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Extreme risk spillover network: application to financial institutions

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Using the CAViaR tool to estimate the value-at-risk (VaR) and the Granger causality risk test to quantify extreme risk spillovers, we propose an extreme risk spillover network for analysing the interconnectedness across financial institutions. We construct extreme risk spillover networks at 1% and 5% risk levels (which we denote 1% and 5% VaR networks) based on the daily returns of 84 publicly listed financial institutions from four sectors—banks, diversified financials, insurance and real estate—during the period 2006–2015. We find that extreme risk spillover networks have a time-lag effect. Both the static and dynamic networks show that on average the real estate and bank sectors are net senders of extreme risk spillovers and the insurance and diversified financials sectors are net recipients, which coheres with the evidence from the recent global financial crisis. The networks during the 2008–2009 financial crisis and the European sovereign debt crisis exhibited distinctive topological features that differed from those in tranquil periods. Our approach supplies new information on the interconnectedness across financial agents that will prove valuable not only to investors and hedge fund managers, but also to regulators and policy-makers.

Keywords: Extreme risk spillovers; Financial network; Financial institutions; Financial crisis

JEL Classification: C51, G01, G20

1. Introduction

The recent global financial crisis caused a near collapse of the financial system, and its shocks on the global economy are still being felt (Battiston *et al*. 2016). It has also focused the attention of researchers on the use of complexity theory to understand the behaviour and dynamics of financial markets, because the financial system has shown itself to be a complex system with a great number of interactive agents. Recently, network science has become a leading tool for understanding complex systems, e.g. the Internet (Barabái and Albert 1999), power grid systems (Watts and Strogatz 1998) and various social systems (Borgatti *et al*. 2009). In the financial system, network science has also emerged as a useful tool for describing the interconnectedness between financial agents (Schweitzer *et al*. 2009, Haldane and May 2011).

Much effort has been devoted to mapping the financial system as a financial network in which nodes in the network stand for different financial entities (e.g. companies, institutions and counties) and edges between the nodes corre-spond to their interactions. In the econophysics literature, correlation-based networks, such as the minimal spanning tree (MST) (Mantegna 1999, Onnela *et al*. 2003), the planar maximally filtered graph (PMFG) (Tumminello *et al*. 2005), the correlation threshold network (Onnela *et al*. 2004, Boginski *et al*. 2005) and the partial correlation PMFG (Kenett *et al*. 2010, Kenett *et al*. 2015), are the most extensively used tools for investigating interactions across different financial agents. Using some topological constraints,‖ correlation-based networks filter important correlation information from the correlation matrix of different financial agents. The primary disadvantage of correlation-based networks is that the economic or statistical meanings of their topological constraints are unclear (Výrost *et al*. 2015). More recently, several econometric approaches have been applied to construct the financial network and to uncover contagion sources and spillover effects. For example, Billio *et al*. (2012) propose the use of the Granger causality network to quantify systemic risk in financial institutions in terms of mean spillovers (Granger 1969), a mean

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‖For example, the topological constraint for the MST (PMFG) network is that the graph remains a tree (planar) when a new edge is added.

spillover network.† The mean spillover network also has been applied to other financial systems, e.g. the European electricity market (Castagneto-Gissey *et al.* 2014), world equity markets (Výrost *et al.* 2015) and the Korean financial system (Song *et al.* 2016). In the framework of vector autoregression (VAR) and generalized variance decomposition (Diebold and Yilmaz 2009, 2012), Diebold and Yilmaz (2014) propose a volatility spillover network for measuring the connectedness of financial institutions. Using the least-absolute shrinkage and selection operator (LASSO) method, Hautsch *et al.* (2015) develop a tail risk network for the financial system to detect the systemic importance of financial firms. These econometric-based networks can capture the complex features of interconnectedness across financial entities and identify possible risk contagion mechanisms from different perspectives.

Here, we propose an extreme risk spillover network using the Granger causality risk test proposed by Hong *et al.* (2009) to quantify the risk embedded in the interconnectedness across publicly listed financial institutions in the US market. Because understanding information spillovers in the financial system is crucial in managing asset risk, constructing investment portfolios, monitoring market stability and formulating regulatory policy (Wang *et al.* 2016), in the literature there is a growing field of research on information spillovers across different financial entities. In the framework of the general Granger causality analysis (Granger 1980), information spillovers can be divided into three types, (i) mean spillovers (Granger 1969), (ii) volatility spillovers (Granger *et al.* 1986, Cheung and Ng 1996, Hong 2001) and (iii) risk spillovers (Hong *et al.* 2009). To analyse the systemic spillover interconnectedness across multiple financial agents, the econometric-based networks are applied to the research on information spillovers in the financial system (see, e.g. Billio *et al.* 2012, Diebold and Yilmaz 2014). Inspired by the Granger causality network in terms of mean spillovers presented by Billio *et al.* (2012), we carry out a Granger causality risk test proposed by Hong *et al.* (2009) to build an extreme risk spillover network for studying interconnectedness in the financial system. According to Hong *et al.* (2009), if the past risk information of one institution can contribute to predicting the future risk information of another institution, the first financial institution is said to be a Granger cause of risk in the second institution, i.e. there are extreme risk spillovers from the first institution to the second. In our proposed extreme risk spillover network, a node represents a financial institution, and a directed edge linked from one financial institution to another represents an extreme risk spillover from the former to the latter, where the extreme risk is quantified by the left tail of return distributions of the financial institutions or equivalently by the value-at-risk (VaR).

Our work is related to the systemic risk literature that includes a variety of effective systemic risk measures. For example, using credit default swap (CDS) spreads of financial institutions and equity return correlations across these financial institutions, Huang *et al.* (2009) measure systemic risk by the price of insurance against financial distress. Under the multivariate extreme value theory framework, Zhou (2010) develops two systemic risk indices—systemic impact index (SII) and vulnerability index (VI)—that, respectively, characterize the systemic impact when an institution fails and the impact on a particular institution when the system exhibits distress. Acharya, Pedersen *et al.* (2017) design two measures of systemic risk, (i) the systemic expected shortfall (SES) that measures each financial institution's contribution to systemic risk and (ii) the marginal expected shortfall (MES) that is an institution's losses in the tail of the system's loss distribution. By extending the work of Acharya, Engle *et al.* (2012); Acharya, Pedersen *et al.* (2017) and Brownlees and Engle (2017) develop SRISK to measure the capital shortfall of a financial institution conditional on a prolonged market decline, which can quantify a financial institution's systematic risk contribution. Engle *et al.* (2015) apply the SRISK measure to investigate the systemic risk of 196 largest European financial institutions during the period of 2000–2012. Adrian and Brunnermeier (2016) propose CoVaR, a well-known modelling that measures the systemic risk contribution of a financial institution defined as the VaR of the financial system when the institution is in financial stress. Based on the multivariate GARCH estimation, Girardi *et al.* (2013) introduce a modified CoVaR to study the systemic risk contributions of 74 US financial institutions from June 2000 to February 2008. These measures are widely used in the analysis of systemic risk, but Hautsch *et al.* (2015) point out that such measures as the SES and MES cannot capture the spillover effects driven by the topology of risk networks and may underestimate the systemic risk contribution when financial institutions are highly interconnected. Our approach focusing on the risk spillover interconnectedness across different financial institutions differs from the above systemic risk measures that are market-based indices of systemic distress (Brownlees and Engle 2017). Our work is also closely related to the research of Adams *et al.* (2014), who extend the CoVaR measure and develop a state-dependent sensitivity VaR method to investigate risk spillovers across four financial sectors (commercial banks, investment banks, hedge funds and insurance companies). They find that commercial banks and hedge funds play a prominent role in transmitting shocks to other financial sectors, but their study focuses on sector-wide risk spillovers and our work is based on institution-level risk spillovers.

Here, we analyse 84 publicly listed financial institutions from the Standard & Poors (S&P) 500 index during the period from 2006 to 2015. We employ a new time-adapted VaR estimator, i.e. conditional autoregressive value-at-risk (CAViaR) introduced by Engle and Manganelli (2004) to measure the extreme (downside) risk of each financial institution. Using the Granger causality risk test of Hong *et al.* (2009), we examine extreme risk spillovers between each pair of financial institutions and build an extreme risk spillover network. Our proposed network for investigating the interconnectedness across financial institutions is a new contribution to the literature of econometric-based networks and has three distinctive features that distinguish it from previous networks. First, our network is an extension of the Granger causality network of Billio *et al.* (2012) but focuses on extreme risk spillovers, and this can serve as a new tool for analysing systemic risk to the financial system. Second, unlike most undirected correlation-based networks (e.g. the MST and PMFG), our network is a directed graph that can reflect the lead–lag relationship between financial institutions, i.e. a directed edge in the extreme risk spillover network not only shows the relationship between two financial

†In the literature mean spillover is also known as return spillover.

institutions but also represents which financial institution leads or influences which in terms of risk. Third, our new network allows us to examine which institutions and sectors are systemic (net) senders and which are recipients of extreme risk spillovers. It also allows us to quantify the period of time in which past information will continue to spill over into the entire system. In addition, time-varying extreme risk spillover networks allow us to analyse the dynamic interconnectedness across financial institutions and to detect any abnormal behaviour in the financial system. Thus, the information obtained by our network has important applications in such areas as asset risk management, portfolio optimization, systemic risk evaluation and financial regulation.

We organize our paper as follows. In section 2, we describe the methodologies for constructing an extreme risk spillover network. In section 3, we show the data. We present empirical results in section 4 and state our conclusions in section 5.

2. Methodology

As mentioned above, our proposed extreme risk spillover network for analysing interconnectedness among financial institutions is based on the Granger causality risk test of Hong *et al*. (2009). In this section, we first briefly describe the CAViaR of Engle and Manganelli (2004) for estimating the VaR of each financial institution. We then introduce the Granger causality risk test of Hong *et al*. (2009) for examining extreme risk spillovers between pairs of financial institutions. Finally, we construct the extreme risk spillover network and introduce some topological approaches to quantifying it.

2.1. CAViaR

VaR was developed in the early 1990s and has become a standard tool used by firms and regulators in the financial industry to estimate market or investment risk. It measures how much a set of asset portfolios can lose with a probability θ in a specific time period with a confidence level of $(1-\theta)$ in which $\theta \in (0, 1)$. Let $\{r_t\}_{t=1}^{T}$ be the returns of a given asset portfolio and T be the length of the returns. VaR is defined as the left θ-quantile of the conditional probability distribution of returns of the asset portfolio, which is subject to

$$\Pr[r_t < -V_t|\Omega_{t-1}] = \theta, \tag{1}$$

where V_t is the VaR at time t, and Ω_{t-1} is the information set available at time $t-1$.

The methods proposed for estimating VaR fall into two groups, (i) factor mapping models (e.g. the variance–covariance approach), and (ii) portfolio models (e.g. historical quantiles), but these approaches are often criticized by researchers and practitioners because they assume that the distribution of asset returns is invariable across time. Thus, Engle and Manganelli (2004) propose a time-adapted CAViaR model using an autoregressive process and regression quantiles. The general CAViaR model is defined as follows:

$$V_t(\theta) = \theta_0 + \sum_{i=1}^{p} \theta_i V_{t-i}(\theta) + \sum_{j=1}^{q} \theta_j l(r_{t-j}), \tag{2}$$

where $l(\cdot)$ is a function that depends on a finite number of lagged values of observables and $\{\theta_i V_{t-i}(\theta)\}_{i=1}^{p}$ are the autoregressive terms ensuring that VaR changes smoothly over time. In particular, Engle and Manganelli (2004) develop four CAViaR models, i.e.

Asymmetric slope :

$$V_t(\theta) = \theta_0 + \theta_1 V_{t-1}(\theta) + \theta_2 (r_{t-1})^+ + \theta_3 (r_{t-1})^-, \tag{3}$$

Indirect GARCH(1, 1) :

$$V_t(\theta) = [\theta_0 + \theta_1 V_{t-1}^2(\theta) + \theta_2 (r_{t-1}^2)]^{1/2}, \tag{4}$$

Symmetric absolute value:

$$V_t(\theta) = \theta_0 + \theta_1 V_{t-1}(\theta) + \theta_2 |r_{t-1}|, \tag{5}$$

Adaptive:

$$V_t(\theta_1) = V_{t-1}(\theta_1) + \theta_1 \{[1 + \exp(F[r_{t-1} - V_{t-1}(\theta_1)])]^{-1} - \theta\}, \tag{6}$$

where $(r_{t-1})^+ = \max(r_{t-1}, 0)$, $(r_{t-1})^- = -\min(r_{t-1}, 0)$ and F is some positive finite number. Engle and Manganelli (2004) also propose a dynamic quantile (DQ) test to check the adequacy of the estimated CAViaR models. Following Hong *et al*. (2009), we use the asymmetric slope model to estimate the VaR of each financial institution when its DQ statistic is significant at the 1% level. We otherwise use the indirect GARCH(1,1) model, the symmetric absolute value model and the adaptive model in turn if the corresponding DQ statistic is significant.†

2.2. Granger causality risk test

The Granger causality risk test proposed by Hong *et al*. (2009) is an extension of the general Granger causality test of Granger (1980). The Granger causality risk test is straightforward, i.e. one financial institution can be said to Granger causes risk to a second institution if the ability to forecast the future risk information of the second institution is improved by incorporating the past risk information of the first institution. We follow Hong *et al*. (2009) and introduce a risk indicator

$$Z_{m,t} = \mathbf{1}(r_{m,t} < -V_{m,t}), \; m = 1, 2, \tag{7}$$

where $r_{m,t}$ and $V_{m,t}$ are the returns and VaR of financial institution m, respectively, and $\mathbf{1}(\cdot)$ is an indicator function. When the actual loss exceeds the VaR, $Z_{m,t}$ takes the value of 1, otherwise it takes 0.

Let $\{r_{1,t}\}$ and $\{r_{2,t}\}$ denote the returns of financial institutions 1 and 2. Consider information set $\Omega_{t-1} = \{\Omega_{1,t-1}, \Omega_{2,t-1}\}$, where $\Omega_{1,t-1} = \{r_{1,t-1}, r_{1,t-2}, \ldots, r_{1,1}\}$ and $\Omega_{2,t-1} = \{r_{2,t-1}, r_{2,t-2}, \ldots, r_{2,1}\}$ are the information sets available at time $t-1$ for the two financial institutions 1 and 2, respectively. The Granger causality risk test compares the null hypothesis

$$\mathrm{H}^0: E(Z_{1,t}|\Omega_{1,t-1}) = E(Z_{1,t}|\Omega_{t-1}), \tag{8}$$

against the alternative hypothesis

$$\mathrm{H}^1: E(Z_{1,t}|\Omega_{1,t-1}) \neq E(Z_{1,t}|\Omega_{t-1}). \tag{9}$$

†We also consider two GARCH(1,1) models (i.e. AR(1)-GARCH(1,1)-Gaussian and AR(1)-GARCH(1,1)-Skewed-t) as alternative approaches to estimate time-varying VaRs, but the backtesting results for evaluating the accuracy of VaR estimates computed by the CAViaR model and the two GARCH(1,1) models show that the CAViaR model is a better choice than the two GARCH(1,1) models (see Appendix 1).

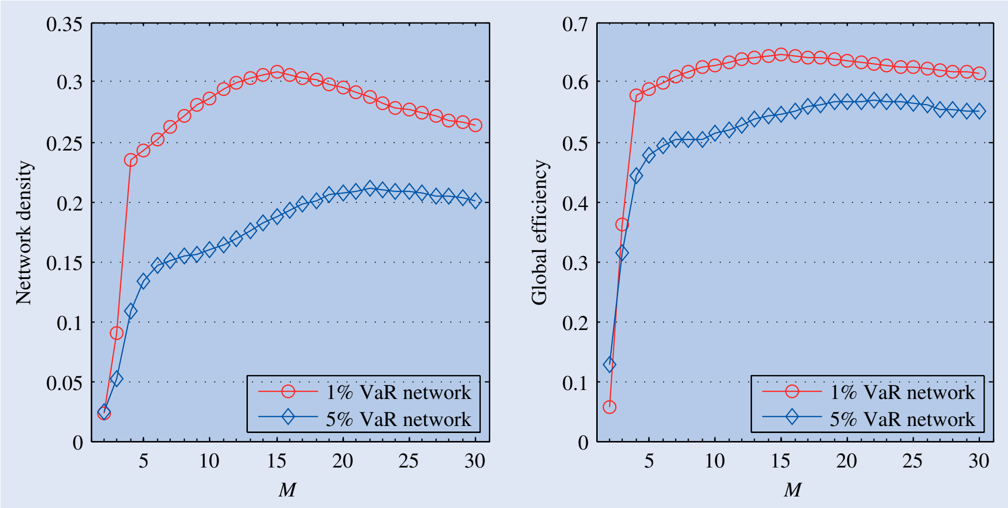

Figure 1. ND and GE of extreme risk spillover networks at 1% and 5% risk levels (i.e. 1% and 5% VaR networks) as functions of lag M. About the interpretation of M, see equations (12)–(14).

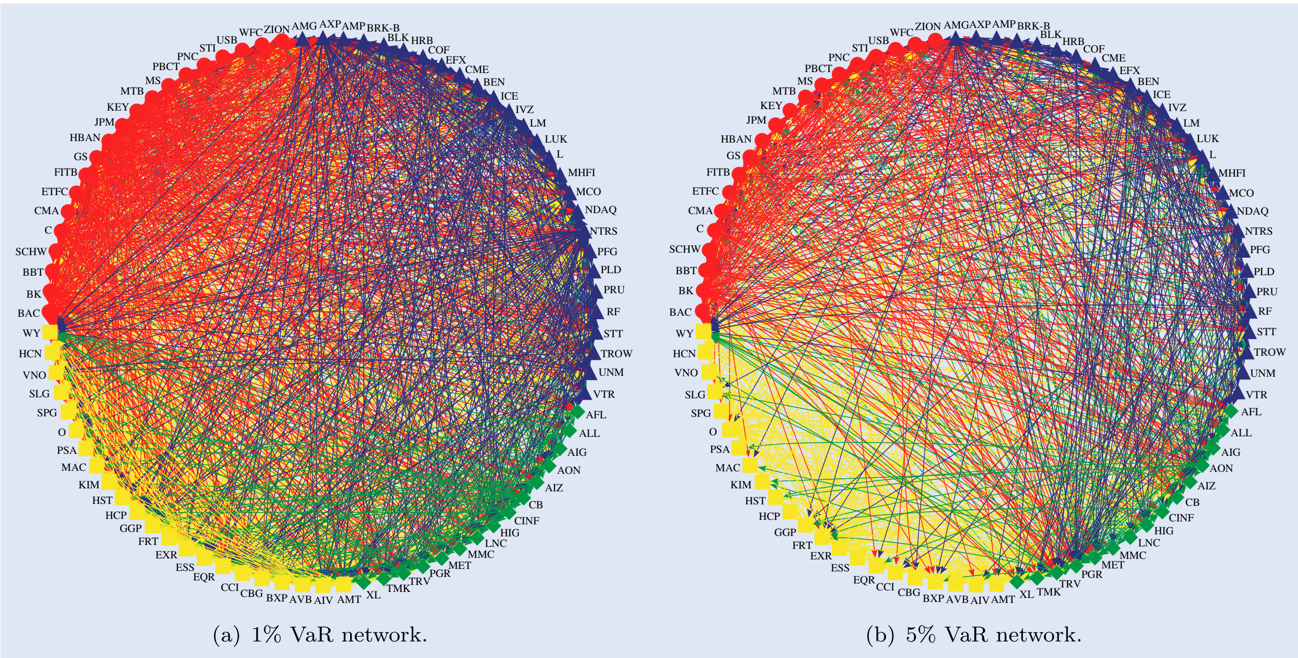

Figure 2. Snapshots of extreme risk spillover networks at 1% and 5% risk levels (i.e. 1% and 5% VaR networks) of 84 publicly listed financial institutions from the S&P 500 index during the period 2006–2015. The lag order M in the calculation of Granger causality test in risk is 10. The sample includes 20 banks, 27 diversified financials, 15 insurance firms and 22 real estate companies. Financial institutions from the same sector are marked by the same colour and shape, and their outgoing edges are indicated by the same colour as their sectors. Coding is: banks, red circles; diversified financials, blue triangles; insurance, green diamonds; and real estate, yellow squares. For the full name of each financial institution, see table 1. Note that the information provided by the network diagram may depend on which lines are first plotted. For example, many yellow lines shown in the 1% VaR network are mostly covered or overplotted by other lines, meaning that real estate companies are actually more connected than that suggested by the network diagram.

Hong *et al.* (2009) use the framework of the cross-correlation function (CCF) to test the null hypothesis. Considering two estimated series of risk indicators $\hat{Z}_{1,t}$ and $\hat{Z}_{2,t}$, their sample cross-covariance function at positive lag j is

$$\hat{C}(j) = T^{-1} \sum_{t=1+j}^{T} (\hat{Z}_{1,t} - \hat{\alpha}_1)(\hat{Z}_{2,t-j} - \hat{\alpha}_2),\ 1 \le j \le T-1, \tag{10}$$

where $\hat{\alpha}_m = T^{-1} \sum_{t=1}^{T} \hat{Z}_{m,t}, m = 1, 2$. Thus, the sample CCF is defined as follows:

$$\hat{\rho}(j) = \frac{\hat{C}(j)}{\hat{S}_1 \hat{S}_2}, \tag{11}$$

where $\hat{S}_m^2 = \hat{\alpha}_m(1 - \hat{\alpha}_m)$. Using the sample CCF, the statistic for testing the unidirectional Granger causality in risk from financial institution 2 to financial institution 1 is defined as follows:

$$Q(M) = \left[T \sum_{j=1}^{T-1} k^2(j/M)\hat{\rho}^2(j) - C_T(M) \right] \Big/ [D_T(M)]^{1/2}, \tag{12}$$

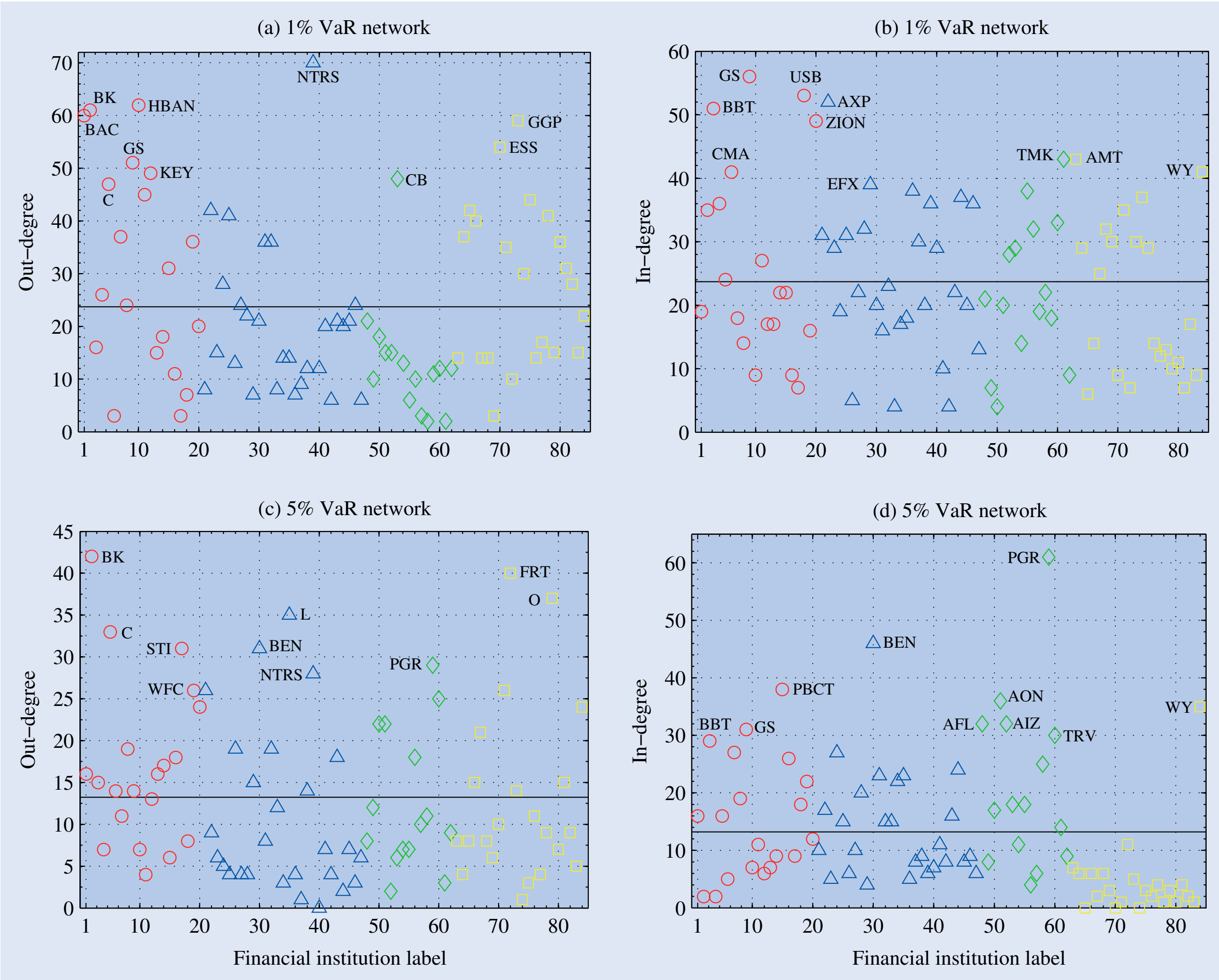

Figure 3. Out-degree and in-degree of each financial institution in extreme risk spillover networks at 1% and 5% risk levels (i.e. 1% and 5% VaR networks) of 84 financial institutions during the period 2006–2015 when M=10. Panels (a) and (b) show out-degree and in-degree of each financial institution in the 1% VaR network, and panels (c) and (d) show those in the 5% VaR network. The solid line shown in each panel is the average out-degree or in-degree. The label order for each institution on the x-axis is shown in table 1. Colours and shapes of sectors are as in figure 2.

where the centring and standardization constants are defined as follows:

$$C_T(M) = \sum_{j=1}^{T-1} (1 - j/T)k^2(j/M), \tag{13}$$

$$D_T(M) = 2\sum_{j=1}^{T-1} (1 - j/T)(1 - (j+1)/T)k^4(j/M). \tag{14}$$

In equations (12)–(14), the kernel function $k(\cdot)$ assigns weights to various lags. Following Hong (2001) and Hong *et al.* (2009) in our study, we employ the Daniell kernel $k(x) = \sin(\pi x)/(\pi x)$. The lag order M represents how many lags are used to examine extreme risk spillovers from financial institution 2 to financial institution 1. Because the domain of the Daniell kernel function is unbounded, M is also the effective lag truncation order, according to Hong (2001) and Hong *et al.* (2009). When $M = 10$, for example, it satisfies the calculation of VaR with a time horizon of 10 days (i.e. the 10-day VaR) required by the Basel Committee on Banking Supervision. In our empirical analysis, we will discuss extreme risk spillover networks under different lag orders.

Under the null hypothesis, Hong *et al.* (2009) show that $Q(M)$ follows an asymptotically standard normal distribution $N(0, 1)$. Thus, the null hypothesis is rejected when $Q(M)$ is greater than the right-tailed critical value of $N(0, 1)$ at a given significance level β (in our case, $\beta = 1\%$), indicating that there is a unidirectional Granger causality in risk from financial institution 2 to financial institution 1.

2.3. *Extreme risk spillover network*

Let $G(V, E)$ be an extreme risk spillover network, where $V = \{1, 2, \ldots, N\}$ is the set of nodes and E is the set of edges. In our network, a node is a financial institution and a directed edge is the Granger causality connectivity in risk from one financial institution to another. For any two financial institutions $i, j \in V$, we draw a directed edge from i to j (i.e. $i \rightarrow j$) if i Granger causes risk to j. In a similar way, we draw a directed edge from j to i (i.e. $j \rightarrow i$) if j Granger causes risk to i. Mathematically, given the confidence level of $(1 - \theta)$, the lag order M, and the significance level β ($\beta = 1\%$), E is a directed binary connection matrix for all i and j such that

$$E_{i \rightarrow j} = \begin{cases} 1, & \text{if } i \neq j \text{ and } i \text{ Granger causes risk to } j \\ 0, & \text{otherwise} \end{cases}. \tag{15}$$

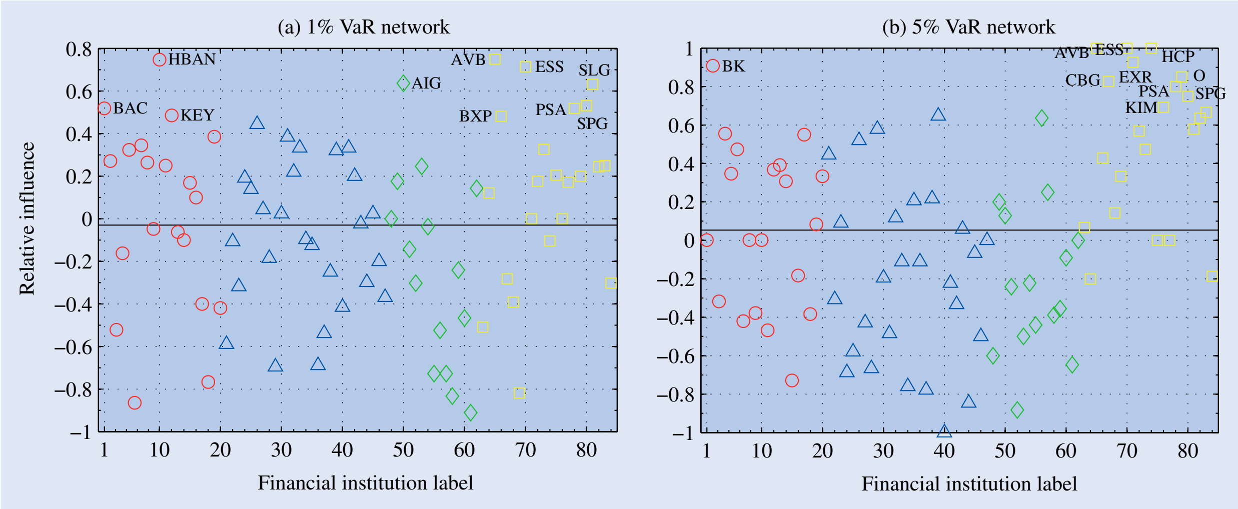

Figure 4. RI of each financial institution in 1% and 5% VaR networks of 84 financial institutions during the period 2006–2015 when M=10. The solid line shown in each panel is the average RI. The label order for each institution on the x-axis is shown in table 1. Colours and shapes of sectors are as in figure 2.

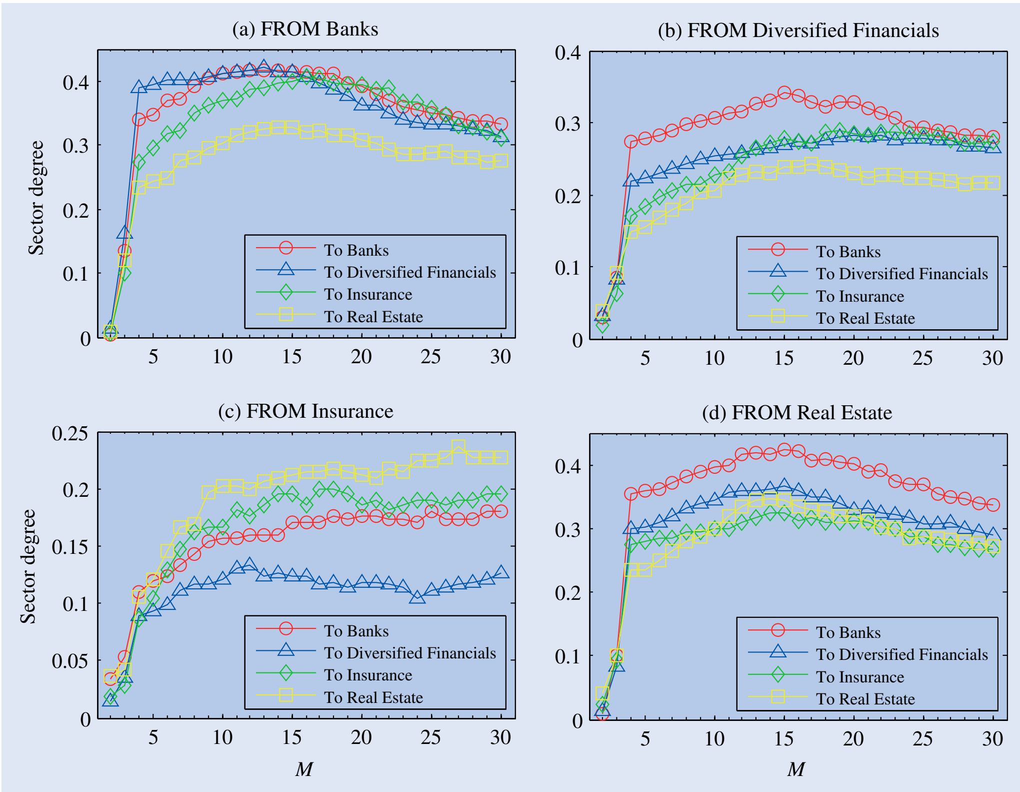

Figure 5. Sector degree from one sector to another or to itself in the 1% VaR network as a function of lag M.

Using rolling windows, we build time-varying extreme spillover networks to investigate the dynamic interconnectedness across financial institutions. Specifically, we divide the empirical data during the 2006–2015 period into $T^{'}$ windows ($t^{'} = 1, 2, \ldots, T^{'}$) with width L and step size δ, where L is the number of daily returns in each window and δ is the step size between two continuous windows. Following Yan *et al.* (2015), we set the width L and step size δ to 250 and 20 trading days, respectively, which are roughly equivalent to a trading year and a trading month. Thus, we obtain dynamic extreme risk spillover networks $G_{t'}(V, E_{t'})$.

To quantify the topological features of (time-varying) extreme spillover networks, we introduce measures from three levels, (i) system-level connectivity, (ii) institution-level connectivity and (iii) sector-level connectivity.

2.3.1. System-level connectivity measures. We introduce *network density* (ND), which is an indicator of network health and functionality. The ND is defined as the ratio between the actual connections (edges) and all possible connections in a network. For N financial institutions, there are $N(N-1)$ possible connections. Mathematically, we have

$$\text{ND} = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j \neq i} E_{i \to j}. \tag{16}$$

Another measure is the *global efficiency* (GE) of a network (Latora and Marchiori 2001), which quantifies how efficiently the information is exchanged over the network and is defined as follows:

$$\text{GE} = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j \neq i} \frac{1}{d_{i \to j}}, \tag{17}$$

where $d_{i \to j}$ is the shortest path length from node i to j. If there is no path in the network from i to j, $d_{i \to j} = +\infty$. Both the values of ND and GE fall in the range [0, 1].

2.3.2. Institution-level connectivity measures. Node degree indicates the number of edges connected to a node. In our extreme risk spillover network, the *out-degree* of financial institution i is the number of outgoing edges from institution i to other institutions. A higher out-degree of a financial institution suggests that it is a more active sender of extreme risk spillovers to other institutions and thus has a greater influence on them. The out-degree of financial institution i is defined as follows:

$$k_{\text{out}}(i) = \sum_{j=1, j \neq i}^{N} E_{i \to j}. \tag{18}$$

Similarly, the *in-degree* of financial institution i is the number of incoming edges from other financial institutions to institution i. When a financial institution has a higher in-degree it is more susceptible to extreme risk spillovers from other institutions. The in-degree of financial institution i is defined as follows:

$$k_{\text{in}}(i) = \sum_{j=1, j \neq i}^{N} E_{j \to i}. \tag{19}$$

Following Kenett *et al*. (2010), we introduce the *relative influence* (RI) of financial institution i, which is the ratio between the difference and the sum of out-degree and in-degree, i.e.

$$\text{RI}_{\text{institution}}(i) = \frac{k_{\text{out}}(i) - k_{\text{in}}(i)}{k_{\text{out}}(i) + k_{\text{in}}(i)}, \tag{20}$$

where $\text{RI}_{\text{institution}} \in [-1, 1]$. A positive (negative) value of RI of a financial institution means that it has greater (less) influence on other institutions than the other institutions have on it, i.e. the intensity of extreme risk spillovers from this institution to other institutions is greater (smaller) than from the others to it.

2.3.3. Sector-level connectivity measures. The sectoral clustering effect is widely found in financial networks (see, e.g. Onnela *et al*. 2003, Kenett *et al*. 2010). In our case, the 84 publicly listed financial institutions fall into four subsectors, banks, diversified financials, insurance and real estate. To investigate how sectors influence each other, we introduce a measure of *sector degree* (SD) from one sector m to another or to itself n, which is defined as follows:

$$\text{SD}_{m \to n} = \frac{1}{N_m N_n} \sum_{i=1}^{N_m} \sum_{j=1}^{N_n} E_{i|m \to j|n}, \tag{21}$$

where m and $n \in$ {banks, diversified financials, insurance and real estate}, N_m and N_n are the number of financial institutions belonging to sector m and sector n, respectively, and $i|m$ ($j|n$) represents financial institution i (j) belonging to sector m (n). Note that when $m = n$ in equation (21), $N_n = N_m - 1$ and $j \neq i$. The SD measures the proportion of extreme risk spillovers from one sector to another or to itself.

The out-degree of sector m is defined as the number of outgoing edges from institutions belonging to sector m to institutions belonging to other sectors. Similarly, the in-degree of sector m is the number of incoming edges from institutions belonging to other sectors to institutions belonging to sector m. Let $k_{\text{out}}(m)$ and $k_{\text{in}}(m)$ denote the out-degree and in-degree of sector m. The RI of sector m is defined as follows:

$$\text{RI}_{\text{sector}}(m) = \frac{k_{\text{out}}(m) - k_{\text{in}}(m)}{k_{\text{out}}(m) + k_{\text{in}}(m)}. \tag{22}$$

In addition to the above measures, we introduce the *survival ratio* (SR), which was proposed by Onnela *et al*. (2003) to examine the connection robustness of dynamic extreme risk spillover networks. SR is defined as the ratio between the number of edges simultaneously found in two continuous networks at times t' and $t' + 1$ and the number of edges in the network at time t',† i.e.

$$\text{SR}_{t'} = \frac{|E_{t'} \cap E_{t'+1}|}{|E_{t'}|}, \tag{23}$$

where $E_{t'}$ is the set of edges of an extreme risk spillover network at time t', $|\cdot|$ is the number of observations in the set and $\cap$ is the intersection operator.

3. Data

Following Billio *et al*. (2012); Diebold and Yilmaz (2014) and Hautsch *et al*. (2015), we apply the proposed extreme risk spillover network to publicly listed financial institutions. We collect the daily closing prices of 84 financial institutions that are component stocks of the financial sector of the S&P 500 index during the period from 3 January 2006 to 31 December 2015. According to the Global Industry Classification Standard (GICS), the financial sector can be subdivided into four GICS industry groups, i.e. banks (GICS code 4010), diversified financials (GICS code 4020), insurance (GICS code 4030) and real estate (GICS code 4040). Our sample is composed of 20 banks, 27 diversified financials, 15 insurance firms and 22 real estate companies.‡ Table 1 shows the list of these financial institutions. We obtain the data from the website of Yahoo Finance (http://finance.yahoo.com). We define the daily return of financial institution i on day t as $r_{i,t} = 100 \times \ln(P_{i,t}/P_{i,t-1})$, where $P_{i,t}$ is the daily closing stock price of financial institution i on day t. There are 2516 observations for each return series.

†Two continuous networks represent two networks at two continuous time windows t' and $t' + 1$. Suppose at the time window t' we have a network $G_{t'}$ and at the next time window $t' + 1$ we have another network $G_{t'+1}$, these two networks $G_{t'}$ and $G_{t'+1}$ are designated two continuous networks.

‡The financial institutions are selected based on their inclusion in the financial sector of the S&P 500 index as of March 2016. Note that on 31 August 2016, stock exchange listed Equity REITs and other listed real estate companies from the financial sector were moved to a new real estate sector in the GICS.

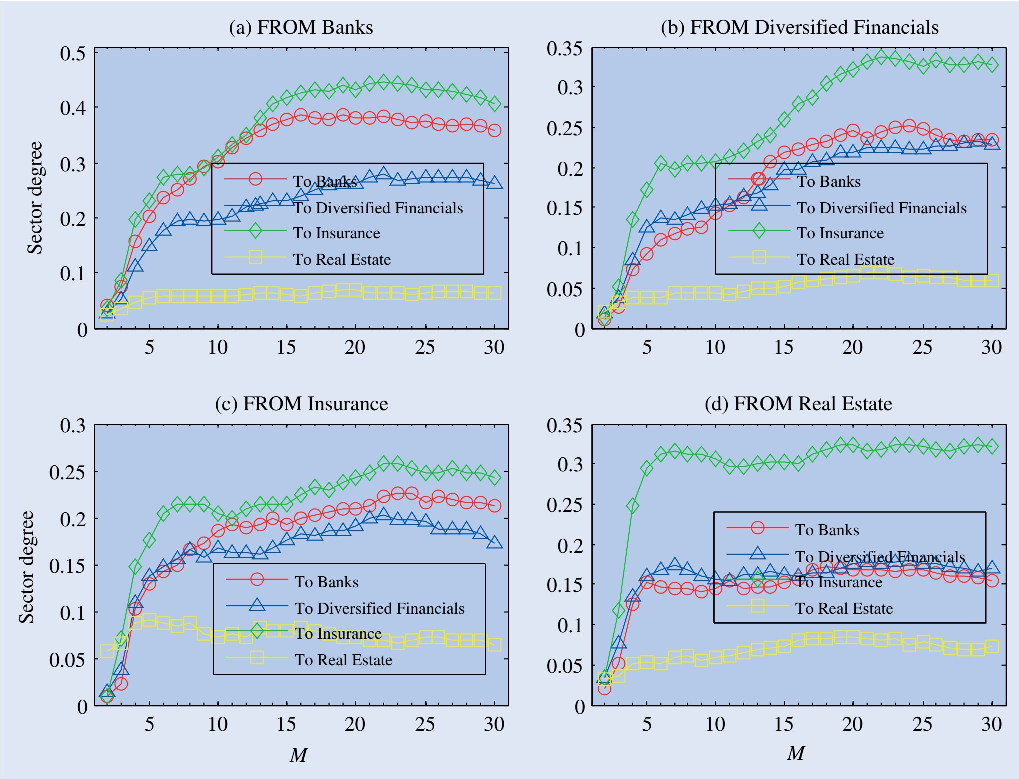

Figure 6. Sector degree from one sector to another or itself in the 5% VaR network as a function of lag M.

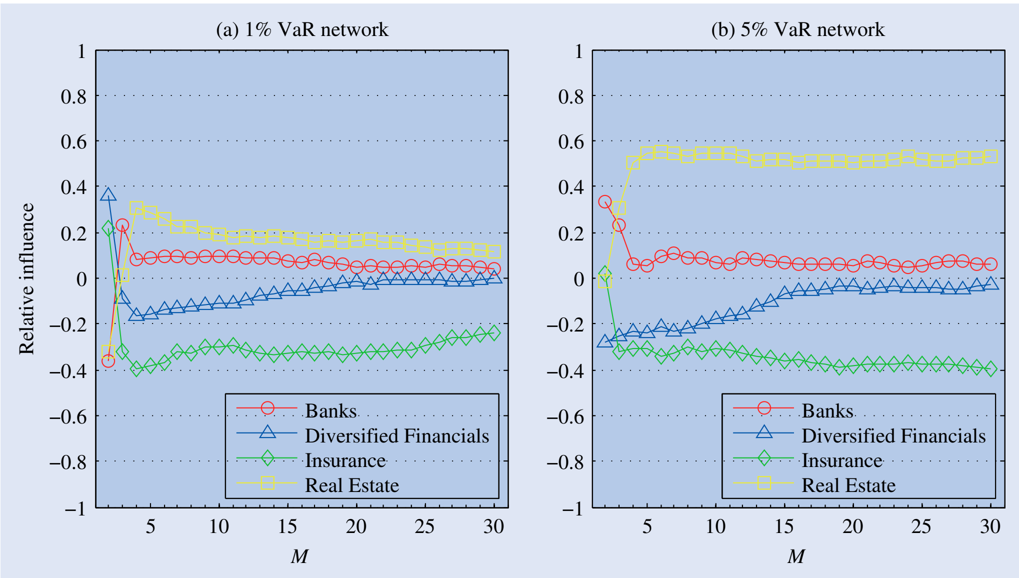

Figure 7. RI of each sector in 1% and 5% VaR networks as a function of lag M.

4. Empirical results

In our empirical study, we consider two commonly used confidence levels (99% and 95%) when calculating VaRs, i.e. 1% and 5% risk levels.† For simplicity, we denote extreme risk spillover networks at 1% and 5% risk levels as 1% VaR network and 5% VaR network.§ As mentioned above, the statistically significance level β for testing the Granger causality in risk from one institution to another is 1%. In the following, we will

†To measure bank capital adequacy, for example, the Bank for International Settlements (BIS) uses the 1% VaR, and the JPMorgan Chase & Co. uses the 5% VaR.

§Due to space limitations, we do not include the results of VaR for each financial institution estimated by the CAViaR models of Engle and Manganelli (2004) and the statistics of the Granger causality risk test of Hong *et al.* (2009) for each pair of financial institutions, but they are available upon request.

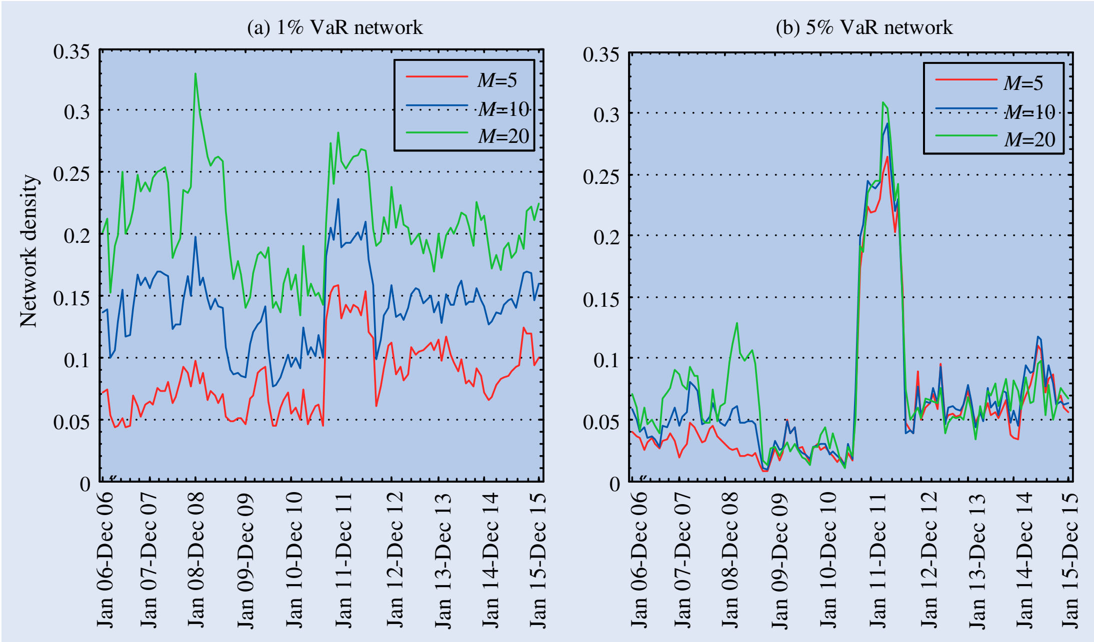

Figure 8. Dynamic ND of time-varying 1% and 5% VaR networks when $M = 5$, 10, and 20. Notes for this and below figures: the time on x-axis represents the period of a 250-day window.

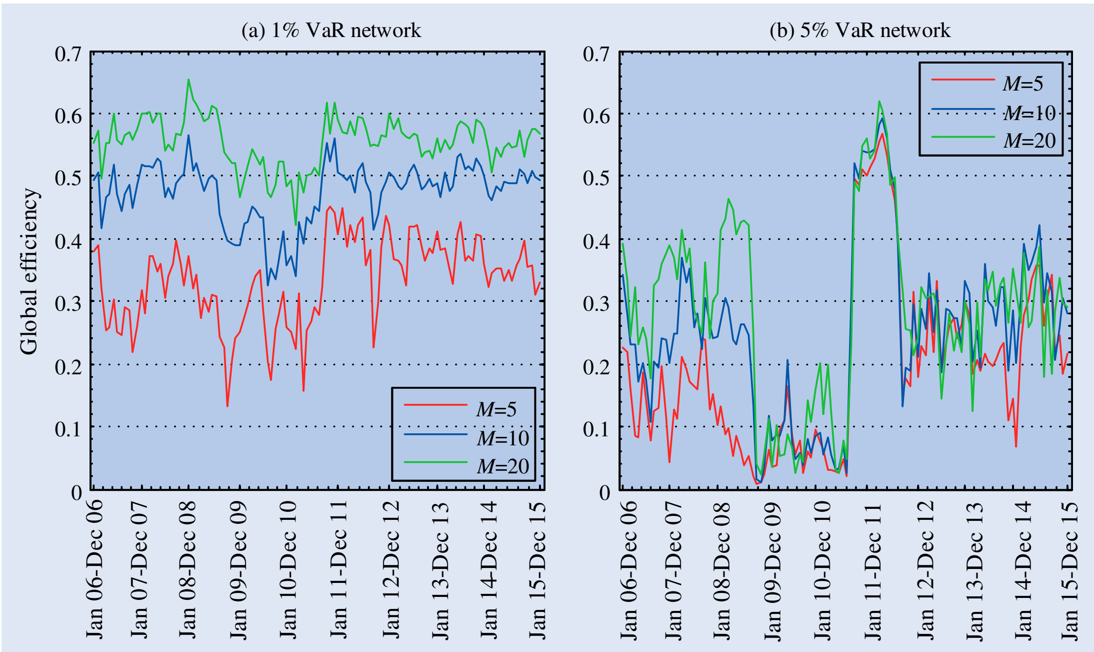

Figure 9. Dynamic GE of time-varying 1% and 5% VaR networks when $M = 5$, 10, and 20.

focus on the extreme risk spillover networks under different lags and time-varying networks.

4.1. Networks under different lags

In real-world economic and financial behaviour, market participants and regulators usually do not respond quickly to past information (e.g. market shocks) but take time to understand the information before taking action and thus contribute to a time-lag effect in extreme risk spillovers. In this subsection, we investigate how extreme risk spillover networks change as the lag order M increases. We construct 1% and 5% VaR networks connecting 84 financial institutions during the period 2006–2015 with varying lags from 2 to 30.

Figure 1 shows the ND and GE of 1% and 5% VaR networks as functions of lag. Both the values of ND and GE of 1% VaR networks at different lags (except for lag 2) are greater than those of 5% VaR networks, suggesting that extreme risk spillovers across financial institutions at 1% risk level have a higher frequency than at 5% risk level. This finding also confirms that when the market is in a more extreme risk condition, the panic of market participants increases and their confidence declines, thus causing ‘herd behavior’ and an increase in the interconnectedness across financial agents in the system.

Table 1. 84 Publicly listed financial institutions and their ticker symbols in alphabetical order within four sectors.

Order	Institution (Ticker symbol)	Order	Institution (Ticker symbol)
Banks (20)		43	Regions Financial Corp. (RF)
1	Bank of America Corp (BAC)	44	State Street Corp. (STT)
2	Bank of New York Mellon Corp. (BK)	45	T. Rowe Price Group (TROW)
3	BB&T Corporation (BBT)	46	Unum Group (UNM)
4	Charles Schwab Corp. (SCHW)	47	Ventas Inc. (VTR)
5	Citigroup Inc. (C)	**Insurance (15)**	
6	Comerica Inc. (CMA)	48	AFLAC Inc. (AFL)
7	E*Trade (ETFC)	49	Allstate Corp. (ALL)
8	Fifth Third Bancorp (FITB)	50	American International Group, Inc. (AIG)
9	Goldman Sachs Group (GS)	51	Aon plc (AON)
10	Huntington Bancshares (HBAN)	52	Assurant Inc. (AIZ)
11	JPMorgan Chase & Co. (JPM)	53	Chubb Limited (CB)
12	KeyCorp (KEY)	54	Cincinnati Financial (CINF)
13	M&T Bank Corp. (MTB)	55	Hartford Financial Svc.Gp. (HIG)
14	Morgan Stanley (MS)	56	Lincoln National (LNC)
15	People's United Financial (PBCT)	57	Marsh & McLennan (MMC)
16	PNC Financial Services (PNC)	58	MetLife Inc. (MET)
17	SunTrust Banks (STI)	59	Progressive Corp. (PGR)
18	U.S. Bancorp (USB)	60	The Travelers Companies Inc. (TRV)
19	Wells Fargo (WFC)	61	Torchmark Corp. (TMK)
20	Zions Bancorp (ZION)	62	XL Capital (XL)
Diversified Financials (27)		**Real Estate (22)**	
21	Affiliated Managers Group Inc. (AMG)	63	American Tower Corp A (AMT)
22	American Express Co (AXP)	64	Apartment Investment & Mgmt (AIV)
23	Ameriprise Financial (AMP)	65	AvalonBay Communities, Inc. (AVB)
24	Berkshire Hathaway (BRK-B)	66	Boston Properties (BXP)
25	BlackRock (BLK)	67	CBRE Group (CBG)
26	Block H&R (HRB)	68	Crown Castle International Corp. (CCI)
27	Capital One Financial (COF)	69	Equity Residential (EQR)
28	CME Group Inc. (CME)	70	Essex Property Trust Inc. (ESS)
29	Equifax Inc. (EFX)	71	Extra Space Storage (EXR)
30	Franklin Resources (BEN)	72	Federal Realty Investment Trust (FRT)
31	Intercontinental Exchange (ICE)	73	General Growth Properties Inc. (GGP)
32	Invesco Ltd. (IVZ)	74	HCP Inc. (HCP)
33	Legg Mason (LM)	75	Host Hotels & Resorts (HST)
34	Leucadia National Corp. (LUK)	76	Kimco Realty (KIM)
35	Loews Corp. (L)	77	Macerich (MAC)
36	McGraw Hill Financial (MHFI)	78	Public Storage (PSA)
37	Moody's Corp. (MCO)	79	Realty Income Corporation (O)
38	NASDAQ OMX Group (NDAQ)	80	Simon Property Group Inc. (SPG)
39	Northern Trust Corp. (NTRS)	81	SL Green Realty (SLG)
40	Principal Financial Group (PFG)	82	Vornado Realty Trust (VNO)
41	Prologis (PLD)	83	Welltower Inc. (HCN)
42	Prudential Financial (PRU)	84	Weyerhaeuser Corp. (WY)

The ND and GE sharply increase at lags 2–4, reach a steady increase when lags $M \geq 5$ and finally achieve stabilization at protracted lags (e.g. lag 15). This trend confirms that the market takes time (at least five trading days as shown in figure 1) to reflect past information, and that extreme risk spillover networks have time-lags.

To investigate the institution-level connectivity of 1% and 5% VaR networks, we select the lag order $M = 10$ as a representation that coheres with the 10-day VaR required by the BIS. Figure 2 shows snapshots of 1% and 5% VaR networks of 84 financial institutions calculated using the daily returns during the period 2006–2015 when $M = 10$. Both of these two networks are highly connected, suggesting a significant danger of extreme risk spillovers across the financial institutions. There are more edges in the 1% VaR network than in the 5% VaR network. Figure 3 shows the out-degree and in-degree of each financial institution in the 1% and 5% VaR networks. In each panel of figure 3, we show the top 10 financial institutions ranked by the out-degree or in-degree in each network. The solid line in each panel is the average out-degree or in-degree.

From the out-degree of 1% VaR network shown in figure 3(a), we find (i) that the Northern Trust Corporation (NTRS) from the sector of diversified financials, which is among the three largest trust companies with total fiduciary assets of \$1,504 billion in Q4 2015, has the largest out-degree (i.e, the largest source of extreme risk spillovers), (ii) that among the top 10 financial institutions ranked by out-degree, there are 6 banks and 2 real estate institutions, and (iii) that the out-degree of most banks and real estate institutions is greater than the average value.† Findings (ii) and (iii) imply that the

†Note that our results based on extreme risk spillover networks are somewhat different from the systemic risk rankings provided by the volatility institute (V-Lab) of NYU Stern School of Business (see http://vlab.stern.nyu.edu/analysis/RISK.USFIN-MR.MES) that are based on the systemic risk measures (SRISK) proposed by Acharya, Engle *et al*. (2012); Acharya, Pedersen *et al*. (2017) and Brownlees

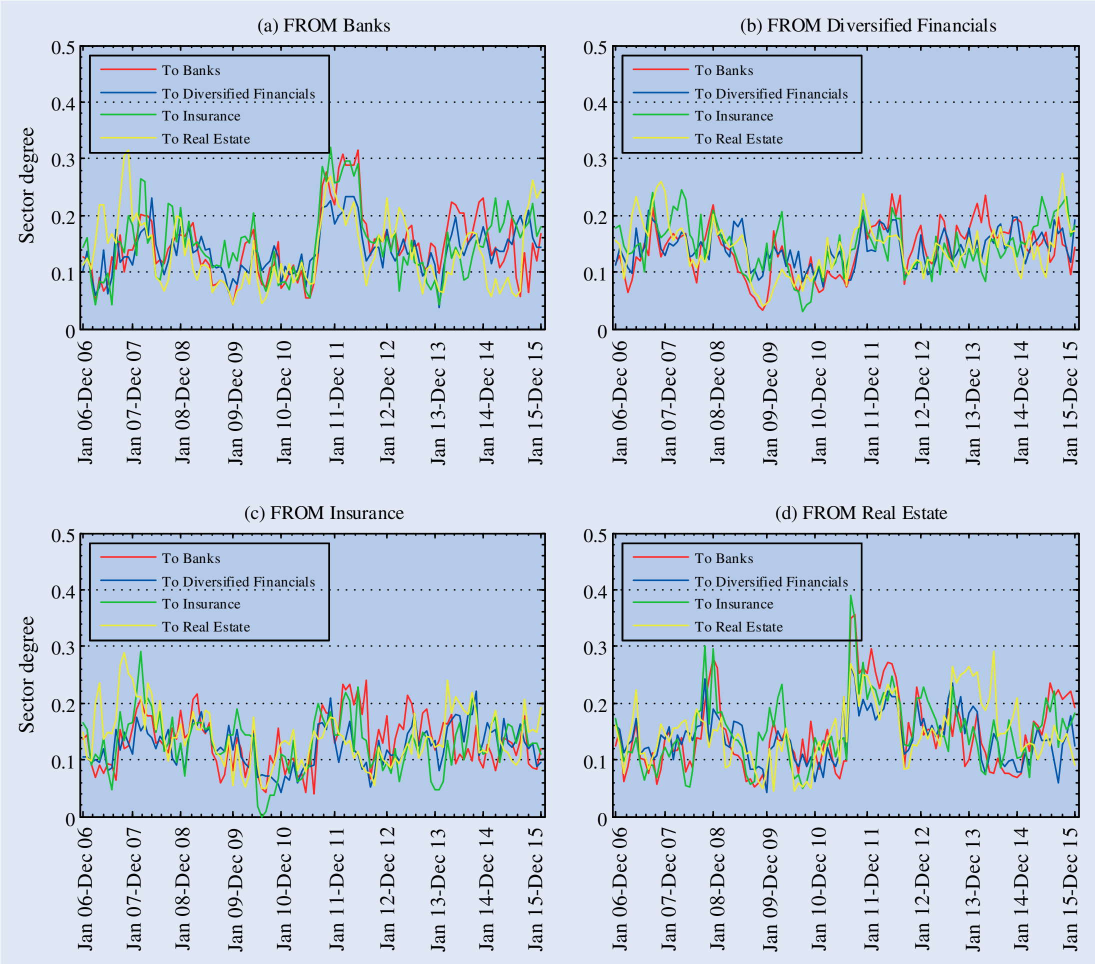

Figure 10. Dynamic SD from one sector to another or to itself in time-varying 1% VaR networks when M=10.

major senders of extreme risk spillovers are from the bank and real estate sectors. Figure 3(b) shows that half of top 10 institutions ranked by in-degree are banks, meaning that bank institutions are also the main recipients of extreme risk spillovers. Comparing figure 3(a) and (b), we find that the in-degree of most diversified financial and insurance institutions is significantly larger than their out-degree, which suggests that most institutions from these two sectors are recipients of extreme risk spillovers. By analysing the out-degree and in-degree of institutions in the 5% VaR network shown in figures 3(c) and (d), we conclude (i) that banks and diversified financial institutions play two roles, i.e. senders and recipients of extreme risk spillovers, (ii) that most real estate institutions are extreme risk spillover senders and (iii) that most insurance institutions are recipients of extreme risk spillovers.

To determine which financial institutions are net senders and which are net recipients of extreme risk spillovers, we investigate the RI of each financial institution in 1% and 5% VaR networks when $M = 10$, and we present the results in figure 4. We also indicate the top 10 financial institutions ranked by the RI in each network. In the list of the top 10 financial institutions in the 1% VaR network [see figure 4(a)], there are 6 real estate institutions, 3 banks and 1 insurance institution, which means that the top net senders of extreme risk spillovers are from real estate and banks, and the US subprime crisis confirmed this finding. It is not surprising that the American International Group (AIG) insurance corporation is listed in the top 10 because it was a key player in the 2008 financial crisis. On 15 September 2008, AIG faced a severe liquidity crisis, and its stock price declined 61% when its credit ratings were downgraded by credit rating agencies. AIG was active in various segments of the US residential mortgage market by selling credit protection through its AIG Financial Products division in the form of CDSs on collateralized debt obligations (CDOs), but these products declined in value and caused the AIG crisis. In the 5% VaR network shown in figure 4(b), nine real estate firms and one bank are among the top 10 institutions

and Engle (2017). According to the V-Lab, for example, the NTRS owning the largest out-degree in the 1% VaR network is only ranked 21 on 31 December 2015, but the high rankings of some banks (e.g. Bank of American, Citigroup and Goldman Sachs) in our network are consistent with the results obtained by the V-Lab. There are two possible reasons for the difference. (i) Our approach differs from the V-Lab measurements (e.g. SRISK) developed by Acharya, Engle *et al.* (2012); Acharya, Pedersen *et al.* (2017) and Brownlees and Engle (2017) that investigate an individual institution's capital shortfall when the system is in distress, and our proposed network focuses on the extreme risk spillover interconnectedness across different financial institutions. (ii) The data samples differ. Our sample includes 22 real estate companies that can influence the interconnectedness across financial institutions and thus may lead to a different ranking. For example, some real estate companies (e.g. General Growth Properties and Essex Property Trust) have high out-degree rankings in the 1% VaR network.

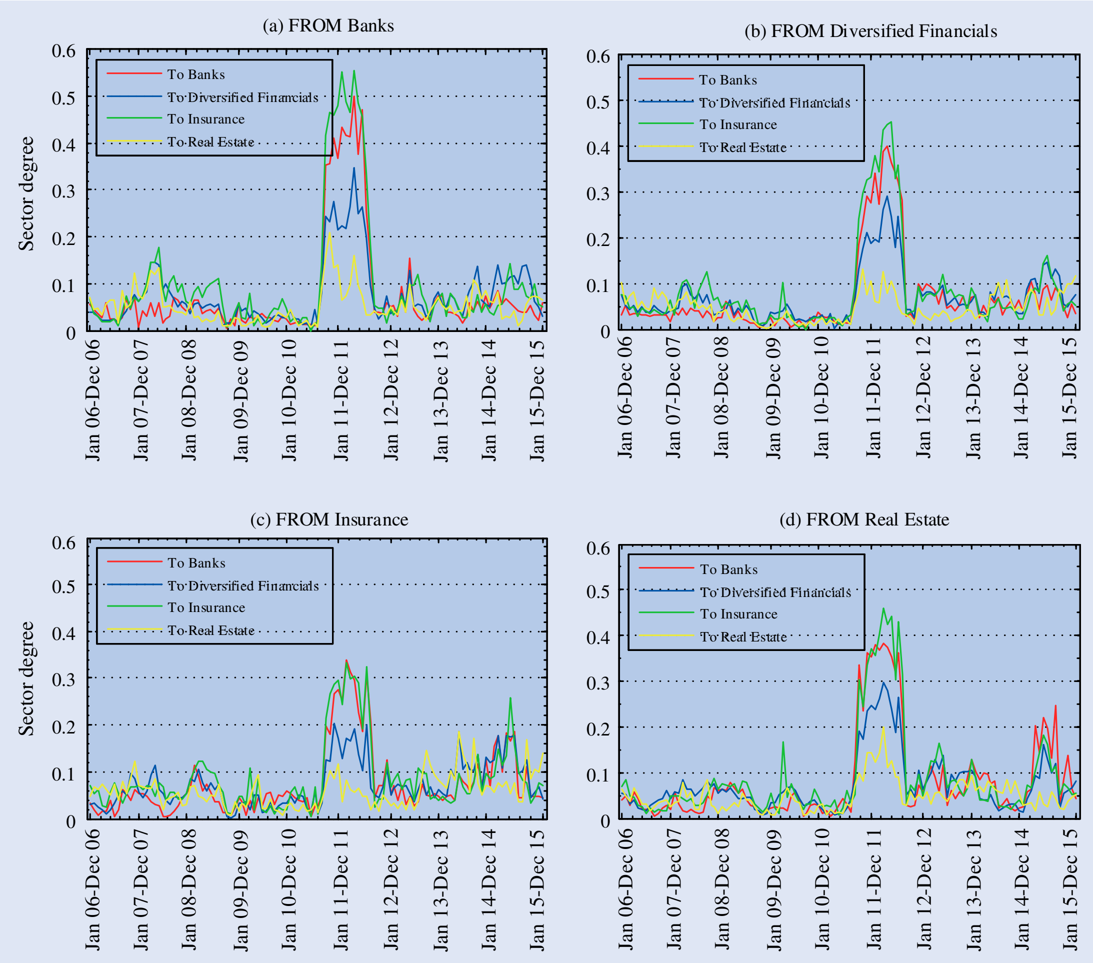

Figure 11. Dynamic SD from one sector to another or to itself in time-varying 5% VaR networks when M=10.

ranked by RI. From the RI of each institution in these two networks, we conclude (i) that most real estate firms are net senders of extreme risk spillovers, (ii) that most insurance companies are net recipients and (iii) that approximately half of the banks and diversified financial institutions are either net senders or net recipients.

To examine the sector-level connectivity of extreme risk spillover networks, in figures 5 and 6 we present the SD from one sector to another or to itself as a function of lag M in 1% and 5% VaR networks. The trend for most SDs is similar to that of ND and GE, i.e. it increases rapidly at first and then rises smoothly as the lag increases. We first look at 1% VaR networks under different lags (figure 5). Figure 5(a) shows that the SDs from banks to banks, diversified financials and insurance are approximately equal (especially when $M \geq 15$), and the SD to real estate is the smallest. This suggests that the most extreme risk from banks spills over to itself, diversified financials or insurance, and the least to real estate. Figure 5(b) shows that the most extreme risks of diversified financials are spilled over to banks, followed by itself, insurance and real estate. Figure 5(c) shows that real estate is the largest recipient of extreme risk spillovers from insurance, followed by insurance itself, banks and diversified financials. Figure 5(d) shows that the bank sector is the largest recipient of extreme risk spillovers from real estate, followed by diversified financials, itself and insurance. The case of 5% VaR networks under different lags (see figure 6) differs from 1% VaR networks. For the recipients of extreme risk spillover from any sector, insurance is the largest, followed by banks, diversified financials and real estate.

Figure 7 shows the RI of each sector in 1% and 5% VaR networks under different lags. By ranking the RI values under different lags, we find that real estate takes the lead, followed by banks, diversified financials and insurance, and that the RI values of real estate and banks are positive and those of the other two sectors are negative. From this we conclude (i) that real estate is the largest net sender of extreme risk spillovers, followed by banks, and (ii) that insurance is the largest net recipient, followed by diversified financials. These findings are in accord with the US subprime crisis, which was trigged by the collapse of real estate bubbles in US that caused mortgage delinquencies and the foreclosure and devaluing of real estate securities, which then spread to banks and insurance companies and finally swept across other financial institutions.

4.2. Time-varying networks

In the financial system, trading and investing strategies made by market participants vary across time, and the interactive behaviour across financial agents, also changes across time. Here, we examine how extreme risk spillover networks vary as time changes. Using 250-day rolling windows, we build

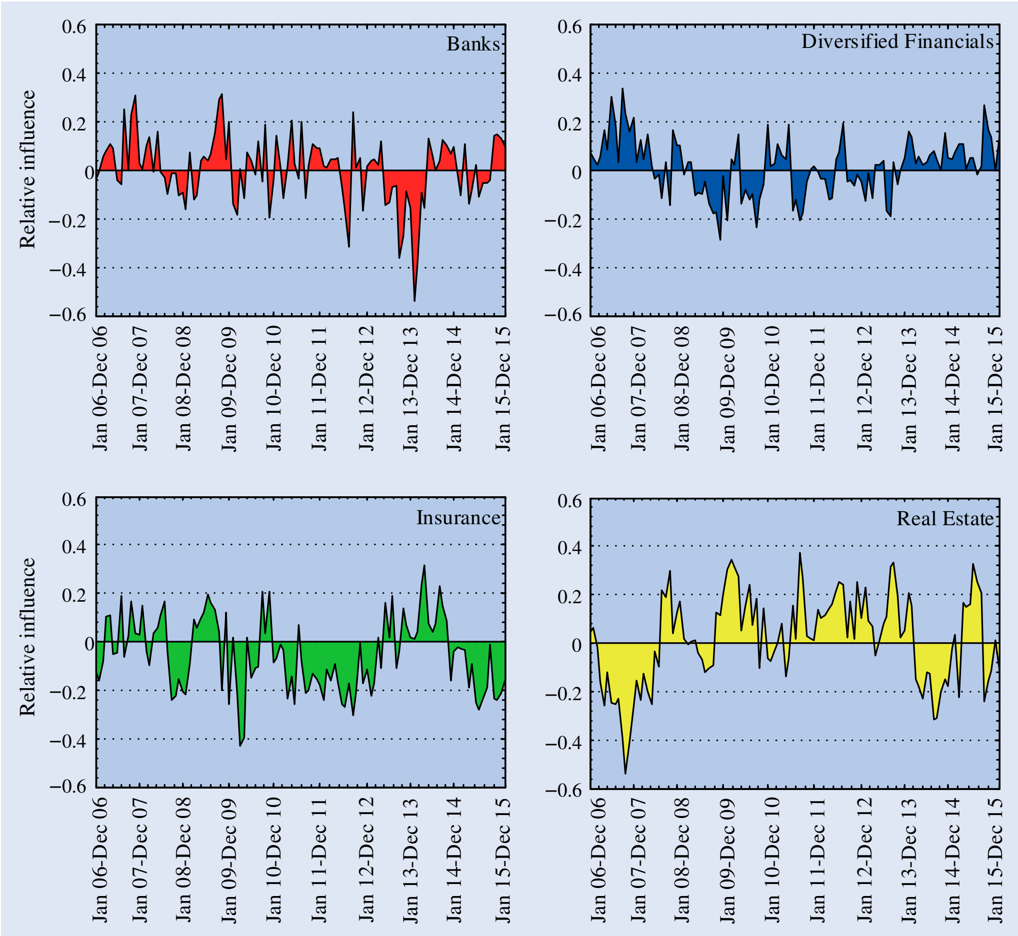

Figure 12. Dynamic RI of each sector in time-varying 1% VaR networks when $M = 10$.

time-varying 1% and 5% VaR networks when $M = 5$, 10, and 20 (lags roughly equivalent to one trading week, two trading weeks and one trading month, respectively).†

Figures 8 and 9 show the dynamic ND and GE of time-varying 1% and 5% VaR networks. The dynamic ND and GE of time-varying 1% VaR networks increase as the lag order increases, but this pattern is not obvious in time-varying 5% VaR networks. For different lags, the trend of dynamic ND (GE) in time-varying 1% and 5% VaR networks shows a similar pattern but with different levels. Figure 8 shows that the ND of time-varying 1% and 5% networks is large and forms a significant peak during two periods, (i) the global financial crisis of 2008–2009 (Period I), and (ii) from mid-2010 to Q4 2011 (Period II). Period II is the worst interval in the European sovereign debt crisis. During this period, there was a 'July-August-2011 stock market crash' across the US, Europe, the Middle East and Asia. This crash was caused by several factors, (i) fears that the European sovereign debt crisis would spread to Spain and Italy, (ii) concerns about France's downgraded credit rating and (iii) concerns about the slow economic growth in the US and its downgraded credit rating from AAA to AA+. These two periods can also be seen in the GE of time-varying 1% and 5% VaR networks (see figure 9). In addition, during the US subprime crisis the GE strongly fluctuated. In summary, when a market faces financial and macroeconomic uncertainty (e.g. a financial crisis and a flash cashing of stocks), ND and the information exchange efficiency of extreme risk spillover networks become strong and large because of the increase in interconnectedness across financial institutions. These findings also imply that measurements of ND and GE can be used to identify when systemic risk and abnormal behaviour will occur in the market.

†In previous research, Wang *et al*. (2010) and Liu and Wan (2011), who, respectively, investigate cross-correlations between the Chinese A-Share and B-Share markets and between crude oil spot and futures markets using rolling windows, explain how to choose the window width. They indicate (i) that one can use a large window width (e.g. four trading years) to study the long-term market dynamics because the evolution of statistic properties is smooth and general tends can be detected and (ii) that to examine the effects of exogenous events (e.g. seasonal factors and financial crisis) on market short-term dynamics, a small window width (e.g. a 250-day trading year) is a better option. They also state that when the window width is too small the statistical measures evolve too rapidly, making local trends difficult to observe. Thus, we choose a small window width of 250 trading days and a step size of 20 trading days to study the dynamic interconnectedness across financial institutions. To test the robustness of our results, we also consider four cases of window width L and step size δ: (i) $L = 250$ and $\delta = 10$, (ii) $L = 250$ and $\delta = 1$, (iii) $L = 225$ and $\delta = 20$ and (iv) $L = 200$ and $\delta = 20$. In (i) and (ii), we keep the same window width $L = 250$ but change the step size. In (iii) and (iv), we keep the same the step size $\delta = 20$ but change the window width. Because when we use these window widths and step sizes the dynamic ND and GE results for the time-varying extreme risk spillover networks are similar to those in figures 8 and 9, we hold that our results are robust. The detailed results are available upon request.

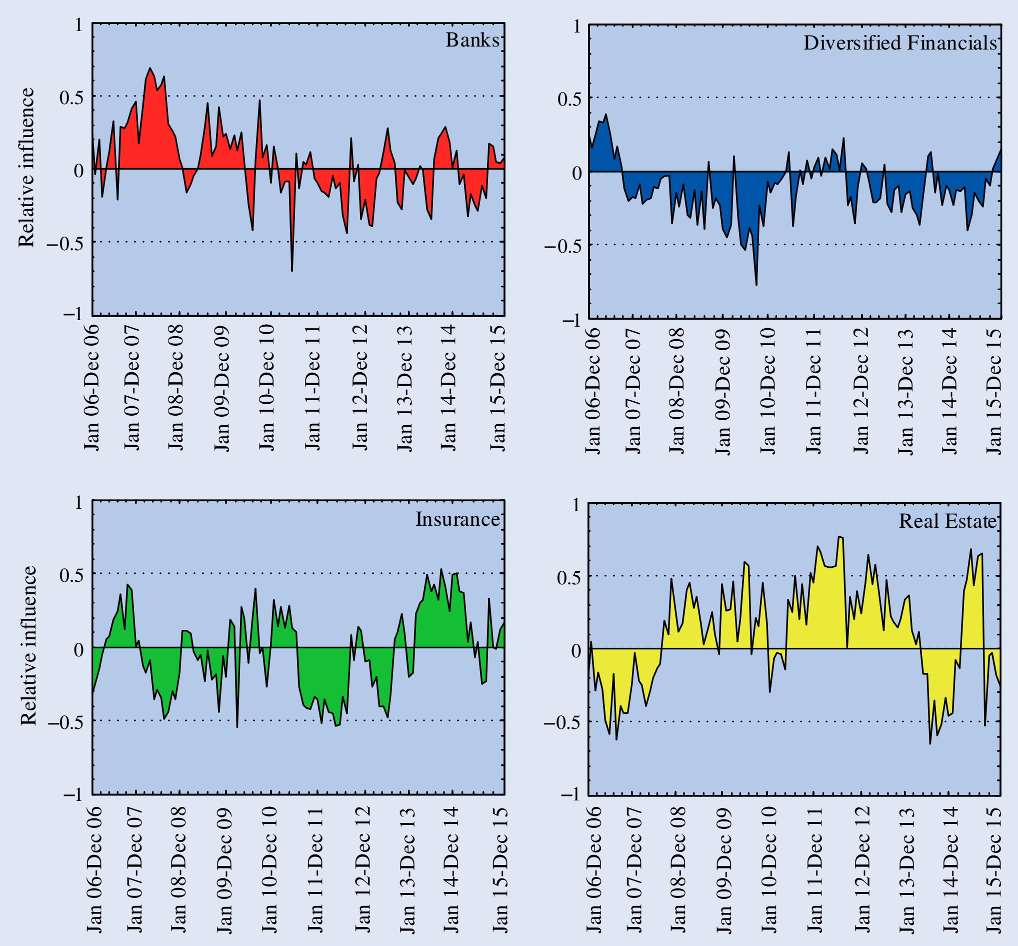

Figure 13. Dynamic RI of each sector in time-varying 5% VaR networks when $M = 10$.

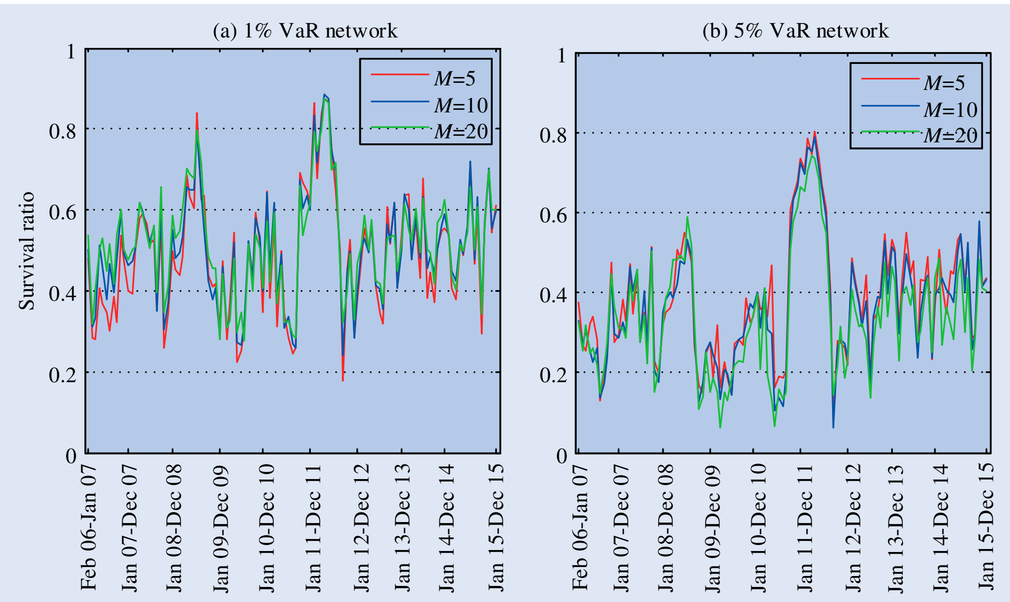

Figure 14. Survival ratios of time-varying 1% and 5% VaR networks when $M = 5$, 10, and 20.

Interestingly, figures 8 and 9 show that the dynamic ND and GE measures of the two networks (especially of the 5% VaR network) have a spike in 2011 that is much larger than the increase during the 2008 financial crisis after the Lehman Brothers collapse. The financial crisis in September 2008 was a much more severe systemic event for the US financial institutions than the European sovereign debt crisis in 2011. Our finding is somewhat inconsistent with this, but it is similar to the outcome reported by Adrian and Brunnermeier (2016) who find that systemic risk measures during the Great Depression were smaller than those during the recent financial crisis. Following Adrian and Brunnermeier (2016), we explain this inconsistency in two ways. First, our finding is a consequence of an artefact of the composition of financial institutions because our sample does not include some systemically important firms in 2008 that were bankrupt, merged or acquired. For example, Bear Stearns was sold to JP Morgan Chase in March 2008, Merrill Lynch was acquired by Bank of America on 14 September 2008, Lehman Brothers collapsed on 15 September 2008, Washington Mutual was sold to JP Morgan Chase in September 2008 and Wachovia was fully acquired by Wells Fargo on 31 December 2008. Our sample also does not contain two key players in the 2008 financial crisis, i.e. Fannie Mae and Freddie Mac, because they were dropped in the S&P 500 index in 2008. Second, our results may have a 'survivorship bias' because the extreme risks of our investigated financial institutions that survived the 2008 financial crisis may be lower and thus the dynamic ND and GE values in the 2008 financial crisis were not as large as those during the European sovereign debt crisis in 2011.

We examine the dynamic SD from one sector to another or to itself in time-varying 1% and 5% VaR networks when $M = 10$, and we present the results in figures 10 and 11, respectively. In both time-varying 1% and 5% VaR networks, the SDs for different sectors change over time. In the time-varying 1% VaR network, because the order changes across time we cannot determine the sector ranking order of recipients of extreme risk spillovers from a particular sector, but we find that the SDs reach a high value during crisis periods. The situation in the time-varying 5% VaR network is the same as in the time-varying 1% VaR network, but during Period II the SDs exhibit a huge peak that suddenly increases and then falls rapidly. During Period II, the order of sector recipients of extreme risk spillovers is insurance, banks, diversified financials and real estate, an order identical to that in the full sample.

Figures 12 and 13 show the dynamic RI of each sector in time-varying 1% and 5% VaR networks when $M = 10$. Figure 12(a) shows the dynamic RI of time-varying 1% VaR networks and that the RI of banks changes over time with positive and negative values, but during most crisis periods the RI takes the positive value, i.e. the bank sector acts as the net sender of extreme risk spillovers. Figure 12(b) shows that the RI of diversified financials also changes over time with positive and negative values, but that the RI of diversified financials is positive (i) from Q1 2006 to Q4 2007 and (ii) from mid-2012 to Q4 2015. Figure 12(c) shows that insurance is usually a net recipient of extreme risk spillovers except (i) from Q2 2006 to Q4 2007 and (ii) from mid-2012 to Q4 2013. In contrast, most of the time real estate is the net sender of extreme risk spillovers except for two periods prior to mid-2007 and Q1–Q4 2013 [see figure 12(d)].

Figure 13 shows the dynamic RI of time-varying 5% VaR networks and how their outcomes are similar to those in time-varying 1% VaR networks, but there are two major differences, i.e. (i) prior to 2010 the bank sector nearly always acts as a net sender of extreme risk spillovers and (ii) the sector of diversified financials is usually a net recipient except during the period prior to Q4 2006. Real estate and banks are usually net senders of extreme risk spillovers and diversified financials and insurance are usually net recipients, but note that the roles of net sender and net recipient for sectors may switch at tipping points of financial and macroeconomic uncertainty.

Figure 14 shows the survival ratios (SRs) of time-varying 1% and 5% VaR networks when lag $M = 5$, 10 and 20 in order to explore the interconnectedness robustness of extreme risk spillover networks. For these three lags, the SRs show a similar trend. The SRs for time-varying 1% VaR networks are greater than those for time-varying 5% VaR networks, indicating that more links survive from one period to the next in the 1% VaR network than in the 5% VaR network. We find that the SRs during Periods I and II are significantly different and larger than during other periods, suggesting that during financial crisis periods the interconnectedness across financial institutions becomes stronger but more sensitive because market participants frequently take the same action when facing financial and macroeconomic uncertainty.

5. Conclusions

We have proposed an extreme risk spillover network using the CAViaR method and the Granger causality risk test. This network allows us to analyse the potential channels of extreme systemic risk spillovers across financial agents. We have used our network to analyse the interconnectedness among 84 publicly listed financial institutions from the S&P 500 index.

In our empirical analysis, we have constructed two extreme risk spillover networks at 1% and 5% risk levels, i.e. 1 and 5% VaR networks. Our work has focused on (i) the time-lag effect of extreme risk spillover networks and (ii) time-varying extreme risk spillover networks. We first investigated the relationship between the networks and lag orders M. The empirical results show that our proposed network indeed incorporates the time-lag effect, and we conclude that market participants who want to use extreme risk spillover networks should consider a long lag (e.g. at least five trading days) because past market information can be fully reflected in long lagged networks. We have also examined the dynamic features of time-varying extreme risk spillover networks. Our two key empirical results indicate (i) that real estate and banks usually act as net senders of extreme risk spillovers, and insurance and diversified financials as net recipients, which is consistent with the evidence from the recent global financial crisis and (ii) that the networks' topological characteristics identified the abnormal behaviour of the market during the 2008–2009 financial crisis and the European sovereign debt crisis.

This extreme risk spillover network supplements the literature on econometric-based networks. It provides a new tool for studying the interconnectedness among financial agents from a systemic risk perspective. Our empirical results are valuable to both market participants (e.g. investors and hedgers) and regulators. For example, we find on average that the real estate and bank sectors are the instigators of extreme risk spillovers and the insurance and diversified financials sectors are the victims. Thus, when risk-averse investors in the stock market build an asset portfolio, they should select companies that can defuse extreme risk shocks and not instigate extreme risk spillovers. Market regulators should pay attention to the real estate institutions and banks in order to prevent or ameliorate extreme risk shocks from these two sectors. For example, regulators of the insurance sector should be wary of real estate companies and banks and their extreme risk shocks and should add investment restrictions and raise capital requirements.

Both our work and the Granger causality network of Billio *et al*. (2012) are limited in that the proposed network is unweighted and thus does not provide the information contained in a weighted network. For example, although a financial institution may have few edges, these edges may transmit huge amounts of risk and thus the institution is systemically relevant. In an unweighted network, this extreme phenomenon is undectectable. On the other hand, Barrat *et al*. (2004) investigate the architecture of complex weighted networks and find that the average strength $s(k)$ of nodes with degree k (i.e. k edges) increases with degree and that $s(k) \sim k^{\beta}$. They also find that the relationship between the strength s_i and degree k_i of node i is $s_i = \langle w \rangle k_i$, where $\langle w \rangle$ is the average weight of the network. Their findings suggest that on average a high-degree node (institution) has a higher strength and thus our investigation based on institution-level connectivity measures are meaningful. We leave the topic of building a weighted extreme risk spillover network for future study. One possible direction for constructing a weighted extreme risk spillover network is using the statistical value and p-value or their variations in the Granger causality risk test as the weight between two financial institutions.

Our network can be extended into several applications. Our study focused only on the US financial market, but more international or cross-regional financial institutions could be added in future study, because in an international business environment the interconnectedness among financial institutions is global and the crisis contagion is worldwide. For example, Diebold and Yilmaz (2015) investigate the interconnectedness among US and European financial institutions using a volatility spillover network. Another important application is using network science to predict the systemic influence or risk in the financial system (Hautsch *et al*. 2014). Our proposed network could serve as a tool that provides an early warning when systemic risk is growing in a financial system.

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Appendix 1. Backtesting for CAViaR and GARCH models

Testing whether the CAViaR method is necessary when estimating time-varying VaRs. We use two GARCH(1,1) models (i.e. AR(1)-GARCH(1,1)-Gaussian and AR(1)-GARCH(1,1)-Skewed-t) and the variance–covariance approach to estimate time-varying VaRs for a comparison. The autoregressive (AR) model is included because equity returns of financial institutions are autocorrelated. The AR(1)-GARCH(1,1)-Skewed-t model is chosen because fat-tailed equity returns is a necessary condition for the GARCH modelling and the Skewed-t distribution addresses the fact that the standardized equity returns (or residuals) have fat-tails. We employ three backtesting techniques, i.e. the likelihood ratio (LR) tests of unconditional coverage (uc), independent (ind) coverage and conditional coverage (cc), to assess the accuracy of VaR estimates of 84 financial institutions during the entire period 2006–2015, where LR_{uc} (LR_{ind}) $\sim \chi^2(1)$ and $LR_{cc} \sim \chi^2(2)$ (see, Kupiec 1995, Christoffersen 1998). We compare the accuracy of VaR calculated by the CAViaR model and the two GARCH(1,1) models, and in table A1 show the fraction of financial institutions whose VaR estimates pass the tests of LR_{uc}, LR_{ind} and LR_{cc}. From table A1, we find (i) that the CAViaR model for either 1% VaR or 5% VaR estimates performs the best, (ii) that the accuracy of the AR(1)-GARCH(1,1)-Gaussian is the lowest, indicating that the fat-tailed returns greatly influence the accuracy when estimating VaR and (iii) that in estimating 1% VaR, the performance of the AR(1)-GARCH(1,1)-Skewed-t model is close to that of the CAViaR model. Thus the CAViaR model is the better option. It performs the best and does not require any assumption on the distribution of equity returns.

Table A1. Fraction of financial institutions whose VaR estimates pass the likelihood ratio (LR) tests of unconditional coverage, independent coverage and conditional coverage.

	CAviaR			AR-GARCH-Gaussian			AR-GARCH-Skewed-t		
	LR_{uc} (%)	LR_{ind} (%)	LR_{cc} (%)	LR_{uc} (%)	LR_{ind} (%)	LR_{cc} (%)	LR_{uc} (%)	LR_{ind} (%)	LR_{cc} (%)
1% VaR	100	99	99	56	96	57	99	95	95
5% VaR	100	95	99	75	79	67	99	79	83

Notes: We use three backtesting techniques including the likelihood ratio (LR) tests of unconditional coverage (uc), independent (ind) coverage and conditional coverage (cc) to examine the accuracy of VaR estimates of 84 financial institutions during the entire period from 3 January 2006 to 31 December 2015, where LR_{uc} and LR_{ind} follow the $\chi^2(1)$ distribution and LR_{cc} follows the $\chi^2(2)$ distribution. For VaR estimates of each financial institution at a given risk level, if the estimated statistic of the LR test is lower than the corresponding chi-square critical value at the significance level (corresponds to the risk level), they pass the corresponding LR test. AR-GARCH-Gaussian and AR-GARCH-Skewed-t mean that VaR estimates are computed by AR(1)-GARCH(1,1)-Gaussian and AR(1)-GARCH(1,1)-Skewed-t. For a detailed introduction for these three backtesting techniques, see Kupiec (1995) and Christoffersen (1998). The detailed results are available upon request.