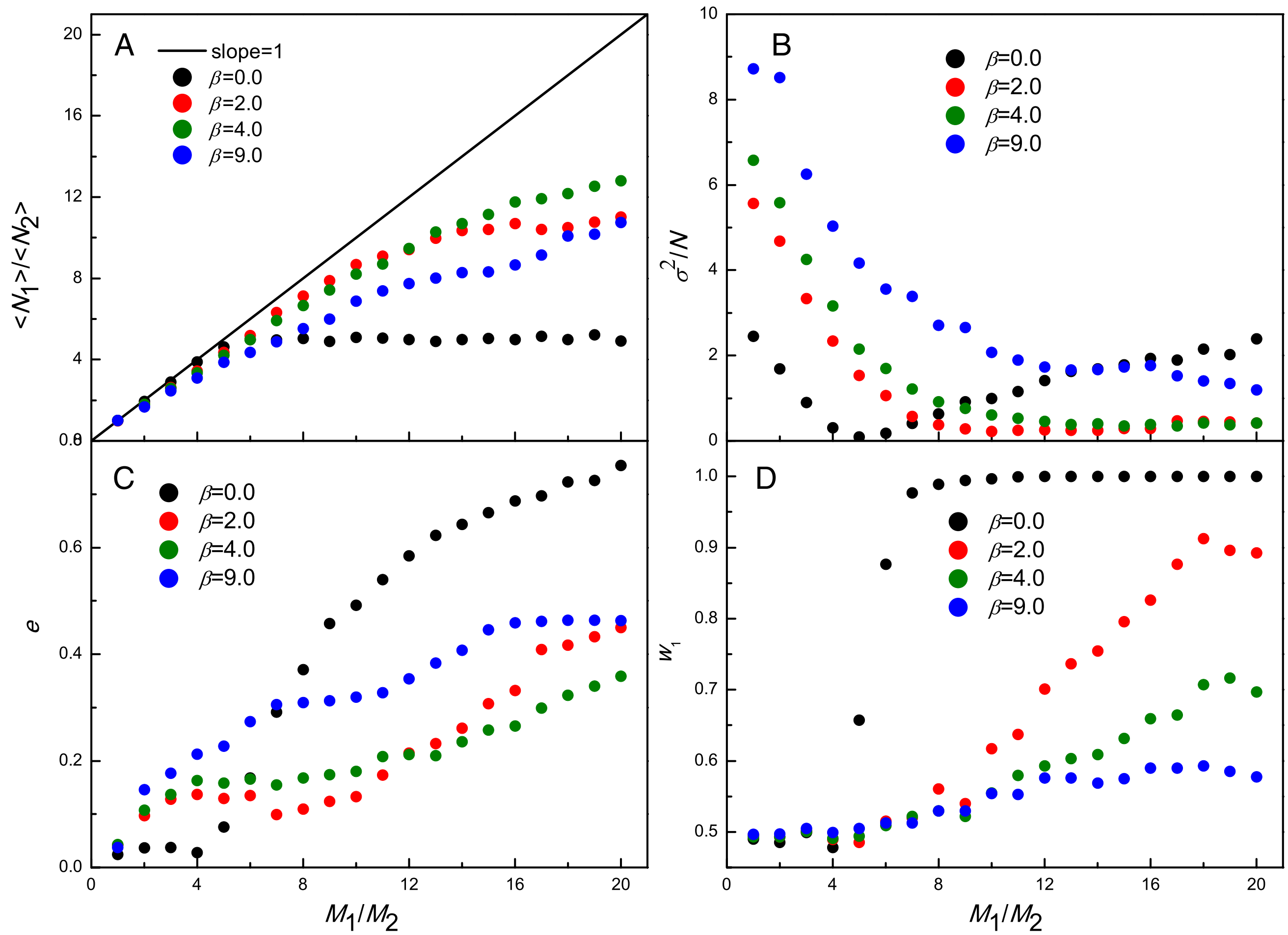

Fig. S5. (*A*) $\langle N_1\rangle/\langle N_2\rangle$, (*B*) e, (*C*) σ^2/N, and (*D*) w_1 as a function of M_1/M_2, for a closed system. Parameters: $N = 150$, $P = 16$, $S = 4$, $k = 5$, and $\beta = 0$,2.0,4.0, and 9.0. Others are the same as those information in Fig. S4.