

Carbon-dioxide emissions trading and hierarchical structure in worldwide finance and commodities markets

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In a highly interdependent economic world, the nature of relationships between financial entities is becoming an increasingly important area of study. Recently, many studies have shown the usefulness of minimal spanning trees (MST) in extracting interactions between financial entities. Here, we propose a modified MST network whose metric distance is defined in terms of cross-correlation coefficient absolute values, enabling the connections between anticorrelated entities to manifest properly. We investigate 69 daily time series, comprising three types of financial assets: 28 stock market indicators, 21 currency futures, and 20 commodity futures. We show that though the resulting MST network evolves over time, the financial assets of similar type tend to have connections which are stable over time. In addition, we find a characteristic time lag between the volatility time series of the stock market indicators and those of the EU CO₂ emission allowance (EUA) and crude oil futures (WTI). This time lag is given by the peak of the cross-correlation function of the volatility time series EUA (or WTI) with that of the stock market indicators, and is markedly different (>20 days) from 0, showing that the volatility of stock market indicators today can predict the volatility of EU emissions allowances and of crude oil in the near future.

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I. INTRODUCTION AND METHOD

In the study of complex systems, there has been much work demonstrating the usefulness of extracting the underlying structure from the correlations found in statistical data [1–14]. Any means of selecting statistically reliable information from correlation matrices has been dubbed a “filtering procedure” [8]. Useful examples of filtering procedures that use a correlation matrix from return time series are hierarchical clustering [1–7], procedures based on the random matrix theory [10–14], and networks from minimum spanning trees [1–5]. Correlation structure studies are not limited only to stock return time series [1–3] but also extend to quasisynchronously recorded time series of worldwide stock exchange market indices [2] and stock return volatility increments [7].

Financial time series can include not only stock price but also many other types of data, such as commodities price, treasury yield, market index, and so on. Investigation of multitype quasisynchronous financial data may yield insights into the interdependent relationships of markets and commodities. Moreover, a relationship map of financial assets can highlight the movement of speculative capital.

We investigate 69 daily financial series from the time period spanning January 2007 to September 2011. The data set includes 21 currency futures, 20 commodity futures which are taken from Ref. [15], and 28 stock market indicators which are taken from Ref. [16] (see the Appendix).

Recently, papers have shown the usefulness of the correlation structure described by an ultrametric space and a corresponding hierarchical organization for financial return time series [1–3]. The approach requires the definition of a metric distance. Because correlation does not fulfill the three [17] axioms that define a metric, the aforementioned papers use the Mantegna-Sornette distance defined by

$$d_{ij} = \sqrt{2(1 - \rho_{ij})} \quad (1)$$

for each pair of elements i and j , where ρ_{ij} is the correlation coefficient of the two time series given by [18]

$$\begin{aligned} \rho_{ij} &\equiv \frac{\langle Y_i Y_j \rangle - \langle Y_i \rangle \langle Y_j \rangle}{\sqrt{(\langle Y_i^2 \rangle - \langle Y_i \rangle^2)(\langle Y_j^2 \rangle - \langle Y_j \rangle^2)}} \\ &= \frac{\langle (Y_i - \langle Y_i \rangle)(Y_j - \langle Y_j \rangle) \rangle}{\sigma_{Y_i} \sigma_{Y_j}}, \end{aligned} \quad (2)$$

where $\langle \dots \rangle$ denotes the mean.

This distance d_{ij} fulfills the three axioms [17] of a metric: (i) $d_{ij} = 0$ if and only if $i = j$, (ii) $d_{ij} = d_{ji}$, and (iii) $d_{ij} \leq d_{ik} + d_{kj}$ [1–3].

In this work, we make a modification to the metric above, based on the reasoning that if two time series have a very large *negative* correlation, they should still be considered close to each other in ultrametric “correlation” space, since this would indicate a strong connection, regardless of the sign. Likewise, weak correlations of either sign indicate a weaker connection. On this basis, strong correlations of either sign should be considered closer than weak ones.

The value of this modification can be readily seen in situations where two entities have strong *anti* correlations, as has been observed in the relationship between bond and stock markets (e.g., between UK gilts and the FTSEMIB), between stocks and currency futures, and between industries and their inputs (e.g., the price of oil and the value of airline stock) [19–22]. Here, use of the conventional Mantegna-Sornette metric would likely result in any of these two entities manifesting on opposite sides of a tree, even though we know them to be very closely linked. Use of our generalized metric ensures that such entities will be placed nearby when such appropriately strong relations exist.

For this reason, we replace the Pearson correlation coefficient in Eq. (1) with the absolute value of the Pearson

correlation coefficient, defining a new distance as

$$d_{ij} = \sqrt{2(1 - |\rho_{ij}|)} \quad (3)$$

with the ρ_{ij} defined as before in Eq. (2) as the correlation coefficient of assets i and j .

This equation also fulfills three axioms of a metric distance for $\rho \in (-1, 1]$. Accordingly, because the set of assets considered has no cases such that $\rho_{ij} = -1$, the first axiom is satisfied on this set in that $d_{ij} = 0$ if and only if the correlation is total ($\rho = 1$, meaning that the stocks perform the same stochastic process). The second axiom (that of symmetry) is trivially satisfied because $\rho_{ij} = \rho_{ji}$ by definition of the Pearson correlation coefficient. For the validity of axiom (iii), consider three time series Y_i , Y_j , and Y_k , which have means equal to 0 and standard deviations equal to 1. All times series have the same length. In order to prove $d_{ij} \leq d_{ik} + d_{kj}$, first, we define two new time series as

$$Y'_i \equiv \begin{cases} Y_i, & \text{if } \rho_{Y_i, Y_k} \geq 0 \\ -Y_i, & \text{if } \rho_{Y_i, Y_k} < 0 \end{cases}, \quad (4)$$

$$Y'_j \equiv \begin{cases} Y_j, & \text{if } \rho_{Y_j, Y_k} \geq 0 \\ -Y_j, & \text{if } \rho_{Y_j, Y_k} < 0 \end{cases}. \quad (5)$$

So we can rewrite $d_{ik} + d_{kj}$ of distance Eq. (3) by using Y'_i and Y'_j as

$$d_{ik} + d_{kj} = \sqrt{2(1 - |\rho_{Y_i, Y_k}|)} + \sqrt{2(1 - |\rho_{Y_j, Y_k}|)} \quad (6)$$

$$= \sqrt{2(1 - \rho_{Y'_i, Y_k})} + \sqrt{2(1 - \rho_{Y'_j, Y_k})} \quad (7)$$

$$\geq \sqrt{2(1 - \rho_{Y'_i, Y'_j})} \quad (8)$$

$$\begin{cases} = \sqrt{2(1 - |\rho_{Y'_i, Y'_j}|)}, & \text{if } \rho_{Y'_i, Y'_j} \geq 0 \\ \geq \sqrt{2(1 - |\rho_{Y'_i, Y'_j}|)}, & \text{if } \rho_{Y'_i, Y'_j} < 0 \end{cases} \quad (9)$$

$$= \sqrt{2(1 - |\rho_{Y_i, Y_j}|)} \equiv d_{ij}. \quad (10)$$

Thus, $d_{ik} + d_{kj} \geq d_{ij}$, and our metric satisfies the three axioms of a metric.

For each of the 22 financial time series, we calculate the return time series, defined as the change of logarithmic price of time series $Y_i(t)$,

$$R_i(t) \equiv \ln[Y_i(t+1)] - \ln(Y_i(t)). \quad (11)$$

Here $Y_i(t)$ is the daily price time series of financial asset i . For each of the 22 time series, we also calculate the volatility time series which is defined simply as the absolute value of the return $|R_i|$

$$V_i(t) \equiv |R_i(t)| = |\ln[Y_i(t+1)] - \ln(Y_i(t))|. \quad (12)$$

Additionally, we define the cross-correlation for our analysis. Consider two time series $\{y_t\}$ and $\{y'_t\}$. The cross-correlation between $\{y_t\}$ and $\{y'_t\}$ is given by

$$C_{y, y'}(n) \equiv \overline{(y_t - \mu)(y'_{t+n} - \mu')}/(\sigma\sigma'), \quad (13)$$

where μ and μ' are the respective means and σ and σ' are the respective standard deviations of the series $\{y_t\}$ and $\{y'_t\}$.

The efficient market hypothesis, a basic tenet of modern economics, states that markets are approximately efficient, meaning that one cannot consistently achieve returns better

than the market because all information about an asset is already incorporated into that asset's price [9,23]. As a result, it is believed that the long-range memory cannot exist in any return time series. Suppose that long-range autocorrelations exist in a return time series: investors may then obtain benefits by using information, which stands in contradiction to the principle of an efficient market. Consider the cross-correlation function [Eq. (13)] between return time series of asset i and asset j . Any significant cross-correlations $C_{R_i, R_j}(n)$ in $n \neq 0$ of two return time series would also contradict the existence of an efficient market. Therefore, we can assume that significant cross-correlations $C_{R_i, R_j}(n)$ will exist only for the case $n = 0$.

However, because trading occurs at different times in different cities, some markets are open when others are closed. The effect of nonsynchronous trading in time-series analysis has been well stated [24,25]. In fact, the highest degree of correlation between different markets may be detected at a 1-day time lag because of the time difference.

Therefore, significant cross-correlations $C_{R_i, R_j}(n)$ may also exist for $n = -1$ or $n = 1$. Additionally, we only care about the magnitude and not the sign of cross-correlation. Thus, we define the absolute correlation coefficient as

$$\rho_{ij} = \max(|C_{R_i, R_j}(n)|) \quad (14)$$

for $n = -1, 0, 1$.

In volatility time series, long-range correlations $C_{V_i, V_j}(n)$ have been shown to exist [10–13]. It follows that significant cross-correlations $C_{V_i, V_j}(n)$ may exist for $n \gg 0$ or $n \ll 0$. Additionally, the existence of long-range negative correlations between past returns and future volatility [26,27], known as the leverage effect, has also been reported. This correlation is moderate and decays exponentially over the long term. However, while both of these types of correlations may help

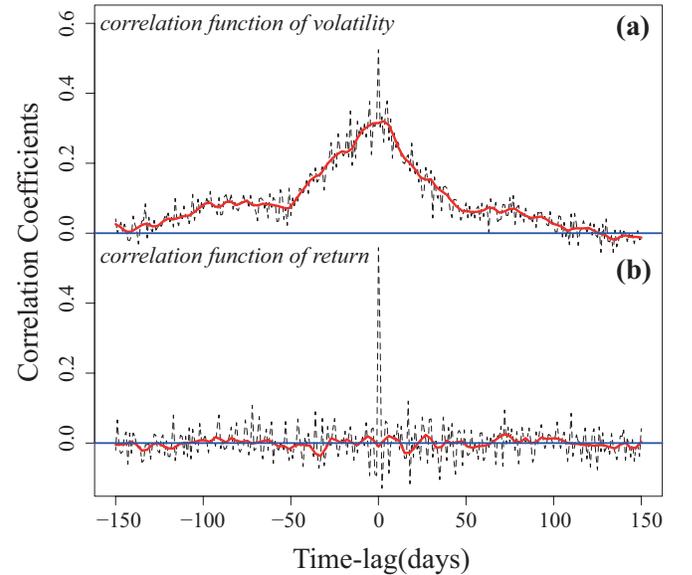


FIG. 1. (Color online) The cross-correlation function $C(n)$ of volatility (a) and return (b) time series between the DJIA and FTSE100. Both show statistically significant correlation coefficients at their maxima near time lag = 0 (dotted curve). Solid lines show the LOWESS (locally weighted scatter plot smoothing) values of $C(n)$, smoothed over a span of 30 days.

predict the financial risks on a long-range time interval, we point out that neither the negative correlation between returns and volatilities nor long-range autocorrelation of volatility can be used to obtain benefits. This is because the price volatility does not include the direction of price changes, and so neither contradicts the efficient market hypothesis.

As a sample, the correlation functions of volatility $C_{V_i, V_j}(n)$ and return $C_{R_i, R_j}(n)$ between the FTSE100 and DJIA are shown in Fig. 1. The peaks (highest correlations) of both correlation functions are near $n = 0$; however, the correlation function for returns is fast decaying, quickly approaching 0 for $n \neq 0$, while the volatility correlation function is slow decaying with $C_{V_i, V_j}(n) > 0$ for $n > -50$ and $n < 50$.

We now turn our attention to the stability and structure of minimal spanning trees (MSTs), which are made using the distance defined in Eq. (3). Following our discussion on MSTs,

we will show the correlation function graphs of volatility that relate the correlation $C(n)$ to the time-lag n , specifically focusing on the value of n that gives the LOWESS (the smoother which uses locally weighted polynomial regression) equal to its maximum value. Here, the LOWESS is defined by a complex algorithm, proposed by W. S. Cleveland [28] in 1981. For each value, we define 10th nearest neighbor values as the local region, which is used to calculate the LOWESS value.

An MST is defined as the set of $n - 1$ links that connect a set of n elements in the smallest possible total distance [29]. MSTs have been used in prior papers [1–5] to connect financial data, illustrating the MST’s usefulness in highlighting the interactions between a number of financial time series. In Fig. 2, we find that, although MSTs show significantly different structures in different calendar years, the same type

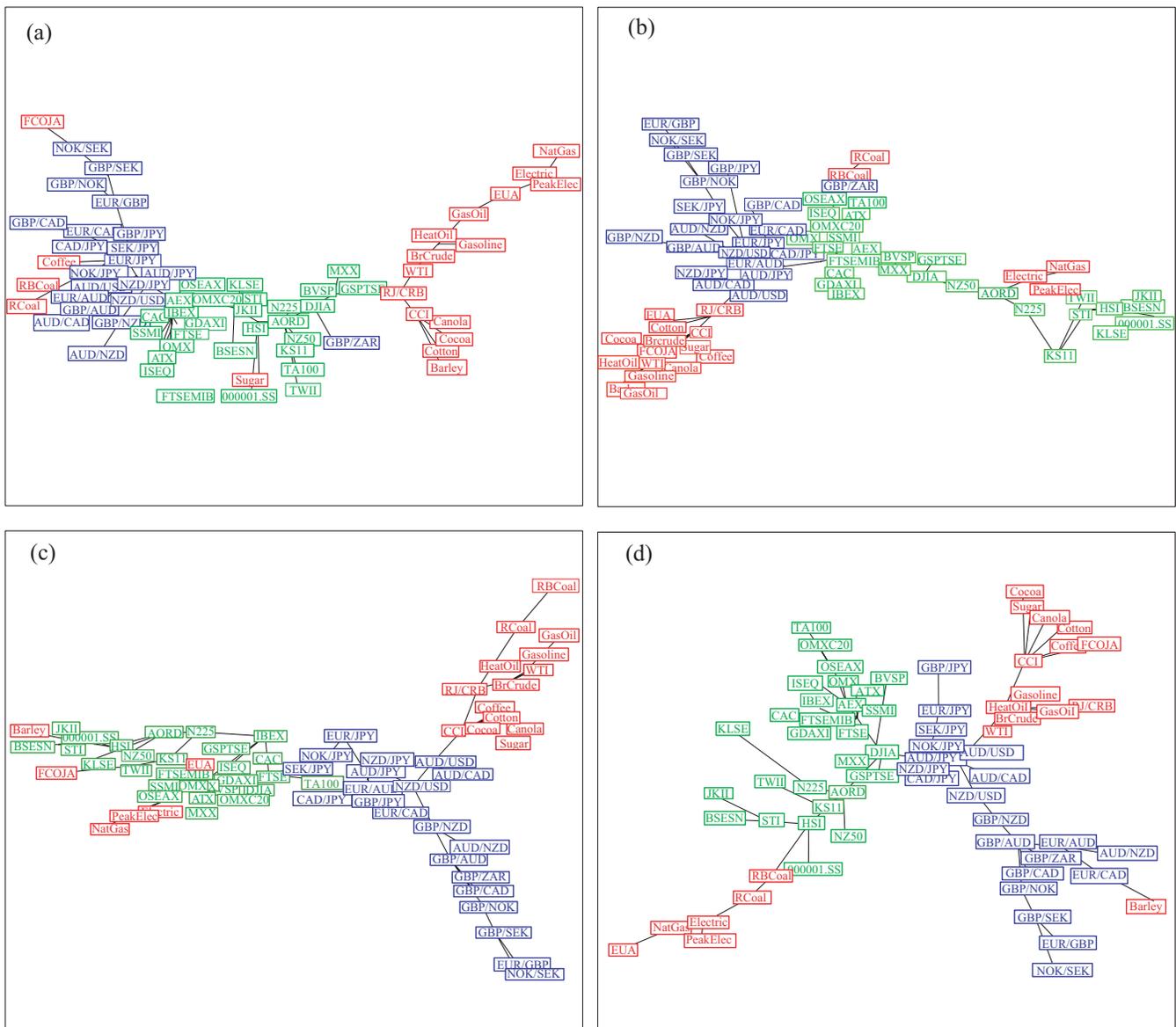


FIG. 2. (Color online) The MST obtained from the absolute correlation coefficients $|\rho_{ij}|$ of the set of 68 return time series during in individual calendar years (a) 2007, (b) 2008, (c) 2009, and (d) 2010. Red indicates commodity futures, blue indicates currency futures, and green represents stock market indicators. See Appendix for a listing of symbols and their meanings.

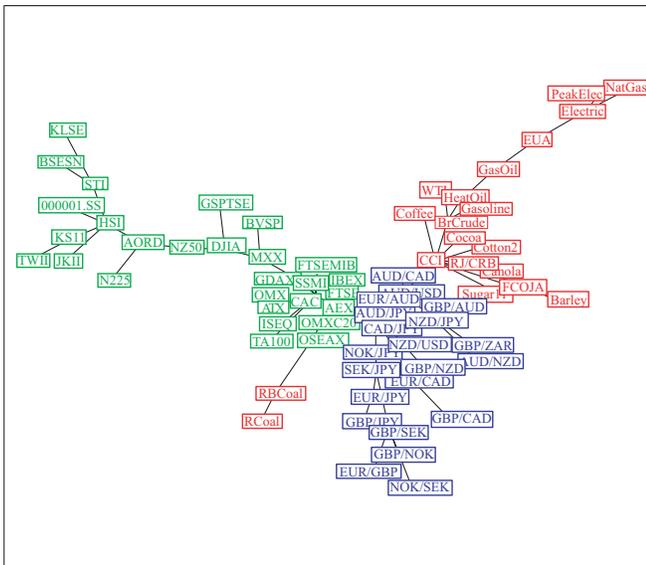


FIG. 3. (Color online) The MST, similar to Fig. 2, but for the longer time span January 2007 to September 2011. See the Appendix for symbol definitions.

of financial assets tend to group together consistently over time.

We also find that the stock market indicators (blue in Fig. 2) and currency futures (green) groups show stronger interconnections than the commodities (red) group. For stock market indicators and currency futures, financial factors are the predominant reason for price changes. On the other hand, commodity futures may be just as much affected by investment as they are by actual supply and demand. Speculation in commodity futures may alter pricing in a way that contradicts the law of supply and demand. The existence of such contradictions depends on commodity type and the calendar year and therefore serve to decrease the stability of the MST. If this reasoning is correct, increasing the time span of the time series for cross-correlation should make the observed connections more stable. In Fig. 3, we show the MST using the time series from January 2007 to September 2011. We note that only two coal futures are not connected to the commodity group and the stability is greater than for single-year time series.

In Fig. 4, we describe the cross-correlation functions of the volatility time series. We show the cross-correlation function of main stock market indicators with EUA in Fig. 4(a) and with WTI in Fig. 4(b). A systemic time shift between EUA or WTI and stock market indicators can clearly be seen. Since the maximum cross-correlation coefficient in most functions is not much greater than 0.2, the connections are not so strong, but certainly they are significant. Further tests of Granger causality also show that the volatility of stock indices is useful in forecasting the volatility of EUA and WTI approximately 20 to 120 days in advance.

As mentioned before, the correlation function of volatility is a slow-decaying function. It is much more slowly decaying than the correlation function of return time series (see Fig. 1), meaning that a long-range cross-correlation relationship exists. If we consider significant cross-correlations between different

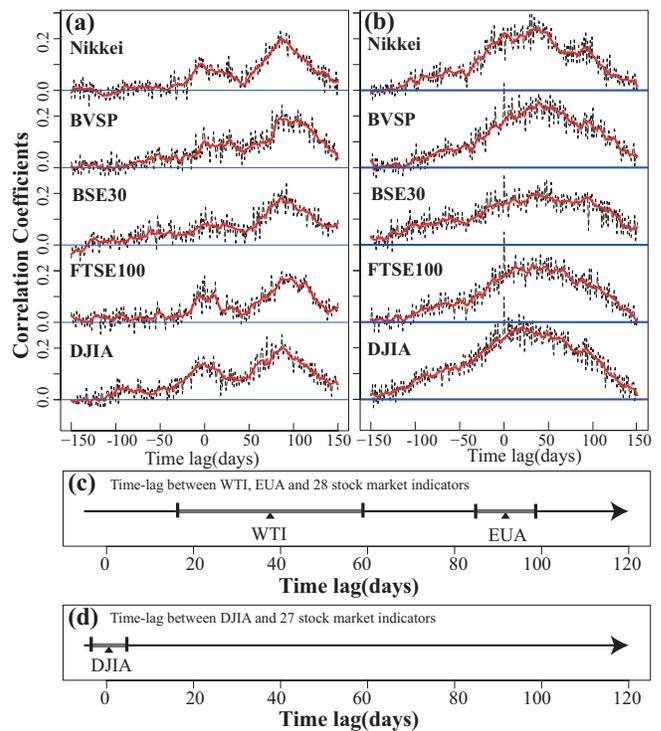


FIG. 4. (Color online) Cross-correlation function $C(n)$ of volatility daily time series between 5 main stock market indices and (a) EUA (European carbon emissions permits) or (b) WTI (light sweet crude oil). Red lines indicate the locally weighted scatter plot smoothing values (LOWESS) of $C(n)$. Graphs show systemic time shift for the highest cross-correlation value. This time shift is observed in most stock markets. (c) The average time-lag between EUA, WTI, and 28 stock market indicators, with error bars showing the standard deviations. (d) The average time lag between the DJIA and other 27 stock market indicators, with error bars showing the standard deviations. The time lags (in days) are calculated from the time lag n of highest LOWESS values.

volatility time series to be an information transfer between different assets, the time lag corresponding to the highest values of correlation gives the time lag of that information transfer. It is worth pointing out that the time lag between each pair of stock markets is approximately 0, such as the time lag between DJIA and other 27 stocks indicators are shown in Fig. 4(d). We show a simple summary of such time lag in Figs. 4(c) and 4(d) If there is information affecting stock markets at a time of 0 days, the EUA and WTI crude oil futures will be affected by this information roughly 30 and 90 days later, respectively. Because both crude oil and the ability to emit carbon are major inputs in the world economy, the existence of this time lag can have strong implications in terms of potential economic feedback loops.

II. DISCUSSION

The drawn MSTs are reflective of a number of easily reasoned underlying economic relationships, both through the stability and specificity of the links.

As expected, we find a stable tendency for like financial assets to cluster. Even during the otherwise anomalous 2007

TABLE I. Stock market indicators.

Symbol	Meaning	Notes	Symbol	Meaning	Notes
000001.SS	SSE Composite Index	Shanghai stocks	ISEQ	Irish Stock Exchange Quotient	
AEX	Amsterdam Exchange Index	Dutch securities	JKII	Jakarta Islamic index	
AORD	All Ordinaries	Australian stocks	KLSE	Kuala Lumpur Stock Exchange	
ATX	Austrian Traded Index		KS11	Seoul Composite (South Korea)	
BSESN	SENSEX	Bombay stocks	MXX	Mexican Stock Exchange IPC	
BVSP	Bovespa Index	São Paulo stocks	N225	NIKKEI 225	Tokyo stocks
CAC	CAC 40	French stocks	NZ50	NZX 50 Index	New Zealand index
DJIA	Dow Jones Industrial Average	American index	OMX	OMX Stockholm 30	
FTSE	FTSE 100	London stocks	OMXC20	OMX Copenhagen 20	
FTSEMIB	FTSE MIB	Italian stocks	OSEAX	Oslo Børs All Share Index	
GDAXI	Deutscher Aktien Index	German blue chips	STI	Straits Times Index	Singapore stocks
GSPTSE	S&P/TSX Composite Index	Toronto stocks	SSMI	Swiss Market Index	
HSI	Hang Seng Index	Hong Kong stocks	TA100	Tel Aviv 100	
IBEX	IBEX 35	Spanish stocks	TWII	TSEC weighted index	Taiwanese stocks

subprime lending crisis and 2008 global financial crisis, this clustering tendency is preserved. This indicates the existence of strong stable connections, which come out of the strong cross-correlations, reflective of basic economic features and interactions. These connections are stable over time and not affected by market conditions.

On the other hand, certain portions of the MSTs are consistently unstable, like those relating to coal. This also may be reflective of economic relationships. Unlike other commodities like oil, speculation in coal is limited, so the movement of the coal futures may be simple supply and demand, as opposed to driven by speculation. Coal's lack of strong connections to other commodities may be a result of investor's low speculation in stock when building their commodities portfolios.

Certain connections in our MSTs reveal underlying relationships that are created by regulation. For example, we find that EUA futures mostly connect with the base electricity and natural gas futures, which show stable correlations among them. EU allowance permits, as a part of the European Union Emission Trading Scheme, are either allocated or auctioned and allow a firm to emit a designated amount of carbon dioxide [30]. Since power generation accounts for about one-quarter of total emissions of carbon dioxide, and natural gas is the most

resource of electricity generation in UK, the stable connections of EUA to UK base electricity futures and UK natural gas futures in our MST graph is reasonable.

Also intuitive is the location specific clustering of stock market indices. AORD, an Australian index, consistently appears closely linked to those of nearby countries like New Zealand, Japan, and China. Similarly, the HSI not only keeps connections with most of the Asian stock indices like the JK11, 000001.SS, BSESN, and TWII, but also keeps connections with or otherwise stays closely connected to indices from America and Australia. Thus, the MST created reflects the common knowledge that Hong Kong is the financial center of Asia.

One benefit of the novelty of our approach is that it connects indices of dissimilar type, yielding new insights. The index that connects most to coal is OSEAX, that of Norway. Norway is rich in oil and natural gas, which explains why the Norway stock index appears as the most "coal-like" of the national indices. Similarly, the most "currency-like" of the national stock indices seems to be AEX, the Dutch securities index.

We also note the relationship between the centrality in the network and real-world geographical knowledge. SOK/SEK is a currency future that is among the furthest from the center. This is intuitive, since currency trading between Norway and

TABLE II. Commodity futures.

Symbol	Meaning	Notes	Symbol	Meaning	Notes
Barley	Western Barley Futures		Gasoline	RBOB Gasoline Futures	
BrCrude	Brent Crude Futures	North Sea crude oil	HeatOil	Heating Oil Futures	
Canola	Canola Futures		NatGas	UK Natural Gas Futures	
CCI	Consumer Confidence Index Futures		PeakElec	UK Peak Electricity Futures	
Cocoa	Cocoa Futures		RBCoal	Richards Bay Coal Futures	
Cotton	Cotton No. 2 Futures		RCoal	Rotterdam Coal Futures	
FCOJA	FCOJ-A Futures	Florida orange juice	RJ/CRB	Thomson Reuters/Jefferies CRB Index	assorted commodities
Electric	UK Base Electricity Futures		Sugar	Sugar No. 11 Futures	
Coffee	Coffee "C" Futures		WTI	West Texas Intermediate	Texas crude oil
GasOil	Gas Oil Futures		EUA	EU emission allowance	

TABLE III. Currency futures.

Symbol	Currency	Traded in units of
AUD	Australian dollar	CAD, JPY, NZD, USD
CAD	Canadian dollar	JPY
EUR	Euro	AUD, CAD, GBP, JPY
GDP	British pound	AUD, CAD, JPY, NOK, NZD, ZAR, SEK
JPY	Japanese yen	
NOK	Norwegian krone	JPY, SEK
NZD	New Zealand dollar	JPY, USD
SEK	Swedish krona	JPY
ZAR	South African rand	

Sweden has little to do with financial activity in the rest of the world. The same principle applies to the trading of Euros with the British pound, shown as EUR/GBP. The Australian dollar, on the other hand, plays a central role in its exchanges with far away currencies like the US dollar, Euro, Japanese yen, and Canadian dollar.

III. CONCLUSION

In this paper, we have analyzed the correlation function of return and volatility time series, constructed MSTs based on return time series, and found consistent time lags in the correlation functions of the volatilities. From these analyses, we have two main conclusions. (i) The stability of MST structure clustering between like commodities reflects a basic rule of economic activity, whereby the interaction between economic actors is not easily affected by capital movement. The method of absolute cross-correlation-coefficient-based MSTs has strong implications in the ongoing debate about the relationships of different financial commodity time series. (ii) We find that a time lag of correlation functions of volatility appears between stock markets and EUA and WTI. From this finding, we hypothesize that there may be systemic differences in the spread of financial risk, most often quantified as volatility. In other words, as concerns risk, different types of markets may have different sensitivities to economic information and other influencing factors. It would be interesting, from a theoretical point of view, to generalize this time lag to predict

the financial risks on a much longer time interval. However, much more would need to be understood first, such as the properties and mechanisms for this time lag. Hence, further empirical study is needed first. We endeavor to address this question more in future work.

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APPENDIX: SET OF DATA

The data under investigation includes 28 stock market indicators, 21 currency futures, and 20 commodity futures.

The stock market indicators investigated are shown in Table I.

The commodity and currency futures investigated are all traded in the markets of intercontinental exchange. The commodity futures are shown in Table II.

Additionally, the currency futures in the form A/B refers to the value of currency A in units of currency B. For example, USD/EUR would be the value of US dollars in units of Euros. The currency futures investigated are shown in Table III.

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