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Money circulation and debt circulation: a restatement of quantity theory of money

Xiaoyun Xing, Wanting Xiong, Liujun Chen, Jiawei Chen, Yougui Wang, and H. Eugene Stanley

Abstract
Both money and debt are products of credit creation of banks. Money is always circulating among traders by facilitating commodity transactions. In contrast, debt is created by borrowing and annihilated by repayment as it is matured. However, when this creation-annihilation process is mediated by banks which are constrained by a credit capacity, there exists continuous transfer of debt among debtors, which can be defined as debt circulation. This paper presents a multi-agent model in which income determination, credit creation, and credit transaction are integrated. A hypothetical economy composed of a banking system and multiple traders is proposed, in which the traders are allowed to borrow money from the bank once their expenditure cannot be financed by their own funds. In order to demonstrate the circulations of money and debt from the micro view, the authors track the transfer processes of them and collect their holding times respectively. When the traders could afford their expenditures, only money circulation can be observed. However, as they are forced to borrow, the money circulation is accelerated and debt circulation emerges. Both distributions of holding times of money and debt are found to take exponential form due to the random nature of exchanges. The velocity of money circulation is determined by the expending behavior of traders, while the velocity of debt circulation is associated with the repayment behavior of debtors. Consequently, the aggregate income can be decomposed into two parts: one comes from money circulation and the other from debt circulation.

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Authors
Xiaoyun Xing, School of Systems Science, Beijing Normal University, Beijing, China
Wanting Xiong, School of Systems Science, Beijing Normal University, Beijing, China
Liujun Chen, School of Systems Science, Beijing Normal University, Beijing, China
Jiawei Chen, School of Systems Science, Beijing Normal University, Beijing, China
Yougui Wang, ✉ School of Systems Science, Beijing Normal University, Beijing, China, ygwang@bnu.edu.cn
H. Eugene Stanley, Boston University, USA

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1 Introduction

Money and debt are the two sides of the same coin. As the credit creation theory of banking argues, both money and debt come from credit creation where commercial banks play the central role (Mcleay et al., 2014; Werner, 2014; Xiong et al., 2017). They are explicitly symmetrical with each other in the process of credit creation, which can be characterized by the concurrent changes in the balance sheet of commercial banks. Once a commercial bank grants a certain amount of loans, it creates the same amount of deposits simultaneously (Mcleay et al., 2014). However, regarding their impacts on macroeconomies, money and debt have gained seriously asymmetrical attention by mainstream economists. Major concerns have been on how money could impact economies for a long time, and the relevant literature is all around. However, there are only a few research works focusing on the role of debt (Bernanke et al., 1998; Bernanke and Blinder, 1988; Bernanke, 1999; Kato et al., 1997). This unequal concern has changed since the recent financial crisis, where debt is the prime culprit to be blamed (Bernanke, 1999; Reinhart and Rogoff, 2011a,b). Due to the extremely inappropriate awarness on debt, integrating financial instituions into the economic modelling becomes an intractable problem (Cecchetti et al., 2011). One of the major attempts of reconstructing macroeconomics is to rebalance the consciousness about the two sides of the same coin, and to remedy the current negligent recognition of the role of debt.

The reason why debt has been neglected for such a long time could be summarized as follows. Firstly, the prevailing paradigm of economics was ever centred on transactions in real economies, where money is a veil and debt is unimportant (Borio and Lowe, 2004). Secondly, with representative agent hypothesis, the increase of debt could not exaggerate the amount of equity, since the liabilities of all borrowers always exactly match the assets of all lenders, it is taken for granted that debt would have little real impacts. However, financial assets and liabilities are assigned to different economic agents, and they would exert their influence on the economy through different channels. It is negligent to look at the economy as a whole, and assess the effect of debt in a net-zero system (Cecchetti et al., 2011). In fact, financial institutions play a central role in credit creation, by which debt and the same quantity of financial asset are created simultaneously (Werner, 2014). Thirdly, focusing on the transactions in goods markets instead of those in credit markets is anther reason why debt is disregarded. Even though credit market has ever been mentioned, much attention is put on its price rather than the credit stock and flow (Godley and Lavoie, 2012; Tobin, 1969).

Although the paradigm where only money matters has long been established for understanding the functioning of macroeconomic systems, a minor group of researchers have already stated that both money and debt play a major role in the working of macroeconomies, especially in the built-up of the crisis. The pioneering economists, such as Joseph A. Schumpeter and Ivring Fisher, made notable contributions (Fisher, 1933; Schumpeter, 1961). One of Post-Keynesian economists, Hyman Minsky, proposed the renowed financial instability hypothesis and interpreted the occurence of the crisis with a structual change of the economic system (Minsky, 1982). In recent decades, many reseachers have developed models to examine the impacts of money and debt equally and specify their roles in promoting the performance of macroeconomies in many aspects (Godley and Lavoie, 2012; Friedman, 1981, 1982; Bezemer and Dirk, 2010; Bezemer, 2012; Keen, 2012, 2014).

The prevailing channel to approach how debt influences the macroeconomic activities is to investigate the relationship between money, debt and the aggregate demand. Ben Bernanke firstly discovered the statitics association between the nonmonetary elements and some macroeconomic indicators (Bernanke, 1999). Since then, several seminal theoretical frameworks were estabilished to intepret how money and debt separately contributed to the aggregate demand (Bernanke and Blinder, 1988; Brunner and Meltzer, 1972; Eggertsson and Krugman, 2012; Bernardo and Campiglio, 2014; Keen et al., 2014). Conerning how debt affects the economy, one of the most general research route is to divide households into the ones with a high marginal propensity to consume (MPC) and the others with a low propensity. Following this route, Thomas Palley found that debt could enhance the economy through enlarging consumption in the beginning because it satisfied part of the demand to spend more. However, debt accumulation gradually comes into being and would increase the burden on debtors. Once they could not afford to repay their debts, the aggregate demand would then decline (Palley, 1994). Moreover, based on this framewoke, Paul Krugman concluded that the distribution of debt mattered rather than the level of it (Eggertsson and Krugman, 2012). In contrast to Krugman's view that aggregate expenditure must be equal to aggregate income, several Post-Keynesian economists including Steve Keen innovatively revealed that aggregate demand is the summation of the aggregate income and the increment of debt during the iterative process of income determination (Keen, 2012, 2014; Bernardo and Campiglio, 2014).

Our understanding on how money and debt affect the aggregate demand is originated from an alternative expression of aggregate demand. A popular way of formulating the aggregate demand in a closed economy is to divide it into consumption, investment and government spending. In such a way the question can then be converted into probing how money and debt affect the behaviors of the corresponding agents, namely households, firms and the government. Most of previous theoretical analyses were conducted on the foundation of this expression of aggregate demand. The aggregate demand can also be expressed by the quantity equation of money which is constituted by the amount of money and its velocity of circulation (Werner, 2012). From this perspective, if we try to figure out the impact of debt, we shall integrate the amount of debt into that of money or we could investigate how debt affects velocity of money circulation (Xiong et al., 2017). The former solution indeed reveals the impact of debt, however, it confuses money and debt as the same thing (Keen, 2012; Keen et al., 2014). Actually, money and debt should not be summed up since they respectively lie in two sides of the balance sheet of commercial banks. At the first glance, the introduction of debt indeed accelerates the velocity of money circulation, and further promoting aggregate demand. Nevertheless, if we start from quantity equation of money, we cannot find out the relationship between debt and velocity of money circulation till we clarify how debt contributes to aggregate income. This reversal of logical sequence and confusion about money and debt indicate that the route mentioned above is not feasible. Therefore, only if we express aggregate demand with money, debt and their corresponding velocities of circulation, can we identify how money and debt respectively exert their impacts on the aggregate demand.

2 Money Circulation versus Debt Circulation

Money is not dropped to the economy by helicopters. As the theory of credit creation argues, the majority of money is created by commercial banks (Mcleay et al., 2014; Werner, 2014). According to this theory, the change of money stock is governed by bank lending and repayment, and the quantity reaches its equilibrium when the inflows and outflows are equal (Xiong et al., 2017). In general, the aggregate amount of money, M, is usually expressed as the multiple of the quantity of base money, M_0, which is given by

$$M = m \cdot M_0, \tag{1}$$

where m is the money multiplier. According to traditional fractional reserve theory of banking, the money multiplier can be simply expressed as the inverse of the reserve requirement ratio and the central bank is the sole authority who can control the money supply. In fact, besides the reserve requirement, prudential regulations could also be binding constraint on commercial bank lending and thus affect the money supply. As a result, some regulation-relevant factors including the risk and maturity structure of bank loans, the relative liquidity and equity position of banks, as well as the run-off ratio of deposit are also found to be main determinants of money multiplier (Li et al., 2017; Xiong and Wang, 2017).

With respect to how the aggregate amount of money constitutes the aggregate income of the economy, the first interpretation is commonly attributed to Irving Fisher's exchange equation:

$$P \cdot Y = M \cdot V, \tag{2}$$

where P is the average price level, Y is the real income, and V is the velocity of money circulation. The two sides of the equation represent the nominal value of aggregate transactions from a monetary perspective and a product perspective respectively. In this way, this equation provides a way to understand how the aggregate income is determined through the channel of money circulation. Here the velocity of money circulation is another aggregate measure, which is defined as the average frequency of one unit of money participating in the exchanges of final goods and services. Obviously, it can be calculated as the ratio of nominal GDP to the stock of money directly from Equation (2), i.e.,

$$V = \frac{PY}{M}. \tag{3}$$

However, the exchange equation is just an identity of exchanges, it does not reveal the intrinsic properties of the velocity. The velocity of money circulation has long been presumed to be determined by monetary system and trading technology, and hence regarded as a constant in normal time. Although many theoretical and empirical researches on the velocity have been carried out to examine its determinants since the birth of the above exchange equation (Friedman and Schwartz, 2008; Friedman, 1956), the understanding of this variable still remains controvertible. As a matter of fact, it is the behaviors of traders that govern the velocity, which can then be derived from a large number of individuals' choices. Based on the agent-based model proposed by some econo-physicists in the studies of money distribution (Dragulescu and Yakovenko, 2000; Chakraborti and Chakrabarti, 2000; Yakovenko, 2010), the process of money circulation was also examined, in which the concept of holding time was introduced. The main conclusion is that the velocity of money circulation can be expressed in terms of average holding time, which provides a micro foundation of this key macroeconomic variable (Wang et al., 2003; Ding et al., 2003; Xi et al., 2005; Wang and Qiu, 2005).

The concept of money circulation has appeared for centuries. However, so far there is little consciousness on the existence of debt circulation even though it is innate in the modern banking systems, the reason of which might be that people concentrate on the role of money in facilitating transactions, while ignore the contribution of debt. In fact, money and debt are the twinborn products of credit creation.

Both of them are created by bank lending, annihilated by repayment (Fischer and Braun, 2003), and hence the determination process of the quantity of debt and that of money are the different sides of the same coin. It is worth noting that money circulation takes place during the time period between its creation and annihilation, but debt circulation comes into being due to its continuous replacements of debt caused by credit creation and its annihilation. Implicit in such a process, the issue of debt circulation has not been noticed by economists.

The definition of money flow is based on the quantity of money and its velocity of circulation. Once the quantity of money is fixed, the flow generated by this stock could be formulated as the product of quantity of money, M, and its velocity of circulation, V_m, which is given by

$$F_m = MV_m. \tag{4}$$

Please note that V_m differs from V given by Equation (3), which measures how frequently one unit of money can participate into the transaction of final goods and services between the moments of its birth and death. Actually, there is a gap between MV_m and MV, which is exactly the monetary flow generated by debt.

Similar to the definition above, the debt induced monetary flow can also be identified as the product of debt stock and the velocity of its circulation, which can be expressed as

$$F_d = DV_d. \tag{5}$$

where F_d denotes the debt induced monetary flow, D is the stock of debt, and V_d is the velocity of debt circulation.

The introduction of debt circulation would transform the long recognized *quantity theory of money*, as Equation (2) shows, which describes aggregate income in terms of only money flow, into a new quantity theory of money, in which both flows generated by money and debt are included. That is to say, the sum of the two flows yields the aggregate income, thus we have

$$PY = F_m + F_d. \tag{6}$$

Substituting Equation(4), (5) into (6), we can get

$$PY = MV_m + DV_d. \tag{7}$$

We have already deduced the traditional expression of velocity of money circulation given by Equation (3) based on the original quantity theory of money given by Equation (2), now we can get a new expression of this velocity by substituting Equation (7) into Equation (3), which takes the following form:

$$V = V_m + \frac{D}{M} V_d. \tag{8}$$

Traditionally, V specifies only how fast money circulates in the economy. In contrast to the prevailing interpretation of this variable, from Equation (8) we can see that it is actually determined by both the velocity of money circulation and that of debt circulation, as well as the ratio of debt to money. Comparing the velocities given by Equations (3) and (8), we can call the former one *Fisher's velocity of money circulation*. Since both money and debt are the twinborn products of credit creation, we can call the latter one *the credit-derived velocity of money circulation*.

Because the creation of money and that of debt take place simultaneously and vice versa for their annihilation, they are therefore collectively defined as *credit creation*. Nevertheless, there is extremely asymmetric concern on those twinborn products of credit creation, that is, much attention on money but thorough neglect of debt, due to the ambiguity in understanding the credit creation process. That is why it is the prerequisite to detect debt circulation to clarify credit creation process and to specify the role of banks in this process.

3 An Agent-based Model

We consider a closed economy which is composed of multiple traders, one commercial bank and one central bank. The central bank issues the base money and sets the required reserve ratio. For the sake of simplicity, all the base money is initially injected into this hypothetical economy by the central bank and exists solely in the form of reserves held by the commercial bank. The bank acts as the credit supplier. Once the bank grants a loan to a borrower, credit is created. As a result, both the assets and liabilities of the bank expand simultaneously with the same amount. It is worth noting that the credit money is the wealth of traders and the loans are their debts, both resulting from the process of credit creation, and being exactly equal to each other. All the credit money together with the base money initially deposited by the traders take the form of deposits for no cash is presumed to be held by the traders. The commercial bank could only lend out its excess reserves to the traders, so the amount of loanable funds depends on the gap between the amount of reserves initially issued by the central bank and the required reserves, which could be obtained by multiplying the actual volume of deposits and the required ratio. As for traders, they are the actors of expenditure, the demanders of credit, and they might also be the debtors who have obligations to repay their debt. Each trader makes a decision on how much he plans to expend according to his income and the amount of money he holds. Comparing the planned expenditure with his amount of money, he would decide whether he needs to borrow money from the bank and how much. The interaction among all the participants in the economy can be described by the following three types of processes: (I) income determination process driven by all traders, (II) credit transaction process carried out by borrowers and the sole lender in the credit market, (III) credit creation process formulated by bank lending and repayment of debtors.

3.1 Income determination process

From the micro-perspective in the income determination process, each trader plans his expenditure according to his wealth and expected income. Supposing that the planned expenditure of trader i at period t is $e_i^p(t)$, the deposit is $m_i(t)$, and the expected income is $\hat{y}_i(t)$, the mathematical relation between them can be expressed by

$$e_i^p(t) = \alpha \cdot m_i(t) + \beta \cdot \hat{y}_i(t), \tag{9}$$

where α and β are are two parameters that characterize the expending behaviors of all traders. To be more specific, α is the marginal propensity to spend with respect to wealth and β is that with respect to expected income. For simplicity, the expected income of the trader is hereafter assumed to be his income in the last period.

After he makes this decision, he considers whether he can afford this purchase. If the amount of money the trader currently has in his account is sufficient to cover his planned expenditure, he would use his money to accomplish this plan; otherwise he would tend to apply for a loan to the commercial bank. Moreover, the demands for loan of all individual traders would be summed up as an aggregate demand of the credit market. Whether this demand can be fulfilled or how much that can be realized depends on the capacity the bank can supply. Thus the realized quantity of loans is determined by the interaction between both demand and supply sides in the credit market. Hence we can see that there are two components constituting the realized expenditure of each trader at time period t, $e_i(t)$, one is the money withdrawn from his deposit account, represented by $a_i(t)$, while the other one is the bank loan, denoted by $b_i(t)$. Then the realized expenditure of trader i takes the following form,

$$e_i(t) = a_i(t) + b_i(t). \tag{10}$$

By summing up the expenditures of all traders, we can obtain the aggregate expenditure at the current period, which is given by

$$E(t) = \sum_i e_i(t). \tag{11}$$

From the macro-perspective, the aggregate income denoted by Y equals the aggregate expenditure in a closed economy where neither import nor export exists, that is to say, the following equation always holds

$$Y(t) = E(t). \tag{12}$$

From Equations (10), (11) and (12), we can draw an inference that the aggregate income has two financing sources: the current savings of traders and the new borrowings from the bank.

Once we obtain the aggregate income, each trader would then get one share of it. As a matter of fact, the traders in the economy not only play the role of expenders, they are also the receivers of income, no matter through what way they get their share of income. Since the allocation of the aggregate income is not our focus in this paper, the crucial point we are concerned with is the updated individual income that each trader needs to move to the next period. For the sake of simplicity, the aggregate income is allocated to the traders randomly, after which trader i would be assigned with an individual income, denoted by $y_i(t)$, so that we have

$$\sum_i y_i(t) = Y(t). \tag{13}$$

Integrating Equations (11), (12) and (13), we may acquire the macroeconomic relationship between those individual receipts and expenses which is given by

$$\sum_i y_i(t) = \sum_i e_i(t). \tag{14}$$

In order to illustrate how the income determination process runs, we present a flowchart in Fig. 1 taking trader i as an example. As shown in Fig. 1, the credit market here plays as a black-box where the input is the demand for credit and the output is the realized loan. Only when the money of trader i cannot cover his planned expenditure, he would ask for a loan from the commercial bank. So we introduce planned borrowing, denoted by $b_i^p(t)$, into the analysis, which is defined by the gap between the planned expenditure, $e_i^p(t)$ and the current deposit of trader i, $m_i(t)$, that is

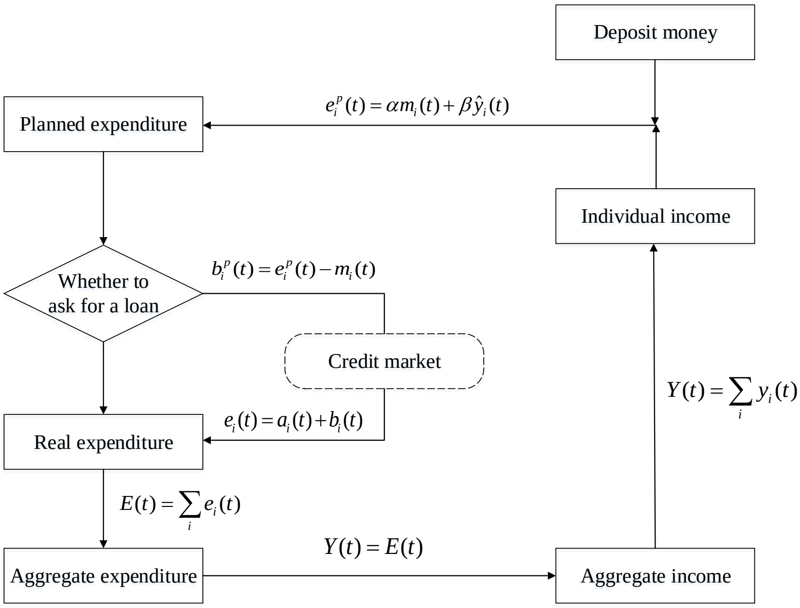

Figure 1: Flowchart of the income determination process

$$b_i^p(t) = e_i^p(t) - m_i(t). \tag{15}$$

Summing up the planned borrowings of all traders, we can obtain the aggregate planned borrowing as an input of the credit market, after which mechanism we would get a realized aggregate loan, then we can deduce the realized expenditure as Equation (10) demonstrates.

3.2 Transaction process

In the elaboration of the income determination process above, the credit market was taken as a black-box. Specifically, when this black-box is disclosed, there are two sides in the credit market: supply and demand, the same thing is true with the goods market. As mentioned in the preceding subsection, the traders whose planned expenditure cannot be financed by their deposits constitute the demand side of the credit market. While the commercial bank who has a capability of creating credit plays the role of supply side in this market. The demand for credit of trader i at period t is defined as the gap between his planned expenditure and his current deposit, which is given by Equation (15) in the above subsection. Then the aggregate demand $B^p(t)$ can be obtained by summing up all the individual demands, which takes the following form,

$$B^p(t) = \sum_i b_i^p(t). \tag{16}$$

This is the aggregate amount of money that the over-spending traders need to borrow from the bank. They are supposed to be submitted to the bank as loan applications.

Once the bank receives these applications, it would estimate its capability of credit supply and then decide whether it can fulfill the quantity demanded. Theoretically, the bank can write contracts of any loans applied by borrowers. In other words, the commercial bank is able to create credit as long as it is demanded, leading to an expansion of its balance sheet on both sides. However, such expansion cannot occur unrestrictedly, there are a few regulations introduced by the central bank and the banking supervisory authorities. The reserve requirement restricts the bank to lend out only the excess reserves to prevent the occurrence of bank runs. The capital adequacy requirement put forward in Basel III accord constraints the ratio between the equity and the total assets of the bank to a certain proportion. Moreover, the liquidity coverage ratio of Basel III requires the commercial bank to hold sufficient quantity of high-quality liquid assets confronted with liquidity risk. In fact, all these regulations limit the capability of a commercial bank in credit creation. In our model, we only take the reserve requirement into account by assuming that the loanable funds of the bank at period t, $F(t)$, are exactly the excess reserves which can be given by

$$F(t) = (1-\gamma)\cdot M(t) - D(t), \tag{17}$$

where γ is the required reserve ratio, and $M(t)$ represents the aggregate amount of deposits at period t, while $D(t)$ represents the aggregate amount of outstanding loans at period t. The former is at the liability side of the bank while the latter is one kind of its assets.

The interaction between the two parties of the credit market yields the realized credit supply, i.e., the equilibrium value of current borrowing, $B^e(t)$, which is assumed to be simply equal to the minimum one between the quantity demanded and the quantity supplied, that is,

$$B^e(t) = \min(B^p(t), F(t)). \tag{18}$$

The determining process of the equilibrium aggregate borrowing in the credit market is shown in Fig. 2. So as to derive the realized loan of each trader from

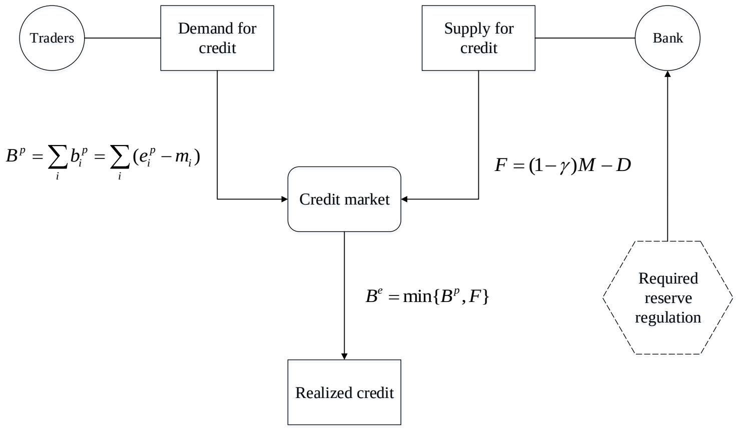

Figure 2: Flowchart of the credit transaction process

the micro perspective, we have to split in the following situations: (i) $B^p(t) > F(t)$, (ii) $B^p(t) \leq F(t)$. If $B^p(t) > F(t)$, then $B^e(t) = F(t)$. In this case, the demand for credit of each trader cannot be fulfilled. We then assume each unit of demand is proportionally achieved, and the ratio k is simply presupposed by

$$k = F(t)/B_p(t). \tag{19}$$

And for each borrower, his realized loan at period t, represented by $b_i(t)$ is assumed to be

$$b_i(t) = k \cdot b_i^p(t). \tag{20}$$

Otherwise if $B^p(t) \leq F(t)$, then we have $B^e(t) = B^p(t)$, and the demand for credit of each trader is certainly to be fulfilled. Therefore, the real loan of each trader is equal to his expected one,

$$b_i(t) = b_i^p(t). \tag{21}$$

3.3 Credit creation process

As the credit creation theory of banking argues, bank creates money through lending. When someone asks for one unit of loan to the bank, two actions will take place simultaneously. The bank not only adds one unit to his asset account, his liability would also be added by the same amount. From such a twinborn variation in the bank balance sheet resulted from behavior of bank lending, we can easily conclude that the aggregate money amount in the economy will increase by one unit, so do the aggregate amount of debt. Conversely, when someone repays one unit of his debt, both the asset and liability sides in the bank balance sheet would be decreased by one unit, after which we can easily see that the aggregate amount of money as well as the aggregate amount of debt is reduced by the same amount. The core of credit creation process is that lending creates credit while repaying annihilates it. Fig. 3 demonstrates how the change in credit stock is governed by an inflow of bank lending and an outflow of repayment, that is

$$\frac{dM}{dt} = \frac{dD}{dt} = B - R, \tag{22}$$

where D represents the amount of credit in the economy, and B is the borrowing or lending flow, R is the repayment flow. Since money and debt are twinborn products of credit creation, both of them are then determined in the same way as described by Equation (22).

This dynamic process holds not only in the case of aggregate level, but also in the case of individual level. Taking trader i as an example, both the income determination process and the credit transaction process would affect his amount of money. To be more specific, his amount of money would be increased by the income while decreased by the expenditure. Besides, it could also be increased by the lending while decreased by the repayment. That is,

$$\frac{dm_i(t)}{dt} = y_i(t) - e_i(t) + b_i(t) - r_i(t), \tag{23}$$

where $m_i(t)$ denotes the amount of money of trader i at period t, and $y_i(t)$ denotes the income, $e_i(t)$ denotes the expenditure, $b_i(t)$ denotes the amount of current loan, $r_i(t)$ denotes the repayment.

As for the dynamics of the debt amount of trader i, represented by $d_i(t)$, it could be increased by the lending while decreased by the repayment, which can be given by

$$\frac{dd_i(t)}{dt} = b_i(t) - r_i(t). \tag{24}$$

The foregoing paragraphs have discussed how the income $y_i(t)$ and expenditure $e_i(t)$ are determined, also the question how the realized loan $b_i(t)$ comes into being

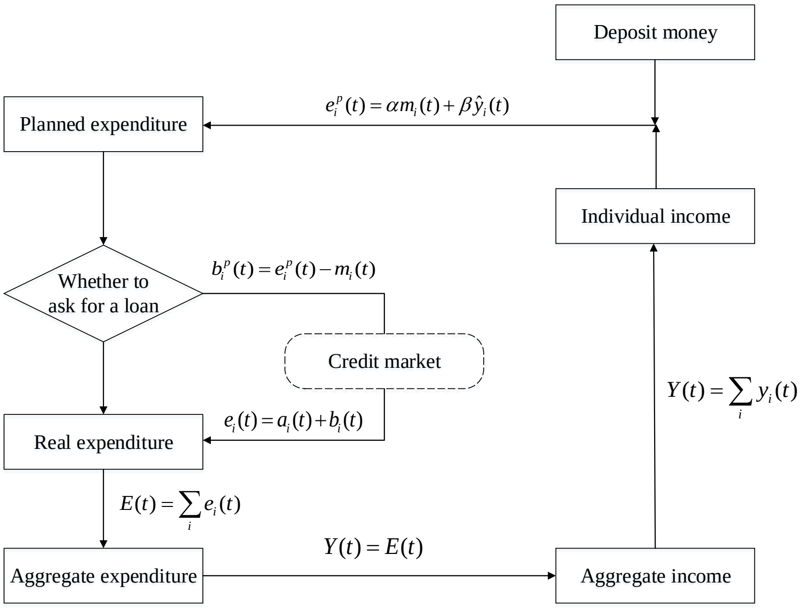

Figure 3: Flowchart of the credit creation process

in the credit market has already been mentioned. However, so far the repayment $r_i(t)$ has not been described in details yet. In order to formulate the repayment, we assume that each trader has the obligation to repay his matured debt at every period. The matured debt of trader i at period t can be depicted by a proportion to his outstanding loan, that is, $\lambda \cdot d_i(t)$. Considering how much money trader i has in his account, we then have to confront two cases: (i) $m_i(t) \geq \lambda \cdot d_i(t)$; (ii) $m_i(t) < \lambda \cdot d_i(t)$. If $m_i(t) \geq \lambda \cdot d_i(t)$, which means the money in trader i's account is enough to cover his obligation of repayment, he would repay this certain proportion of his debt at current period, so

$$r_i(t) = \lambda \cdot d_i(t). \tag{25}$$

Otherwise, i.e.,$m_i(t) < \lambda \cdot d_i(t)$, in which case even if trader i wiped out his account he could not accomplish his obligation of repayment. We then assume that trader i would do as much as he can to repay his debt, and under this circumstance, the repayment $r_i(t)$ can be given by

$$r_i(t) = m_i(t). \tag{26}$$

4 Simulation Results

Based on the model described above, we performed several computer simulations. The initial setting of the system is as the following: the number of traders $N = 1000$ and each trader has a given amount of money $m_i(0) = 10$. Then the traders will decide how much to spend and make transactions with each other. During this money transferring process, the amount of money for each agent will change over time.

4.1 Equilibrium volume of credit

Firstly, we would like to observe the evolution process of macroeconomic variables in each simulation including the aggregate amount of money, the aggregate amount of debt and total income. Setting $\alpha = 0.5$, $\beta = 0.8, \gamma = 0.1$, and $\lambda = 0.1$, we run the 1000-agent system, and the evolutions of these variables over time are respectively illustrated in Fig. 4 and Fig. 5. As shown in Fig. 4, both money and debt grow over time in the beginning, later attain their corresponding equilibria. Please note that the difference between the equilibrium value of money and debt is actually equal to the amount of base money.

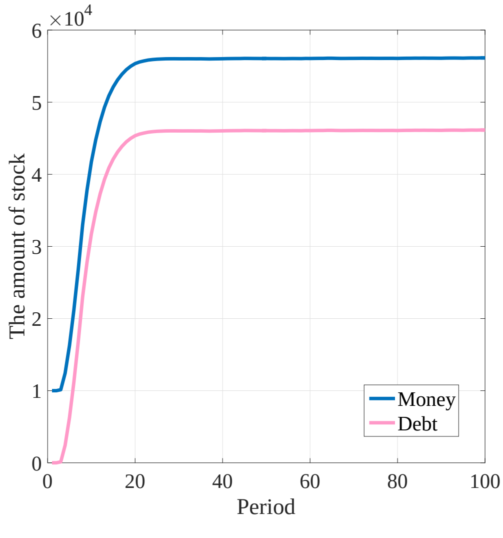

Figure 4: The evolution process of both money and debt, with $\alpha = 0.5$, $\beta = 0.8$, $\gamma = 0.1$, $\lambda = 0.1$. The blue curve represents the evolution of money over time while the pink curve represents that of debt.

With the same initial settings, we also followed the evolution of the aggregate income or that of expenditure and plot it in Fig. 5. It is obvious that there also exists an equilibrium for total income or expenditure.

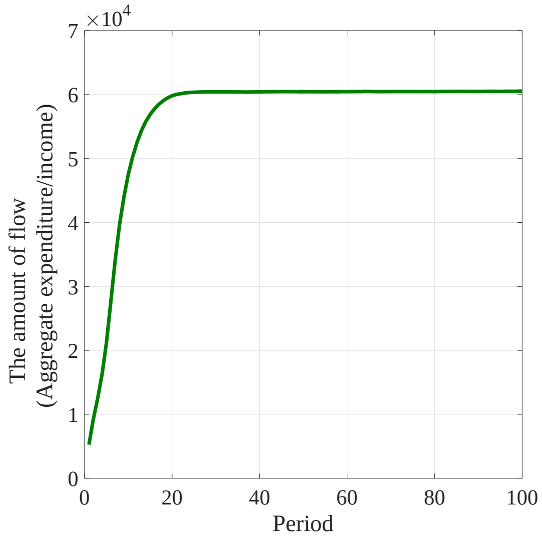

Figure 5: The evolution process of aggregate expenditure or income, with $\alpha = 0.5$, $\beta = 0.8$, $\gamma = 0.1$, $\lambda = 0.1$. The green curve represents the evolution of aggregate expenditure or aggregate income over time.

Secondly, we shift our focus from the evolutions of economic variables to their final equilibrium values. In particular, we would check under what conditions the traders would eventually resort bank to finance their expenditures. We identify the conditions by varying the expenditure propensities and see how the values of the two parameters of α, and β determine the number of people who may borrow money from the bank under the given settings. To be more specific, we set $\gamma = 0.1$ and $\lambda = 0.1$, then we run the system for discrete values of α, β within the interval $[0,1]$ and record the number of borrowers once the system reaches its equilibrium state. As shown in Fig. 6, the color in each grid indicates the number for the corresponding setting. From this figure we can see the whole space prescribed by the two parameters is clearly divided into two phases: white and black areas. In the white area, none of the traders would have the needs to borrow, so we name it as no-indebtedness area. Similarly, the black area means that all the agents would have the desire to hold a debt, we call it all-indebtedness area correspondingly.

In the no-indebtedness area, both the marginal propensity to spend with respect to wealth and that with respect to income have small values, in which case the agents' propensities to consume are not adequate to prompt them to apply for bank loans. As a result, only the money issued initially circulates in the system. Nevertheless, in the all-indebtedness area, high propensities to consume have motivated agents to hold bank loans in order to finance their much higher expenditures. In this situation, money and debt are simultaneously created for agents initiate their borrowing, and both of them are circulating in the system, facilitating the transactions.

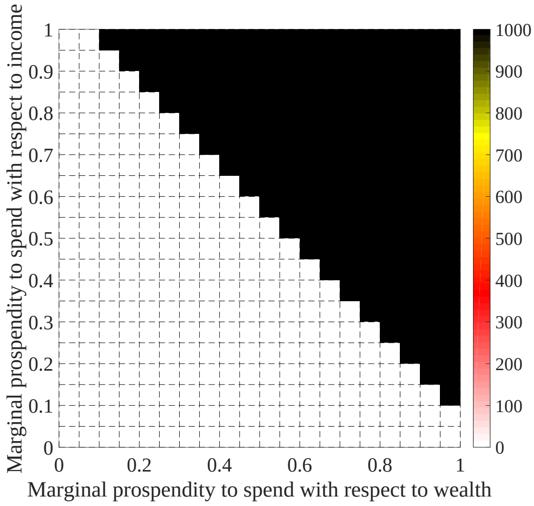

Figure 6: Number of agents who borrow as a function of α and β with $\gamma = 0.1$, $\lambda = 0.1$. The system has been separated into two areas, the white one is called no-indebtedness area while the black one is called all-indebtedness area.

4.2 Holding time and its distributions

No matter how much of money is initially put into the system or how much additional money is later created by the borrowing behavior of the traders, they are always circulating among the agents. If we track anyone unit of money, you can see it is constantly transferred from one to another. During the two consecutive transfers, it will be held in one agent's hand. As illustrated in Fig. 7, the holding time of each unit of money is defined by the time interval between the moment it reaches the hand of one agent and when it leaves his pocket or account (Wang et al., 2003; Ding et al., 2003; Xi et al., 2005). It is obvious that the holding time of money changes over time and varies among agents. The continuous transfers of all money in this system form the circulation of money.

Similarly, the holding time of debt is defined by the time interval between the moment one unit of debt is created while one agent is thus indebted and when it is repaid to the bank meanwhile it is destroyed. The illustrative definition of holding time of debt is shown in Fig. 8, which obviously shows that the transfer of debt must be implemented by the bank instead of traders themselves. Likewise, the continuous transfers of all debts in the system yield the circulation of debt, during which the holding time of debt also changes over time and among debtors. It is worth noting that the intermediation of banks plays an essential role in debt circulation process.

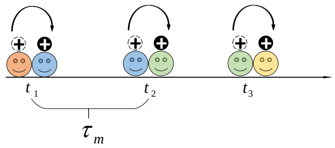

Figure 7: An illustrative definition of holding time of money, τ_m.

We have put forward a prototype model where both the quantity of money and its velocity are determined by expenditure behaviors and credit creation of banks (Xiong et al., 2017). In the microscopic view, we can follow that the transferring of money among agents and obtain the holding times of them. In the macroscopic view, the speed of these transfers can be characterized by a macro variable called the velocity of money, which can be expressed in terms of the holding time as follows,

$$V_m = \frac{1}{\overline{\tau_m}}, \tag{27}$$

where $\overline{\tau_m}$ represents the average value of all the holding times of money (Wang et al., 2003). Due to differences between individuals' choices and randomness of

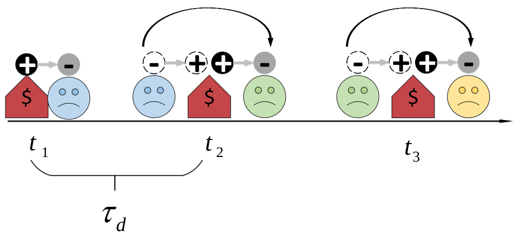

Figure 8: An illustrative definition of holding time of debt, τ_d.

exchanges, the average holding time can be formulated generally by the statistical distribution of holding times $P(\overline{\tau_m})$ in the following way

$$\overline{\tau_m} = \sum_{0}^{\infty} P(\tau_m)\tau_m d\tau_m, \tag{28}$$

where τ_m represents the holding time of money. The Equations (27) and (28) indicates the way that the velocity of money as one presentation of money circulation in the macro perspective connects with the holding time of money which characterizes the circulation of money in the micro one.

Following the line of our previous works, we can also look into the debt circulation by demonstrating its holding time distribution. Similar to the formula of the velocity of money presented above, the velocity of debt can also be expressed as the reciprocal of the average holding time of debt, that is

$$V_d = \frac{1}{\overline{\tau_d}}, \tag{29}$$

where the average holding time $\overline{\tau_d}$ is given by

$$\overline{\tau_d} = \sum_{0}^{\infty} P(\tau_d)\tau_d d\tau_d, \tag{30}$$

where τ_d represents the holding time of debt. The circulation of debt obviously has its both macro and micro representations.

In order to investigate money circulation in the no-indebtedness area, and debt circulation in the all-indebtedness area respectively, we choose certain pairs of α, β according to the two distinct areas in Fig. 6, and perform three groups of simulations.

The way of money circulation can be demonstrated by the distribution of holding time of money in the no-indebtedness area. In the first group of simulations, we track every unit of money during the transaction process and collect the data of holding times regardless whose hand it is staying on. We choose several different pairs of α and β, and fix both the repayment behavior and the reserve ratio to run the model. As shown in Fig. 9, each curve with different color corresponds to each given pair of α and β, and the inset demonstrates the same curves in the logarithmic coordinates. In the semi-logarithmic plot, all the holding time distributions are almost straight lines, indicating they take exponential form given by

$$P(\tau_m) = exp(-a \cdot \tau_m), \tag{31}$$

where the exponent a, the slope of the line, is actually the velocity of money circulation. Thus it can be obtained by fitting the corresponding exponential distribution for each setting. We show the values of velocity of money circulation under different pairs of α and β obtained from simulation results in Table 1 of Appendix. We can draw a straight conclusion that the velocity of money circulation increases as α or β gets larger. Since both α and β denote the propensities to expend for all traders, the larger α and β are, the larger the volume of transaction is. As a result, money would change hands in a higher rate, the velocity would be thereby higher.

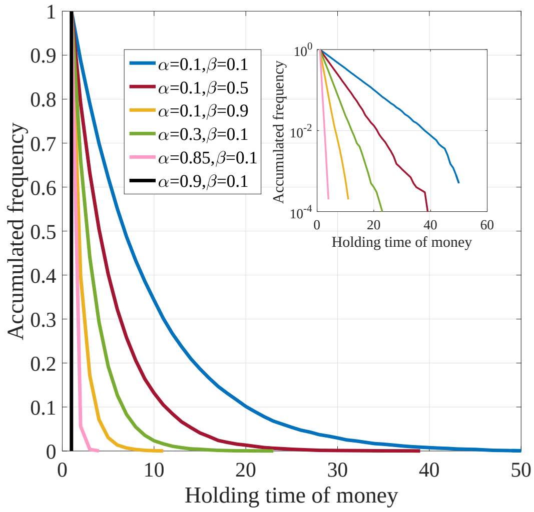

Figure 9: Holding time distribution of money as a function of α and β with $\gamma = 0.1$, $\lambda = 0.1$. Each curve with different color represents the result for each pair of α and β, and the inset demonstrates the same curves in the semi-logarithmic coordinates.

Similar to the approach to observing the money circulation from micro perspective presented above, debt circulation in the all-indebtedness area can also be exhibited by the distribution of holding time of debt. In the second group of simulations, we choose different pairs of α and β, and fix both the repayment behavior and the reserve ratio to run the model, while tracking every unit of debt and collecting the data of holding times. The holding time distributions for different settings are shown in Fig. 10, where the inset demonstrates the same curves in the semi-logarithmic coordinates. It is obvious that the curves of holding time distribution also appear as straight lines in the inset panel, indicating that they take the following expression

$$P(\tau_d) = exp(-b \cdot \tau_d), \tag{32}$$

where the exponent b corresponds to the velocity of debt circulation. By fitting the holding time distribution curves we can obtain the corresponding exponents, which are displayed in Table 2 of Appendix. From Fig. 10 and results in Table 2, we can easily perceive that the curves for different settings almost collapse into the same line, and the simulated velocities of debt circulations are pretty close with each other. This fact implies that expenditure propensities are not the major factors that drive debt circulation, which can be interpreted from the following two aspects. Firstly, expenditure propensities are proved to affect money circulation process through expending channels. Since money circulation is a direct transfer of money, increasing α and β is equal to expanding the scale of each transfer. Secondly, although higher expenditure propensities may cause the traders to borrow money from the bank, this would result in not only more borrowing but also more debt, yielding the same velocity of its circulation. In the equilibrium state, the stock of debt is eventually determined by the required reserve ratio and the amount of monetary base. On the other side, the expenditure aroused by borrowing is governed by the repayment of debtors, which is prescribed by a certain proportion of the volume of debt. Both are not relevant with the expenditure propensities of the traders. Thus we can exclude them from our consideration as main factors of debt circulation, because here expenditures only play as a stimulus to borrowing of traders. This cognition has motivated us to do several other simulation experiments to find the main driving force of debt circulation.

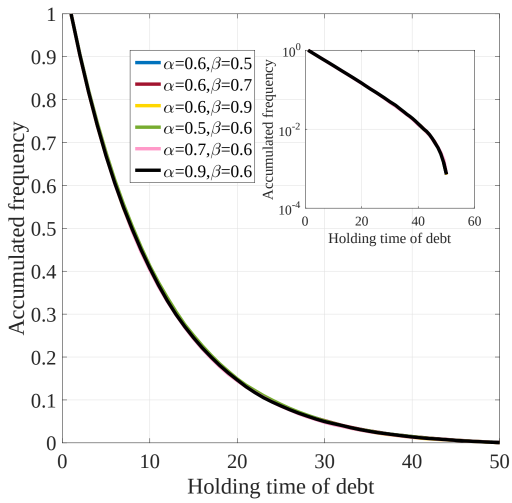

Figure 10: Holding time distribution of debt as a function of α and β with $\gamma = 0.1$, $\lambda = 0.1$. Each curve with different color represents the result for each pair of α and β, and the inset demonstrates the same curves in the logarithmic coordinates.

When we turn to the credit creation process, as we have mentioned above, the amount of money that traders could borrow from the bank is constrained by its capability of credit supply. As the system falls into the equilibrium state, where the total volume of outstanding loans reaches a constant level, and the borrowing is exactly equal to the repayment of debtors. This fact tells us that as the transaction continues, the bank will eventually run out of its capacity of credit and then resorts to only the money that is currently paid back from the debtors. Therefore, the expenditure financed by debt strongly depends on the repayment, implying the repayment function plays significant part in debt circulation. In our model, λ is set to describe the repayment behavior, specifically, it represents the mature rate of debt, which means how much each agent has to repay with regard to his outstanding debt. According to this implication, in our third group of simulations, we choose different values of λ, and fix both the expenditure behavior and the required reserve ratio to run the model and collect the data of holding time of debt. As shown in Fig. 11, each curve with different color represents the corresponding curve for different values of λ, and the inset demonstrates the same curves in semi-logarithmic coordinates. It is obvious that the holding time distributions of debt have also exhibited exponential form. Following the same method of calculation, we can obtain the corresponding simulated velocity of debt, which are displayed in Table 3.

All these fitting results of simulations indicate that the velocity of debt circulation increases as λ gets higher. As elaborated above, debt creation takes place when traders borrow money from the bank while debt annihilation happens when they repay them back. The length of this time interval characterizes how fast that debt moves from the "old" borrowers to the "new" borrowers even though this transfer is accomplished with the intermediation of the bank. Hence, the repayment behavior dominates debt circulation, so it can be understood that λ has a significant effect on the holding time of debt.

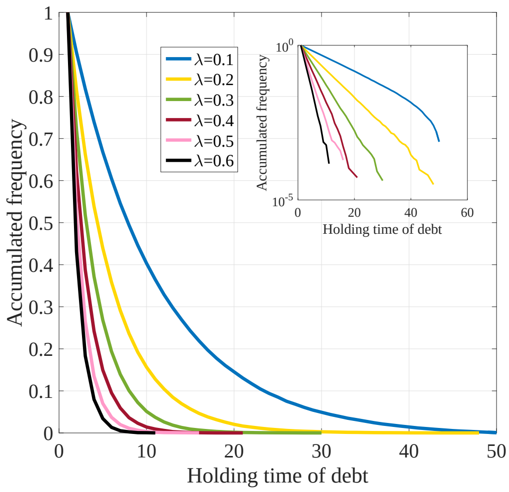

Figure 11: Holding time distribution of debt as a function of λ with $\alpha = 0.8$, $\beta = 0.9$, and $\gamma = 0.1$. Each curve with different color represents the curve for different value of λ, and the inset demonstrates the same curves in the semi-logarithmic coordinates.

5 Comparison between theoretical analysis and simulation results

As mentioned in Section 2, we have proposed the new *quantity theory of credit* given by Equation (7). In the following theoretical analysis, we set $P = 1$ for the sake of simplicity, then we have

$$Y = M \cdot V_m + D \cdot V_d. \tag{33}$$

In the case of no-indebtedness, no trader needs to borrow money and thus no debt is generated, so we have $D = 0$, and the above equation can be simplified as

$$Y = MV_m. \tag{34}$$

In this case, the realized expenditure of each trader is equal to his planned amount, so we can have the following form from Equation (9),

$$e_i(t) = \alpha m_i(t) + \beta y_i(t-1). \tag{35}$$

Summing up over all traders we can get

$$E(t) = \sum_{i=1}^{N} e_i(t) = \alpha \sum_{i=1}^{N} m_i(t) + \beta \sum_{i=1}^{N} y_i(t-1), \tag{36}$$

so

$$E(t) = \alpha M(t) + \beta Y(t-1). \tag{37}$$

Consider both equilibrium conditions of $Y = E$ and $Y(t) = Y(t-1)$, we can obtain the expression of aggregate income at equilibrium state as follows

$$Y_e = \frac{\alpha M}{1-\beta}. \tag{38}$$

Combining Equations (34) and (38) yields the equilibrium value of the velocity of money circulation denoted by V_m, given by

$$V_m = \frac{Y_e}{M_e} = \frac{\alpha}{1-\beta}. \tag{39}$$

This result indicates that the velocity of money is governed by the expending behavior of traders which is characterized by the two parameters α and β.

When we look into the money circulation process in the case of no-indebtedness from the micro-perspective, the simulations show that the distribution of holding time of money presents an exponential form, which can be formulated as Equation (31) shows. Then the expected value of holding time, also the average holding time of traders ($\overline{\tau_m}$), can be calculated as Equation (28) demonstrates, and the expected value should be equal to the reciprocal of the exponent a as follows,

$$\overline{\tau_m} = \frac{1}{a}. \tag{40}$$

In our simulations in the no-indebtedness area, the velocity of money circulation can be expressed in terms of the average holding time of traders ($\overline{\tau_m}$) according to Equation (27). Substituting Equation (40) to (27), we can get

$$V_m' = a. \tag{41}$$

We vary the value of α and β, and plot the theoretical results of the velocity of money given by Equation (39) and the simulation results given by Equation (41) in the same figure, as shown in Fig. 12. It is obvious that each point almost falls on the diagonal line, implying the simulation results match the theoretical results very well.

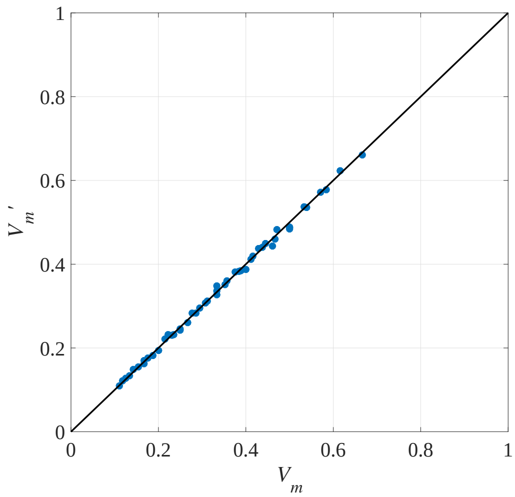

Figure 12: Comparison between theoretical results and simulation results with $\gamma = 0.1$, $\lambda = 0.1$. $V_m = \frac{\alpha}{1-\beta}$, and is calculated in different values of α and β as the horizontal value of each blue point. $V_m' = \frac{1}{\overline{\tau_m}}$, and is the expected value of holding time based on a number of simulations as the vertical value of each blue point. The dark line is the diagonal line in this coordinate.

As for the all-indebtedness area, the aggregate expenditure can be obtained by summing up those of all individual traders, which takes the following form

$$E(t) = \sum_{i=1}^{N} e_i(t) = \sum_{i=1}^{N} m_i(t) + \sum_{i=1}^{N} b_i(t). \tag{42}$$

Thus we have

$$E(t) = M(t) + B(t). \tag{43}$$

When the economy falls into an equilibrium state, the aggregate borrowing of traders comes from the aggregate repayment of the debtors, that is, $B_e = R_e$, and the aggregate income can be thereby written as

$$Y_e = M_e + R_e. \tag{44}$$

Comparing Equation (44) with (33), we can apparently find that in this case, the velocity of money circulation is explicitly

$$V_m = 1, \tag{45}$$

which implies that the holding times of all units of money in this situation are equal to one. We further testify this theoretical prediction by collecting holding times of money in the corresponding simulations. We choose one pair of α and β from the all-indebtedness area, and then we track the money. In this case, the holding times of money are shown in Fig. 13, from which we can easily draw a conclusion that all the holding times are located at one point of 1. The concentration of holding time distribution is obviously caused by the rule of borrowing of traders. Once a trader finances his expenditure by the bank loan, he must have run out of his own account in the first place, indicating that if a unit of money reaches the hand of a trader, it would move to the hand of another trader in the next time period immediately. Accordingly, money would not stop moving from one trader to another one in the all-indebtedness area, which can also be expressed by

$$\tau_m = 1. \tag{46}$$

That is to say, the velocity of money is accelerated to the extreme case where its value is given by

$$V'_m = \frac{1}{\overline{\tau_m}} = 1. \tag{47}$$

Regarding the velocity of debt circulation, if all debtors could repay their principles as requested by the contract they have signed, according to Equations (33) and (44), we can get

$$V_d = \frac{R_e}{D_e} = \lambda. \tag{48}$$

In fact, the velocity of debt circulation can be derived from the holding time distribution of debt as shown in Fig. 11. Since the distribution exhibits an exponential form, it can also be expressed by Equation (32). Then the expected value of holding time of debt, also the average holding time of it ($\overline{\tau_d}$), can be calculated as Equation

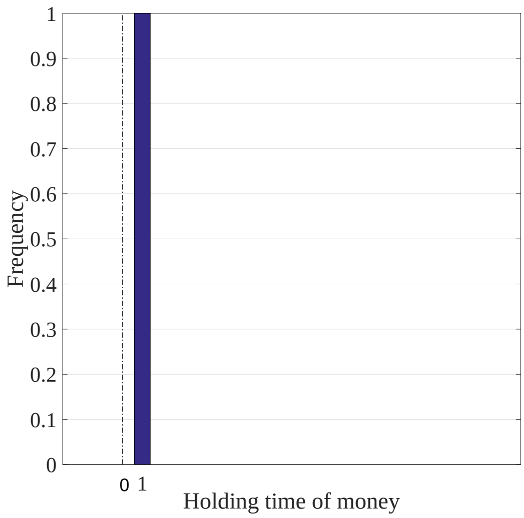

Figure 13: Holding time distribution of money with $\alpha = 0.8$, $\beta = 0.9$, $\lambda = 0.1$ and $\gamma = 0.1$. The holding time of each unit of money is equal to one.

(30) demonstrates, and the expected value should be equal to the reciprocal of the exponent b as follows,

$$\overline{\tau_d} = \frac{1}{b}. \tag{49}$$

Thus the simulated velocity of debt circulation could be expressed in terms of the average holding time ($\overline{\tau_d}$) according to Equation (29). Substituting Equation (49) to (29), we can get

$$V_d' = b. \tag{50}$$

By varying the value of λ, we again plotted the theoretical results and the simulation results in the same figure, as shown in Fig. 14. We can see that each point almost falls below the diagonal line, implying the simulated velocity of debt is always less than the theoretical one.

The reason for these deviations is that not all the debtors have the capability to make repayment given by Equation (25). In certain cases, some debtors could not receive enough income to cover their required repayment, and they use all receipt to repay a portion of their obligations corresponding to Equation (26). Then the real aggregate repayment is less than the required one. In order to describe the degree that the debtors have fulfilled their debt services, we introduce the ratio of realized repayment (R) to the required repayment (λD), denoted by c, calculated by the following

$$c = \frac{R}{\lambda D}. \tag{51}$$

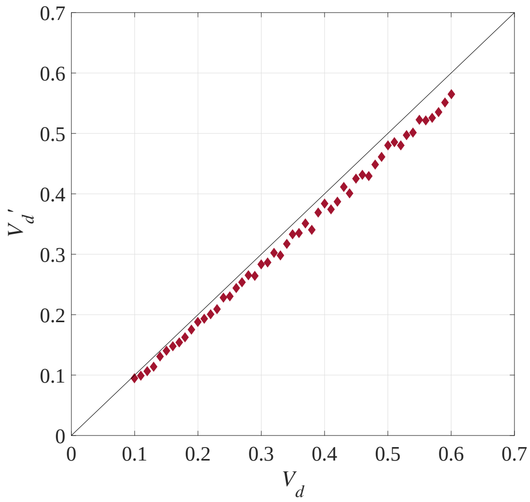

Figure 14: Comparison between theoretical results and simulation results with $\alpha = 0.8$, $\beta = 0.9$ and $\gamma = 0.1$. $V_d = \lambda$, and is calculated in different values of λ as the horizontal value of each brown point. $V_d' = \frac{1}{\tau_d}$, and is the expected value of holding time based on a number of simulations as the vertical value of each brown point. The dark line is the diagonal line in this coordinate.

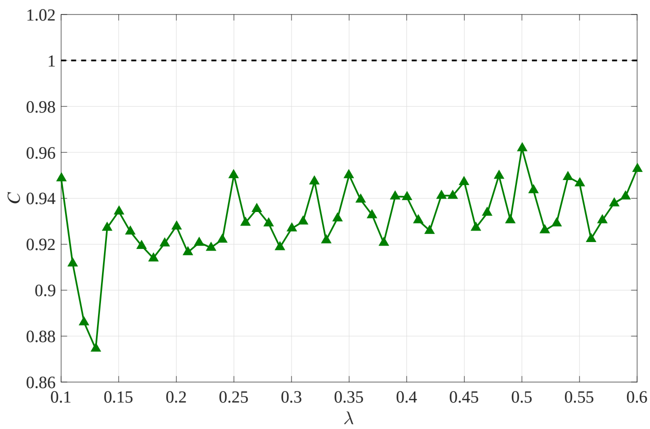

Figure 15: Ratio of realized repayment to the required repayment as a function of λ with $\alpha = 0.8$, $\beta = 0.9$ and $\gamma = 0.1$. Here, c can be obtained by $c = \frac{R}{\lambda D}$, which is always below 1.

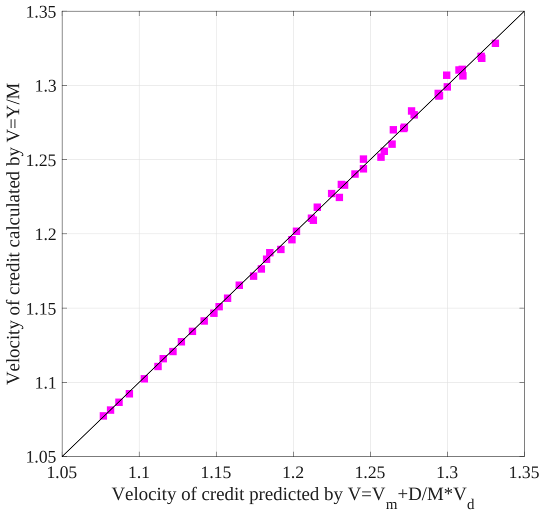

Figure 16: Comparison between two expressions of velocity with $\alpha = 0.8$, $\beta = 0.9$ and $\gamma = 0.1$. One of the expression is that $V = V_m + \frac{D}{M} V_d$, taking as the horizontal value of each purple point, and the other expression is that $V = \frac{Y}{M}$, taking as the vertical value of each purple point.

Again, we set $N = 1000$, $m_i(0) = 10$, $\alpha = 0.8$, $\beta = 0.9$, $\gamma = 0.1$, and we value different values of λ to run the model. As Fig. 15 shows, c would fluctuate while λ differs, but it is always below one, suggesting that real repayment could not reach the required one at all times. We can see about $6-7\%$ gap always remains in the figure.

In Section 2, we have obtained the mathematical expression of the credit-derived velocity of money circulation, as shown in Equation (8), which provides a way to predict the credit-derived velocity of money circulation if the corresponding velocities of money and debt are already known. We set $\alpha = 0.8$, $\beta = 0.9$, $\gamma = 0.1$ and differ λ to run several simulations. As shown in Fig. 16, the horizontal axis shows the credit-derived velocity of money circulation given by Equation (8), while the vertical axis demonstrates Fisher's velocity of money circulation given by Equation (3). It is obvious that each purple point almost falls below the diagonal line, implying that those two expressions of velocity are approximately equivalent.

6 Conclusion

Money circulation has long been recognized by economists and used to formulate aggregate economic activity. In this paper, we analogously put forward the concept of debt circulation and demonstrate its existence. A multi-agent model is developed in which money can be transferred among traders, while debt can be transferred among debtors. The money circulation is driven by continuous earning and spending of traders, and the circulation of debt is formed by iteration of loan granting and repayment. Just like money circulation, debt circulation also contributes to the aggregate income.

In this agent-based system, there exist the credit creation process and the income determination process, the former is a process of stock accumulation and the latter is a process of flow iteration. Both stock equilibrium and flow equilibrium could be eventually achieved. In this equilibrium state, the expenditure habits of the households determine the extent to which the economy relies on debt. When the traders' propensities are high enough, almost all of them would be indebted. As a result, the aggregate equilibrium income comes from not only money circulation but also debt circulation, which is actually generated by the iteration of credit creation and annihilation. The introduction of debt circulation calls for reinterepting *the quantity theory of money*, which originally states that the nominal income is the aggregate amount of money multiplies the velocity of money. In fact, as banks create debt by lending, the income turns out to have two sources: one is product of the aggregate amount of money and its velocity of circulation, the other one is the product of the aggregate amount of debt and its corresponding velocity of circulation. The theoretical hypothesis on the decomposition of aggregate equilibrium income into these two kinds of flows is verified by the computer simulations.

Following the money that is transferred among traders we can measure the holding times of money of anyone trader. Likewise, following the debt that is created and then annihilated, we can measure the holding times of debt for any given debtor. The holding time of money is defined as the time interval between the receipt of money and dispense of money, while the holding time of debt is defined as time interval between credit creation and its annihilation. The concept of holding time is another way to characterize the circulations of money and debt from micro perspective. By computer simulations with certain settings, the data of holding times of money and debt are collected. It is found that both holding time distributions of money and debt take exponential form, due to the random nature of exchanges. The velocities of money and debt circulations can be obtained respectively from the corresponding holding time distributions. It is found that the velocity of money circulation is governed by the expenditure habits of traders, while the velocity of debt circulation is determined by the repayment behavior of debtors.

Although we presume that the borrowing is motivated by the over-expenditure behavior of traders in this work, this assumption can be of course relaxed to include over-consumption induced debt and investment-financing induced debt. The main conclusions we have drawn in this model will still be true and can be applied to study more realistic macroeconomic issues.

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Appendix

Table 1: Comparison between simulation results of velocity of money circulation and theoretical ones calculated from Equation (39).

Parameters	Simulated V_m	Calculated V_m
$\alpha = 0.1$, $\beta = 0.1$	0.1117	0.1111
$\alpha = 0.1$, $\beta = 0.5$	0.2035	0.2000
$\alpha = 0.1$, $\beta = 0.8$	0.4781	0.5000
$\alpha = 0.3$, $\beta = 0.1$	0.3400	0.3333
$\alpha = 0.85$, $\beta = 0.1$	0.9432	0.9444
$\alpha = 0.9$, $\beta = 0.1$	1.0000	1.0000

Table 2: Simulation results of velocity of debt circulation.

Parameters	Simulated V_d
$\alpha = 0.6$, $\beta = 0.5$	0.0969
$\alpha = 0.6$, $\beta = 0.7$	0.0975
$\alpha = 0.6$, $\beta = 0.9$	0.0976
$\alpha = 0.5$, $\beta = 0.6$	0.0913
$\alpha = 0.7$, $\beta = 0.6$	0.0981
$\alpha = 0.9$, $\beta = 0.6$	0.0976

Table 3: Comparison between simulation results of velocity of debt circulation and calculated ones according to Equation (48).

Parameters	Simulated V_d	Calculated V_d
$\lambda = 0.1$	0.0981	0.1
$\lambda = 0.2$	0.1857	0.2
$\lambda = 0.3$	0.2807	0.3
$\lambda = 0.4$	0.3806	0.4
$\lambda = 0.5$	0.4835	0.5
$\lambda = 0.6$	0.5712	0.6