

# Exacerbated vulnerability of coupled socio-economic risk in complex networks

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**Abstract** – The study of risk contagion in economic networks has most often focused on the financial liquidities of institutions and assets. In practice the agents in a network affect each other through social contagion, *i.e.*, through herd behavior and the tendency to follow leaders. We study the coupled risk between social and economic contagion and find it significantly more severe than when economic risk is considered alone. Using the empirical network from the China venture capital market we find that the system exhibits an extreme risk of abrupt phase transition and large-scale damage, which is in clear contrast to the smooth phase transition traditionally observed in economic contagion alone. We also find that network structure impacts market resilience and that the randomization of the social network of the market participants can reduce system fragility when there is herd behavior. Our work indicates that under coupled contagion mechanisms network resilience can exhibit a fundamentally different behavior, *i.e.*, an abrupt transition. It also reveals the extreme risk when a system has coupled socio-economic risks, and this could be of interest to both policy makers and market practitioners.

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**Introduction.** – Can the failure of a single element threaten the stability of an economic system? This is the key question when authorities deal with predicting and reducing risk propagation in highly complex economic systems [1]. All complex systems are at risk of cascading failure in which one failed part can cause the failure of linked fractions such that failure propagates throughout the entire system [2–4]. Systemic risks associated with cascading failures have been present in recent and earlier economic crises, *e.g.*, the world financial crisis, the Asian economic crisis, and dot-com bubbles [5,6].

Because network science can use both the behavior of economic system participants and the relationships among them to model contagion mechanisms [7–9], it has attracted enormous attention among researchers studying risk propagation in such economic systems as banking networks, buyer and seller credit systems, international trade, capital markets and stock markets [10–14]. In network

science vertices represent agents in economic systems and links represent connections among them. The networks can be generated by using real data [15,16] or by developing theoretical networks in which properties are assumed, *e.g.*, random networks or scale-free networks [17]. Two types of network failure mechanism are used in this research. The first is the generic context failure approach based on a network social contagion epidemiological model in which one node fails or survives depending on the state of its neighbors [18–20]. The other is the liquidity approach in which each node is assigned a simplified balance sheet state, and a default in asset results in cascading failures through liquidations. When its asset value drops below its liability value it either fails or survives with an assumed tolerance probability [17,21,22].

Most studies of systemic risk in economic networks focus on economic/liquidity contagion. Yet real-world systems have both economic and social contagion risks [23]

that have different mechanisms but that strongly interact [24]. Thus the crucial aspect not appearing in previous studies is social contagion among market players: how information-sharing and panic-spreading affect the collective actions of the market players and can trigger systemic risk to the overall market. Social contagion can be related to the risk propagation that occurs through the spreading of panicked sentiment caused by market shock that gets further amplified [25]. If we are to understand the risk propagation mechanisms in a complex economic system, both asset-shock transmission and social-panic contagion must be taken into consideration [26]. In addition, most risk contagion studies have focused on banking systems and stock markets. Here we study a critical sector of the financial system: the venture capital market. The venture capital market is fragile when facing external shocks, *i.e.*, the typical venture-backed project has negative earnings and few tangible assets and is thus susceptible to failure. The venture capital market is also closely related to the stock market and since 1999, 60 percent of the IPOs that have occurred in the US stock market have been venture-backed [27]. Thus the instability of venture capital market frequently induces damage to the overall financial market and to the entire economic system.

We propose a unified cascading model based on a coupled socio-economic network that allows us to quantify the contribution of social contagion to systemic risk. We find the economic system shows exacerbated risks of a different nature when the social contagion effect is present. In particular, the transition to a large-scale market failure is abrupt instead being smooth, implying a first-order phase transition that generates extreme risk.

**Model.** – We first build a multiplex network  $G = \{I, N\}$  that represents a socio-economic network in which venture capital firms (VCs) are embedded. Here  $I$  is the economic network and  $N$  the social network.

Equity connections between venture capital investors (VC) and portfolio projects are the major channels by which asset shocks are transmitted from one VC to another. We use a weighted bipartite network to describe these equity connections. Hence  $I = \{C, V, E^c, W\}$ , where  $C$  denotes venture capital investor vertex set  $c_i \in C$  and  $V$  denotes portfolio project nodes set  $v_j \in V$ . Once VC  $i$  invests portfolio project  $j$ , there is  $e_{ij}$  between node  $c_i$  and  $v_j$ .  $E^c$  is the edge set and  $e_{ij} \in E^c$ . In this network, weight  $w_{ij}$  of each link  $e_{ij}$  represents the equity amount of portfolio project  $j$  owned by VC  $i$ . Because this network is undirected,  $e_{ij} = e_{ji}$  and  $w_{ij} = w_{ji}$ . There is no edge between the VC nodes or the portfolio project nodes.

In addition to equity connections, social connections are a critical channel for the spread of attitude and behavior, and they can amplify threat. Because in the venture capital industry investors tend to syndicate their investment and build co-investing relationships, we use a co-investing network to represent the social connections among VCs. In this network two VC nodes are connected only when

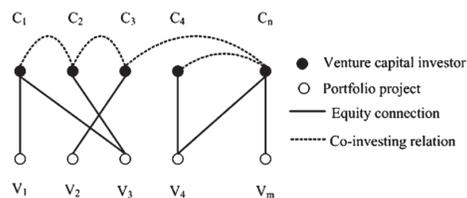


Fig. 1: Coupled socio-economic network of a venture capital market.

they both invest in the same portfolio project in the same round. Co-investors in the same round are more likely to exchange information and to share risk. Here we do not consider the weight and direction of links. Thus  $N = \{C, E^s\}$ , where  $C$  also denotes the VC node set and  $E^s$  the co-investing edge set. Once two VCs co-invest in one portfolio project in the same round,  $e^s$  connects the two VC nodes.

Thus for each VC node there are two types of link, i) equity connections to portfolio projects and ii) co-investing links to other VCs. Figure 1 shows the structure of a coupled socio-economic network of venture capital investors.

To take the social contagion effect into consideration when we simulate the cascading failure process, we apply the bipartite network model proposed by Huang *et al.* in 2012 [21] and extend it by considering two types of social contagion, i) the herd behavior effect and ii) the leader-following effect. The simulation process consists of the following steps:

- 1) The initial shock hits portfolio projects belonging to industry  $m$ . The total market value of industry  $m$  is reduced to a  $p$  fraction of its original value,  $p \in [0, 1]$ . The smaller the value of  $p$ , the stronger is the shock level. When  $p = 0$ , the total market value of the industry is wiped out. When  $p = 1$ , there is no shock imposed.
- 2) As the market deteriorates, each VC  $i$  that owns portfolio projects in the shocked industry  $m$  will experience a  $S_i^m(1 - p)$  reduction in value, where  $S_i^m$  is the total asset value of the shocked portfolio projects that are on the balance sheet of VC  $i$ , denoted  $S_i^m = \sum_J w_{ij}^m$ .
- 3) We assume that when the total asset value of any VC  $i$  is lower than its liability  $L_i$ , *i.e.*,  $S_i < L_i$ , where  $S_i^m = \sum_J w_{ij}$ , then VC  $i$  faces liquidity distress or default risk. Any portfolio projects on the balance sheet of a VC with liquidity distress will suffer a corresponding market value devaluation because of a fire sale by the VC, which leaves the market to avoid the risk of bankruptcy. We define VCs with an asset value lower than its liability to be in a failed state and otherwise in a surviving state.
- 4) When we take the social contagion phenomena into consideration the fire sale behavior of VC  $i'$  is

dependent on its balance sheet condition and is influenced by the actions of its co-investing partners. Even when VC  $i$  does not have liquidity distress, it can still choose to fire sale its assets because its partners have already exited. Thus we use  $s \in [0, 1]$  as a measure of the social contagion effect.

The  $s$  value functions differently in different social contagion mechanisms. In the case of herd behavior, social contagion is when market participants follow the actions of the majority of the VCs. There is a limitation in the amount of available information, and a market participant can only know the business decisions of its closest partners. Thus surviving VC  $i$  observes the fire-sale behavior of its co-investing partners (in our case its neighbor nodes in the co-investing network), and if a fraction larger than  $s$  of its partners chooses to fire sale their assets and exit the market, VC  $i$  will also fire sale its assets. Thus  $s$  is the threshold value of percolation. When  $s = 1$ , no social contagion occurs and VCs deciding whether to fire sale their assets can be only depend on their balance sheet. The lower the  $s$  value, the more easily each VC  $i$  will be influenced by the behavior of its partners, and the stronger will be the social contagion effect.

The other prevailing social contagion phenomenon is that market players tend to adopt the benchmark business strategy of market leaders, especially those within its co-investing partners group. So when VC  $i$  is without liquidity distress they still can choose to fire sale their asset with a probability  $1 - s$  when the focused market leader has made a fire sale decision. The focused market leader here is defined as the one whose total amount of investment is the largest of all the neighbor nodes of VC  $i$  and is also larger than the total investment amount of VC  $i$ . Also the lower the value of  $s$ , the stronger will be the social contagion effect.

- 5) Let  $a$  represent the illiquidity parameter that determines the degree to which the market values of the portfolio projects are devalued after the fire sales have occurred. The VCs with liquidity distress must exit to meet their liabilities. The ability of the market to absorb this sale is not perfect, which leads to a price decrease of the affected portfolio projects. Depending on the liquidity of the market,  $a$  can be between 0 and 1. When the market is extremely liquid, the value of the equity will not be adversely affected by VC  $i$ 's fire sales, so  $a = 1$ . When the market is extremely illiquid, the equity value could potentially be very close to zero.
- 6) A further deterioration of the market can then contribute to the failure of more VCs and portfolio projects. Thus the damage in the bipartite network spreads between VCs and portfolio projects bidirectionally until the cascading failure stops, which

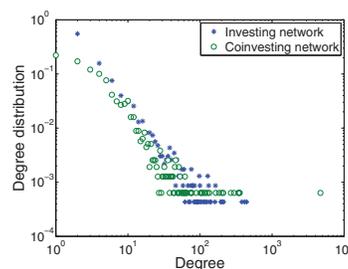


Fig. 2: (Colour online) Log-log degree distribution of investing network (blue dot) and co-investing network (green dot).

is for all VCs when  $S_i > L_i$ , and there are no more VCs in failure and the simulation stops iterating.

**Data: China venture capital market as an empirical case.** – We obtain our venture capital investment data from the China Venture Source database [28], which covers more than 90% of all venture investments in China. The data we use is from 1 January 2011 to 31 December 2014, covering 2310 venture capital firms, 5893 portfolio projects, and 19892 investment events. We use investment data to generate a bipartite network of venture capital investors and portfolio projects. For each venture capital firm we use the fund-raising amount as its liability, because a venture capital firm works as a financial intermediary to raise capital from such institutions as pension funds, banks, and insurance corporations. Once a fund has matured, the investment amount goes back to the investors accompanied by the return that the venture capital firm has promised. Each portfolio firm dataset is categorized by industry, and there are ten industry categories. Thus for a bipartite equity connection network, the average degree of VC nodes and portfolio project nodes are 8.61 and 3.38, respectively. Within the recorded data, 66.62% of the sample venture capital firms participate in syndicated investment. We use the same round of syndication investment to build a co-investing network of VCs. Thus in our co-investing network there 1571 vertices and 4468 links, making the average degree 5.69. Figure 2 shows the degree distribution of the bipartite equity connection network and the co-investing network. Note that the degree of the VC equity connection network and the co-investing network shows a distribution with a heavy tail and exhibits power-law properties, which means few VCs have a large number of portfolio projects and co-investing partners and that the majority of VCs have only a few equity-connection and social-relation linkages.

**Results.** – There are many different definitions of market resilience  $R$  [29–31]. In order to measure how  $R$  is dependent on the model parameters, we define it here as the fraction of venture capital firms surviving an external shock.

We begin by investigating the relationship between market resilience and the controlling parameters  $p$  (external

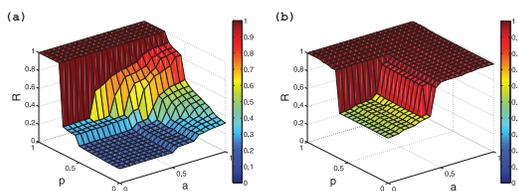


Fig. 3: (Colour online) Market resilience, fraction of surviving venture capital firms ( $R$ ) as a function of external shock level ( $p$ ) and the market liquidity degree ( $a$ ) in no-social-contagion scenario for two different shocked industry sectors: Internet industry (a) and Logistics industry (b).

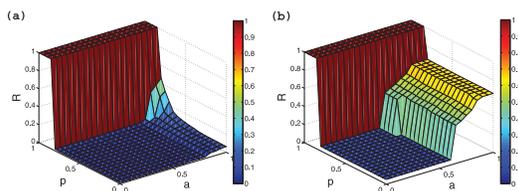


Fig. 4: (Colour online) Market resilience,  $R$  as a function of  $p$  and the market  $a$  in herd behavior scenario for two different shocked industry sectors: Internet industry (a) and Logistics industry (b).

shock level) and  $a$  (market liquidity level) for three different scenarios, i) assuming no social contagion, ii) assuming herd behavior social contagion, and iii) assuming leader-following social contagion. We also consider  $s = 0.5$  for ii) and iii) (see Model section). The largest sector is the Internet industry, which has  $\approx 49\%$  market share, and the smallest is the logistics industry, which has  $\approx 2\%$  market share. Both of these sectors are initially shocked.

Figure 3 shows that if a VC makes a fire sale decision by relying only on its balance sheet condition without being influenced by any counterparty behavior, there is no social contagion and the overall venture capital market is resilient. Figure 3(a) shows that even when the initially shocked industry sector is the largest, approximately 30% of the VCs survive. When a shock hits the smallest sector and the shock level is extremely high ( $p < 0.3$ ) and market liquidity is almost frozen ( $a < 0.3$ ), the fraction of surviving VCs is still greater than 50% (see fig. 3(b)).

When there is social contagion among the VCs, however, the system is dominated by an opposite cascading mechanism. Figures 4(a) and (b) show the market resilience dynamics under an external shock when herd behavior dominates. Note that it does not matter whether the initially shocked industry sector is large or small, when  $p < 0.8$  and  $a < 0.6$  almost all of the VCs fail. This indicates that when panic prevails market players are eager to liquidate their equity to maintain a certain liquidity ratio and avoid further losses even when they do not face actual liquidity distress. This irrational herd behavior can cause a huge amount of damage and activate a sudden freeze of market liquidity, even when asset price fluctuations are

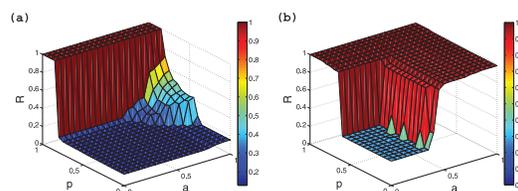


Fig. 5: (Colour online) Market resilience,  $R$  as a function of  $p$  and the  $a$  in leader-following scenario for two different shocked industry sectors: Internet industry (a) and Logistics industry (b).

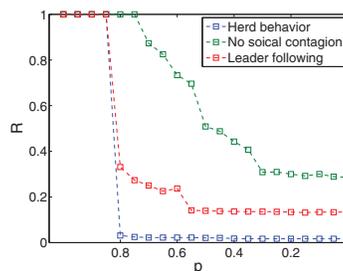


Fig. 6: (Colour online) Market resilience,  $R$  as a function of  $p$  with given  $a = 0.8$  for three scenarios: without social contagion (green line), herd behavior (blue line) and leader following (red line).

slight. For example, in the 2008 world financial crisis after the bankruptcy of Lehman Brothers most banks and financial institutions stopped lending, tightened credit terms for borrowers, and increased the stress on market liquidity.

Figure 5 shows that, when leader following is the type of social contagion that dominates the market, if the initially shocked industry is the largest, the amplifying effect of the contagion produces a risk level similar to that of herd behavior but the damage incurred is less serious, even when the level of shock is the same. Figure 5(b) shows that when the initial shock hits the smallest sector, if the shock level is moderate ( $p > 0.7$ ) and the market liquidity is relatively high ( $a > 0.5$ ), more than 80% of the VCs survive, which means that the leader following does not dramatically amplify the risk level. If the shock level is severe and the market liquidity is extremely low, leader-following effects will increase the market fragility much more than when there is no social contagion present.

Note that when the external shock level and market liquidity conditions are the same, social contagion increases market fragility. This finding is consistent with previous studies that found that the financial system exhibits a robust-yet-fragile behavior [32–34].

Figure 6 depicts the market resilience as a function of  $p$ , the magnitude of the external shock for fixed  $a = 0.8$  when the Internet sector is the initially shocked industry for three cases (no social contagion, herd behavior contagion, and leader-following contagion).

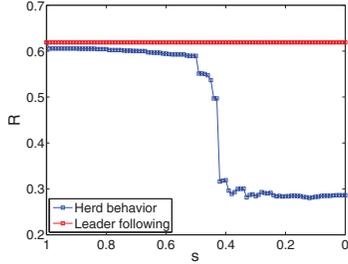


Fig. 7: (Colour online) With given  $p = 0.8$  and  $a = 0.9$ , compare market resilience,  $R$  as a function of social contagion degree ( $s$ ) in herd behavior (blue line) and leader-following scenarios (red line).

There are two cascading mechanism regimes that the herd behavior contagion effect has, one in which no VC faces liquidity distress at the end of the model, which means  $R \rightarrow 1$ , and one in which the number of surviving VCs is below 10%. When there is no social contagion these two separate phases do not form and market resilience decays slowly as external shocks increase. This indicates that if every market player were rational and made decisions independently the market would deteriorate gradually. This would allow sufficient time to warn policy makers who would then have time to respond. However, when the herd behavior effect is strong, the market can rapidly deteriorate so that policy makers have no time to react. If  $p$  is defined such that the fraction of surviving VCs is below 20% at a critical point  $p_c$ , the herd behavior case  $p_c = 0.8$  is higher than the leader-following case  $p_c = 0.55$ . Thus herd behavior is more likely to contribute to systemic risk than leader following. Because approximately 60% of the VCs have fewer than five co-investing partners, fire-sale actions of even a small fraction of the VCs can spread to the whole market and endanger the entire system. When there is no social contagion, the fraction of surviving VCs is above 30% as the market deteriorates, indicating that the market can sustain a huge decrease in asset value.

To quantify the impact of social contagion on market resilience, we investigate herd behavior and leader following for a shock level of  $p = 0.8$  and a liquidity condition of  $a = 0.9$ . Figure 7 shows that when the Internet is the initially shocked industry and herd behavior dominates, the market resilience undergoes a phase transition as  $s$  reaches a critical value of 0.5. When  $s > 0.5$  the level of herd behavior is relatively limited, and the fraction of healthy VCs is  $\approx 60\%$  as the degree of social contagion increases. But when  $s < 0.5$ , market resilience sharply decreases to less than 30%, cascading failures occur, and the system undergoes an abrupt first-order transition, which indicates that the market condition has suddenly changed from stable to a crisis situation.

Under the same shock level and liquidity conditions, however, when the leader-following effect dominates, the

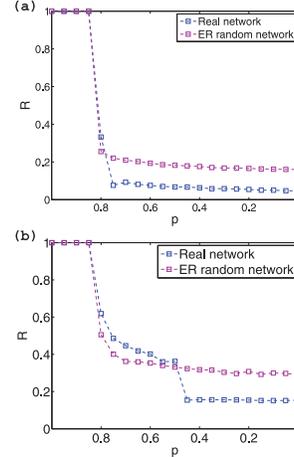


Fig. 8: (Colour online) With given  $a = 0.9$ , compare market resilience,  $R$  as a function of  $p$  based on two different co-investing networks: real network (blue line) and  $ER$  random network with same  $\langle k \rangle$  as real network (purple line). (a) shows herd behavior scenario and (b) shows leader-following scenario.

fraction of surviving VCs remains at 60% as the social contagion degree increases. Because the shock level is moderate and market liquidity is sufficiently high, the majority of market leaders do not have a liquidity problem, and this is critical in maintaining the overall health of the system.

Previous studies indicate that network topology impacts risk propagation. We compare the dynamics of market resilience with social contagion in a real-world co-investing network with those in an  $ER$  random network with the same average degree  $\langle k \rangle$ . Here the  $ER$  network is only a social contagion network. The underlying bipartite network of liquidity contagion remains the same, such that only the social contagion effect is considered. Figure 8(a) shows that in herd behavior social contagion, market resilience in a real-world co-investing network and a random co-investing network both exhibit a first-order phase transition as the shock level increases, but when  $p < 0.8$  the fraction of surviving VCs in the real-world network is much less than that in an  $ER$  random network with the same  $\langle k \rangle$ . Figure 8(b) shows the leader-following scenario in which market resilience exhibits a first-order transition in random networks and a second-order phase transition in real-world co-investing networks.

**Summary and conclusions.** – Our study has shown that adding social contagion to liquidity contagion completely changes the nature of network resilience. In particular, the coupling of the two contagion mechanisms causes a first-order transition phenomenon not observed when there is only a single contagion mechanism. These results may be related to a recent study of cooperative contagion [35] in which a first-order phase transition was also observed. An analysis of the empirical network of

the Chinese Venture Capital market indicates that social contagion changes both the nature of network resilience and increases overall risk. Our analysis on the bipartite network using two coupled mechanisms reveals a significant aspect of considering multiple mechanisms in such networks. For further academic research, more than two mechanisms can be studied in the same framework and investigate the coupled effect. In addition, it is valuable for practitioners, especially governments, to be informed that the market is much more vulnerable when social contagion is taken into account. In particular, this analysis framework could be used to more effectively predict the damage that could be inflicted on the whole economy when financial crisis occurs. One possible future work is to take into account nodes' spontaneous recovery mechanism [36], and model the temporal bipartite network dynamics in the data. Such an analysis will be able to provide a more realistic picture of risk contagion with multiple mechanisms.

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#### REFERENCES

- [1] SCHWEITZER F., FAGIOLO G., SORNETTE D., VEGA-REDONDO F., VESPIGNANI A. and WHITE D. R., *Science*, **325** (2009) 422.
- [2] CRUCITTI P., LATORA V. and MARCHIORI M., *Phys. Rev. E*, **69** (2004) 045104.
- [3] BULDYREV S. V., PARSHANI R., PAUL G., STANLEY H. E. and HAVLIN S., *Nature*, **464** (2010) 1025.
- [4] GAO J., LIU X., LI D. and HAVLIN S., *Energies*, **8** (2015) 12187.
- [5] BILLIO M., GETMANSKY M., LO A. W. and PELIZZON L., *J. Financ. Econ.*, **104** (2012) 535.
- [6] ACEMOGLU D., OZDAGLAR A. and TAHBAZ-SALEHI A., *Am. Econ. Rev.*, **105** (2015) 564.
- [7] WATTS D. J., *Proc. Natl. Acad. Sci. U.S.A.*, **99** (2002) 5766.
- [8] BORGATTI S. P., MEHRA A., BRASS D. J. and LABIANCA G., *Science*, **323** (2009) 892.
- [9] GARAS A., ARGYRAKIS P., ROZENBLAT C., TOMASSINI M. and HAVLIN S., *New J. Phys.*, **12** (2010) 113043.
- [10] ALLEN F. and GALE D., *J. Polit. Econ.*, **108** (2000) 1.
- [11] HALDANE A. G. and MAY R. M., *Nature*, **469** (2011) 351.
- [12] GEORG C.-P., *J. Bank. Finance*, **37** (2013) 2216.
- [13] ZHANG X., PODOBNIK B., KENETT D. Y. and STANLEY H. E., *Physica A: Stat. Mech. Appl.*, **415** (2014) 43.
- [14] CALDARELLI G., CHESSA A., PAMMOLLI F., GABRIELLI A. and PULIGA M., *Nat. Phys.*, **9** (2013) 125.
- [15] BOSS M., ELSINGER H., SUMMER M. and THURNER S., *Quant. Finance*, **4** (2004) 677.
- [16] CAINELLI G., MONTRESOR S. and MARZETTI G. V., *J. Evol. Econ.*, **22** (2012) 711.
- [17] PODOBNIK B., HORVATIC D., BERTELLA M. A., FENG L., HUANG X. and LI B., *EPL*, **106** (2014) 68003.
- [18] TOIVANEN M., Bank of Finland Research Discussion Paper No. 19 (2013).
- [19] FISHER E., *Anal. Econ.*, **28** (2013) 109.
- [20] PECKHAM R., *J. Public Health* (2013) 1.
- [21] HUANG X., VODENSKA I., HAVLIN S. and STANLEY H. E., *Sci. Rep.*, **3** (2013).
- [22] LEVY-CARCIENTE S., KENETT D. Y., AVAKIAN A., STANLEY H. E. and HAVLIN S., *J. Bank. Finance*, **59** (2015) 164.
- [23] JACKSON M. O. *et al.*, *Social and Economic Networks*, Vol. **3** (Princeton University Press, Princeton) 2008.
- [24] ZHANG X., SHAO S., STANLEY H. E. and HAVLIN S., *EPL*, **108** (2014) 58001.
- [25] HIRSHLEIFER D. A. and TEOH S. H., *Thought and Behavior Contagion in Capital Markets* (Elsevier) 2009.
- [26] SCHERER C. W. and CHO H., *Risk Anal.*, **23** (2003) 261.
- [27] TOWNSEND R. R., *Manag. Sci.*, **61** (2015) 2782.
- [28] VENTURE C., *China Venture Source*, <http://source.chinaventure.com.cn>.
- [29] STARR R., NEWFROCK J. and DELUREY M., *Strategy and Bus.*, **30** (2003) 70.
- [30] KAUË DAL'MASO PERON THOMAS, DA FONTOURA COSTA L. and RODRIGUES FRANCISCO A., *Chaos*, **22** (2012) 013117.
- [31] TUCKER P., *Barclays Annual Lecture, London*, No. 22 (2009).
- [32] CIFUENTES R., FERRUCCI G. and SHIN H. S., *J. Eur. Econ. Assoc.*, **3** (2005) 556.
- [33] GAI P. and KAPADIA S., *Proc. R. Soc. London, Ser. A: Math. Phys. Eng. Sci.*, **466** (2010) 2401.
- [34] CALLAWAY D. S., NEWMAN M. E., STROGATZ S. H. and WATTS D. J., *Phys. Rev. Lett.*, **85** (2000) 5468.
- [35] CAI W., CHEN L., GHANBARNEJAD F. and GRASSBERGER P., *Nat. Phys.*, **11** (2015) 936.
- [36] MAJDANDZIC A., PODOBNIK B., BULDYREV S. V., KENETT D. Y., HAVLIN S. and STANLEY H. E., *Nat. Phys.*, **10** (2014) 34.