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Spurious detection of phase synchronization in coupled nonlinear oscillators

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Coupled nonlinear systems under certain conditions exhibit phase synchronization, which may change for different frequency bands or with the presence of additive system noise. In both cases, Fourier filtering is traditionally used to preprocess data. We investigate to what extent the phase synchronization of two coupled Rössler oscillators depends on (1) the broadness of their power spectrum, (2) the width of the bandpass filter, and (3) the level of added noise. We find that for identical coupling strengths, oscillators with broader power spectra exhibit weaker synchronization. Further, we find that within a broad bandwidth range, bandpass filtering reduces the effect of noise but can lead to a spurious increase in the degree of phase synchronization with narrowing bandwidth, even when the coupling between the two oscillators remains the same.

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In recent years both theoretical and experimental studies of coupled nonlinear oscillators have demonstrated that such oscillators can exhibit phase synchronization [1–5]. Analysis of experimental data has also indicated the presence of phase synchronization in a range of coupled physical, biological, and physiological systems [6–17]. In many of these studies, an important practical question is how multivariate time series characterized by a relatively broad power spectrum are phase synchronized in a specific frequency range [18–24]. The presence of internal or external noise may also be an obstacle when quantifying phase synchronization from experimental data [18,19,25–27]. In both cases a bandpass filter is traditionally applied either to reduce the noise effect or to extract the frequency range of interest. Thus, it is important to know to what extent the width of the bandpass filter influences the results of the phase synchronization analysis, as well as what is the range of the index values obtained from the analysis that indicate a statistically significant phase synchronization.

To address these questions, we consider a system of two coupled Rössler oscillators (1,2) defined as

$$\dot{x}_{1,2} = -\omega_{1,2}y_{1,2} - z_{1,2} + C(x_{2,1} - x_{1,2}),$$

$$\dot{y}_{1,2} = \omega_{1,2}x_{1,2} + ay_{1,2},$$

$$\dot{z}_{1,2} = f + z_{1,2}(x_{1,2} - b) \qquad (1)$$

with parameter values $a=0.165$, $f=0.2$, and $b=10$. For the mismatch of natural frequencies, we choose $\omega_{1,2}=\omega_0 \pm \Delta\omega$, with $\omega_0=0.6$ and $\Delta\omega=0.005$ [Fig. 1(a)]. The time step in our simulation is $\Delta t=2\pi/10^3$, and the signal length $n=\mathrm{int}[t/\Delta t]$ with $t=10^4$, where int$[x]$ denotes the integer part of x.

We first investigate the characteristics of the system defined in Eq. (1) by comparing them with the characteristics of a second set of two coupled Rössler oscillators (3,4) studied in [3]. The system (3,4) is also described by Eq. (1), and has the same values for the parameters a, f, and b as system (1,2). The only differences are the natural frequency $\omega_0=1$ and the frequency mismatch $\Delta\omega=0.015$ [Fig. 1(b)]. We observe a significantly broader power spectrum for system (1,2) with $\omega_0=0.6$ and frequency mismatch $\Delta\omega=0.005$ [Fig. 1(c)]. Further, we observe that the instantaneous phase differences $\Delta\psi_{1,1}=[\phi_{x_1}(t)-\phi_{x_2}(t)] \bmod (2\pi)$ for system (1,2) exhibits larger fluctuations [Fig. 1(d)], described by a broader distribution [Fig. 1(e)], compared to system (3,4), suggesting a weaker 1:1 phase synchronization for system (1,2). To quantify the degree of phase synchronization in the two Rössler systems we use the synchronization index $\rho=(S_{\max}-S)/S_{\max}$ [18], where $S\equiv-\Sigma_{k=1}^{N}P_k \ln P_k$ is the Shannon entropy [28] of the distribution $P(\Delta\psi_{1,1})$ of $\Delta\psi_{1,1}$, and $S_{\max}=\ln N$, where $N=\mathrm{int}\{\exp[0.626+0.4\ln(n-1.0)]\}$ is the optimized number of bins over which the distribution is obtained [29]. For system (3,4) with a narrow power spectrum we obtain a significantly larger value of ρ compared to the system (1,2) characterized by a broader power spectrum [Fig. 1(f)]. Varying the values of the coupling strength C, we find that the phase synchronization index ρ is consistently higher for system (3,4) characterized by the narrower power spectrum. Thus, for the same coupling strength C and for identical other parameters, system (1,2) with $\omega_0=0.6$, which has a broader power spectrum, exhibits weaker synchronization compared to system (3,4) with $\omega_0=1$, which has a narrow power spectrum. These findings are complementary to a recent study indicating a different degree of phase synchronization for the spectral components of coupled chaotic oscillators [30].

Recent work has shown that coupled Rössler oscillators may exhibit different degrees of synchronization for different ranges of time scales obtained via wavelet transform [31]. Here, we ask to what extent the width of a bandpass filter affects the degree of phase synchronization between two coupled Rössler oscillators. While the output observables $x_1$and x_2 of system (1,2) are clearly not in phase [Fig. 2(a)], after Fourier bandpass filtering in the range of $\Delta f=0.01$ centered at the peak of the power spectrum $2\pi f\approx 0.54$ [Fig. 1(c)], the observables x_1 and x_2 appear 1:1 synchronized with well-aligned peaks [Fig. 2(b)]. The effect of the bandpass filter can be clearly seen in the behavior of the instantaneous

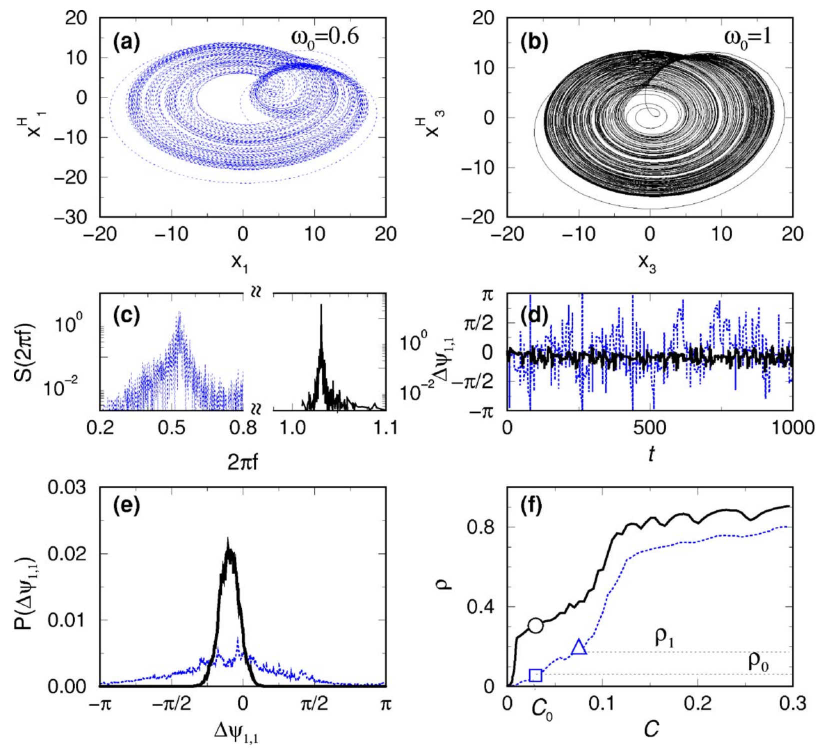

FIG. 1. (Color online) Differences in the synchronization of two Rössler systems with identical coupling strengths and different power spectra. Phase plot trajectories of the variables x vs their Hilbert transform x^H for (a) system (1,2), with x_1 corresponding to $\omega_1=\omega_0+\Delta\omega$, where $\omega_0=0.6$ and $\Delta\omega=0.005$; (b) system (3,4), with x_3 corresponding to $\omega_3=\omega_0+\Delta\omega$, where now $\omega_0=1$ and $\Delta\omega=0.015$. For both Rössler systems $C=0.03$. (c) Power spectra of the time sequence x_1 (dashed line) and x_3 (solid line). A broader spectrum is observed for system (1,2) compared to system (3,4). (d) Instantaneous phase difference $\Delta\psi_{1,1}\equiv(\phi x_1(t)-\phi x_2(t))\ \mathrm{mod}(2\pi)$ for system (1,2) (dashed line), and $\Delta\psi_{1,1}\equiv(\phi x_3(t)-\phi x_4(t))\ \mathrm{mod}(2\pi)$ for system (3,4) (solid line), and (e) their corresponding distributions $P(\Delta\psi_{1,1})$. System (1,2) exhibits larger fluctuations in $\Delta\psi_{1,1}$ and is characterized by a broader distribution $P(\Delta\psi_{1,1})$. (f) Synchronization index ρ as a function of the coupling strength C. For identical values of C, system (3,4) (solid line), which is characterized by a narrower power spectrum, exhibits stronger synchronization (larger index ρ) compared to system (1,2) with a broader power spectrum. Specifically, for identical coupling strength $C=C_0=0.03$, the index $\rho=\rho_0$ (□) for system (1,2), while $\rho=0.3>\rho_0$ (∘) for system (3,4) although the frequency mismatch for system (3,4) is much larger. The effect of a Fourier bandpass filter applied to the system (1,2) while keeping $C=0.03$ fixed is equivalent to an increase of the coupling strength of the system leading to a larger index $\rho_1>\rho_0$ (Δ) as also shown in Fig. 2(e).

phase difference $\Delta\psi_{1,1}$ [Fig. 2(c)] and in the shape of the probability density function $P(\Delta\psi_{1,1}(t))$ [Fig. 2(d)]. After bandpass filtering, $\Delta\psi_{1,1}$ becomes smoother with fewer fluctuations, and the distribution $P(\Delta\psi_{1,1})$ exhibits a more pronounced peak. To quantify how the degree of synchronization changes with the width Δf of the bandpass filter, we calculate the synchronization index ρ [Fig. 2(e)]. We find that for very large values of the bandwidth Δf, the index ρ is the same as the value ρ_0 obtained for the system (1,2) without any filtering, and that ρ remains unchanged for intermediate values of Δf. However, for decreasing Δf, the index ρ increases rapidly from the expected value ρ_0 [Figs. 1(f) and 2(c)]. Such deviation to higher values of $\rho>\rho_0$, while the coupling constant C in Eq. (1) remains fixed, indicates a spurious effect of synchronization due to the bandpass filter. Thus, applying a bandpass filter with a too narrow bandwidth when preprocessing empirical data may lead to overestimation of the phase synchronization (as defined by index ρ) between two empirical systems where the coupling strength is not known *a priori*.

Many physical and biological systems are influenced by external noise, which can mask their intrinsic properties. Recent studies have shown that noise can bias the estimation of the driver-response relationship in coupled nonlinear oscillators leading to change in synchronization measures [32]. Specifically, external noise may weaken the detection of the coupling and reduce the synchronization between two coupled dynamical systems. To address this problem, we next test the effect of external noise on the degree of phase synchronization of the two coupled Rössler oscillators defined in Eq. (1). Adding uncorrelated and unfiltered Gaussian noise η to the output observables x_1 and x_2, while keeping the coupling constant C in Eq. (1) fixed, we find that the synchronization index ρ decreases with increasing noise strength σ_η (i.e., higher standard deviation σ_η compared to the standard deviation σ of the output signals x_1 and x_2) [Fig. 3(a)]. The dependence of ρ on the value of the coupling constant C for different noise strength is shown in Fig. 3(b). We find that the transition to the state of maximum degree of synchronization [indicated by a horizontal plateau for ρ in Fig. 3(b)] occurs at decreasing values of the coupling constant C for increasing noise strength σ_η. For very strong noise ($\sigma_\eta=\sigma=8.3$), the two Rössler oscillators in Eq. (1) appear not to be synchronized, characterized by low values for the index ρ, even for very large values of the coupling

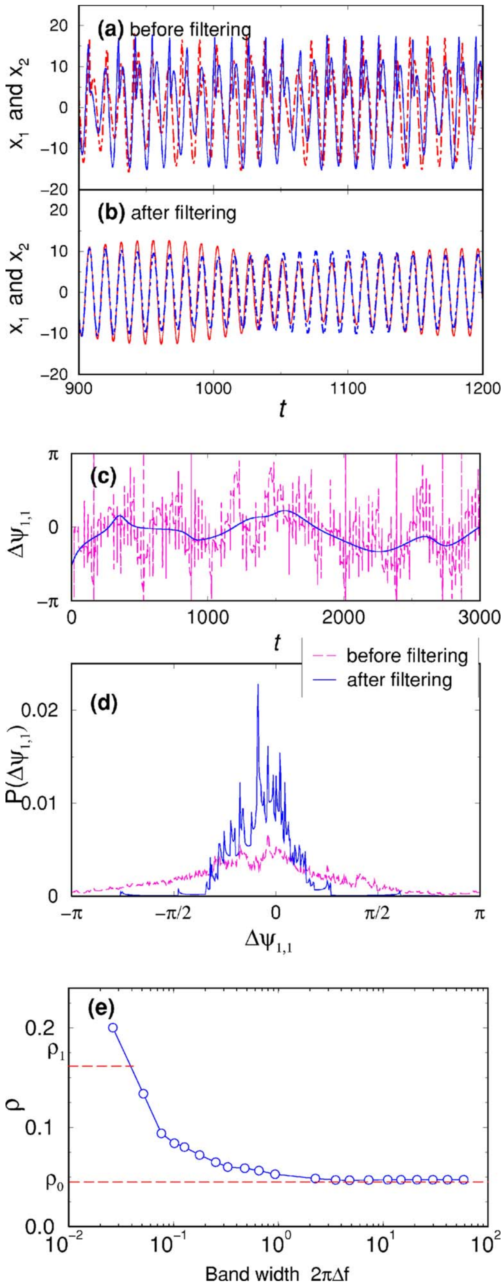

FIG. 2. (Color online) Effects of bandpass filtering on synchronization. Time sequence of the variables x_1 and x_2 of system (1,2) (a) before and (b) after applying a bandpass Fourier filter with bandwidth $\Delta f=0.01$. After bandpass filtering the sequences x_1 and x_2 are better aligned in time (with almost matching peaks). (c) Instantaneous phase difference $\Delta\psi_{1,1}$ and (d) the distribution $P(\Delta\psi_{1,1})$ before (dashed line) and after (solid line) the Fourier bandpass filtering. After filtering, $\Delta\psi_{1,1}$ is characterized by fewer fluctuations and a much narrower distribution $P(\Delta\psi_{1,1})$, indicating a stronger synchronization, although the coupling strength $C=0.03$ remains constant. (e) Dependence of the index ρ on the bandwidth $2\pi\Delta f$ for fixed $C=0.03$. A filter with a relatively broader bandwidth ($2\pi\Delta f > 1$) leaves the synchronization index ρ practically unchanged, $\rho = \rho_0$, where ρ_0 characterizes the synchronization between x_1 and x_2 before filtering. Narrowing Δf leads to a sharp increase in ρ, which is an artifact of the Fourier filtering as the coupling C and all other parameters remain unchanged, e.g., for $\Delta f=0.005$, $\rho=\rho_1 \approx 4\rho_0$.

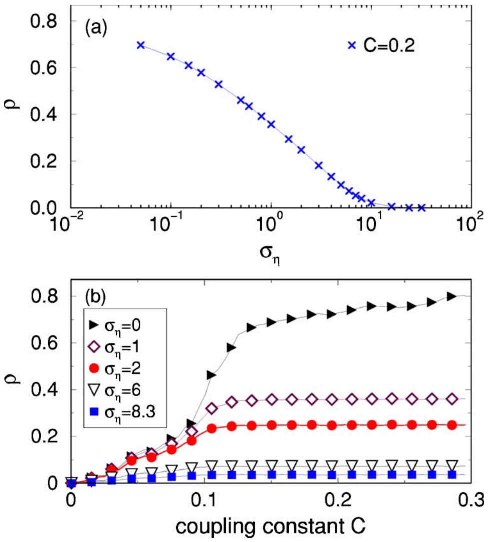

FIG. 3. (Color online) Effect of external additive white noise on phase synchronization for system (1,2) defined in Eq. (1). (a) Dependence of the synchronization index ρ on the noise strength σ_η for fixed value of the coupling constant C. (b) Dependence of the synchronization index ρ on the coupling strength C for different levels of white noise which are defined through the standard deviation σ_η.

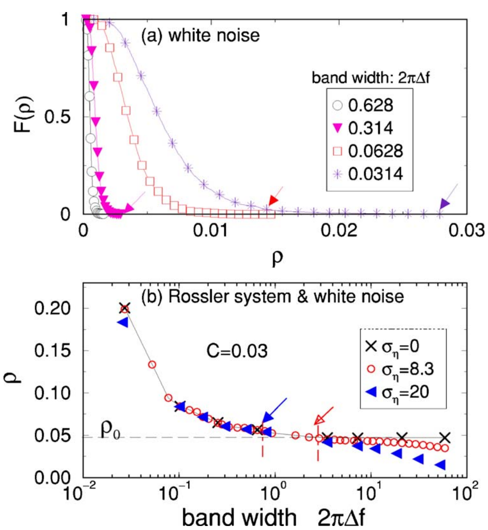

FIG. 4. (Color online) Combined effects of external noise and Fourier bandpass filtering on the synchronization. (a) Cumulative distribution function $F(\rho) \equiv 1-\int_0^\rho P(\rho')d\rho'$ for the synchronization index ρ obtained from 100 different realizations of pairs of white noise signals without coupling. The length of the noise signals is $\mathrm{int}[10^7/2\pi]$. Tails of the distributions for each bandwidth indicate the maximum values of ρ one can obtain simply as a result of bandpass filtering when there is no synchronization between two white noise signals. (b) Synchronization index ρ obtained for system (1,2) defined in Eq. (1) with additive white noise as a function of the bandwidth Δf for $C=0.03$. While the effect of noise is gradually reduced by the Fourier bandpass filter with decreasing bandwidth Δf, there is an artificially increased synchronization (sharp increase in ρ) when $2\pi\Delta f < 1$, as also shown in Fig. 2(e).

constant C [Fig. 3(b)]. We note, that with increasing noise strength σ_η the position of the crossover to the plateau of maximum synchronization shifts to smaller values of C in Fig. 3(b), indicating that with increasing σ_η the level of the plateau drops faster compared to the decline in the growth of ρ with increasing coupling C.

To reduce the effect of noise in data analysis, a common approach is to apply a bandpass filter. In the case of the coupled Rössler oscillators defined in Eq. (1), we next ask to what extent a bandpass filter can reduce the effect of external noise while preserving the expected "true" phase synchronization as presented by ρ_0 in Fig. 1(e). To answer this question, we first need to determine what are the limits to which spurious phase synchronization can be obtained purely as a result of bandpass filtering of two uncorrelated and not coupled Gaussian noise signals. Our results for the synchronization index ρ obtained from multiple realizations of pairs of uncoupled white noise signals show that the synchronization index ρ can reach different maximum values ρ_{max}, indicated by arrows in Fig. 4(a), for different bandwidth Δf—with decreasing bandwidth ρ_{max} increases. The values of ρ_{max} provide an estimate of the maximum possible effect additive noise may have on the spurious "detection" of phase synchronization in coupled nonlinear oscillators. Thus, empirical observations of synchronization index $\rho > \rho_{max}$ may indicate presence of a genuine phase synchronization between the outputs of two coupled oscillators, which is not an artifact of external noise. Our simulations show that the value of ρ_{max} does not change significantly with the length of the uncorrelated noise signals. In Fig. 4(b) we show how the synchronization index ρ for system (1,2) depends on the strength of the added noise and on the width Δf of the bandpass filter. For very broad bandwidth Δf the noise is not sufficiently filtered, and the synchronization between the two oscillators decreases (ρ decreases) with increasing noise strength σ_η. With decreasing band width Δf, i.e., applying a stronger filter, the effect of the noise is reduced, and correspondingly the index ρ increases—approaching the value ρ_0 expected for the system (1,2) without noise. On the other hand, applying a filter with too narrow bandwidth Δf leads to a spurious synchronization effects with $\rho > \rho_0$ [Fig. 4(b)], following closely the dependence of ρ on Δf shown in Fig. 2(e) for a Rössler system without noise.

In summary, our results indicate that phase synchronization between coupled nonlinear oscillators may strongly depend on the width of the power spectrum of these oscillators. Further, we find that while external noise can affect the degree of phase synchronization, bandpass filtering can reduce noise effects but can also lead to a spurious overestimation of the actual degree of phase synchronization in the system. This is of importance when analyzing empirical data in specific narrow frequency ranges, for which the coupling strength may not be known *a priori*.

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