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Statistical approach to partial equilibrium analysis

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ABSTRACT

A statistical approach to market equilibrium and efficiency analysis is proposed in this paper. One factor that governs the exchange decisions of traders in a market, named willingness price, is highlighted and constitutes the whole theory. The supply and demand functions are formulated as the distributions of corresponding willing exchange over the willingness price. The laws of supply and demand can be derived directly from these distributions. The characteristics of excess demand function are analyzed and the necessary conditions for the existence and uniqueness of equilibrium point of the market are specified. The rationing rates of buyers and sellers are introduced to describe the ratio of realized exchange to willing exchange, and their dependence on the market price is studied in the cases of shortage and surplus. The realized market surplus, which is the criterion of market efficiency, can be written as a function of the distributions of willing exchange and the rationing rates. With this approach we can strictly prove that a market is efficient in the state of equilibrium.

1. Introduction

In the past decade, econophysicists have devoted many efforts into understanding the performance of markets, in particular, of financial markets [1–10]. The efforts on empirical data yield more and more universal stylized facts, such as heavy tails of return distribution, volatility clustering, volume/volatility correlation etc. [11]. Moreover, various theoretical models have also been proposed and investigated in depth to explore the mechanism behind these facts [12–19]. Among these studies, agent-based modeling, as a popular approach to simulating the behavior of a complex system [20], occupies a prominent place. Although these results are intriguing and helpful, the econophysicists have been accused of ignoring the existing economics literature when developing such kinds of models [21,22].

Traditionally the study of markets has been the territory of economists. Once we open a textbook of economics to see what has been stated about markets, we will find a story of supply and demand. This story can be traced back to Alfred Marshall, who put forward the theory of markets and constituted the foundation of modern economics more than one hundred years ago [23]. Even though the statement of Marshall is not a perfect abstraction of real markets, it has been dogmatized and told through generations with little alteration. Stirred by this fact, econophysicists aim to rebuild economics, for instance, starting from proposing a new dynamic theory of markets to replace the main economics version [3]. However, "economics is not at all an empty box", as argued by Ormerod [22]. The underlying ideas of traditional analysis on the markets should not be put aside or discarded when we start to set up a new one.

The core theory of markets in economics is called partial equilibrium analysis (PEA) [25]. The PEA applies to one market of a specific private good or service. The traders involved in the market are divided into two groups: consumers and producers. On the one hand, a consumer values each unit of the good he would like to consume by maximizing his utility subject to his

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budget constraint, yielding an individual demand curve. The total market demand, which relates the quantity demanded in the market to a given price, can be obtained by summing up all the consumers' demand curves. On the other hand, whether producers are willing to produce the good depends on the cost of inputs and technology. By assuming profit-maximizing behavior and complete competition, one individual producer's marginal cost curve becomes its supply curve. Likewise, all producers' marginal cost curve becomes the supply curve of the market.The actual exchange quantity and the level of price are thought of being determined by the equilibrium point at which the supply and demand curves intersect.

As a merit of PEA, supply and demand collectively represent two sides of traders in a market, so the consequence of the interaction between them can be easily figured out by getting the equilibrium point. As a result, the causality between any change of relevant factors and the market outcome can be understood very well through following the shifts of supply or demand [24]. Actually, the change in prices of financial assets has been mimicked by taking the imbalance of supply and demand into account [16–19]. This tool is easy to grasp due to its simplicity and concision. However, this kind of representation inclines us to overlook the individual features of traders and the essence of interactions among them behind the supply and demand. Such defect greatly limits the scope of the applications of PEA. In order to apply PEA efficiently into the problems of markets with rapid change or volatile fluctuation, we need to equip it with individual characteristics of traders.

Indeed, PEA is erected on the basis of individual analysis. The basic properties of supply and demand are derived from typical producer's and consumer's optimization behaviors respectively. This representative agent approach may commit the so-called fallacy of composition [26]. It also faces several other difficult dilemmas. Firstly, this individual analysis is incomplete, in which only consumers are regarded as representative of all buyers, and producers are used to cover all possible sellers. The properties of supply and demand should be derived from common characteristics of broad sellers and buyers, instead of these two specific ones. Secondly, the diversity of traders on the same side is neglected. Actually, as indicated by our analysis in the next section, the essential laws of markets originates from this kind of diversity. Moreover, most of dominating individual analysis is static, which makes it impossible to apply this approach properly to the dynamic process of markets.

Aiming to mimic the dynamic process of markets and involve different types of interactive agents, considerable attempts have been made in the past [12–19]. These achievements provide us more insights about performance of markets than traditional PEA. However, concerning how to evaluate the performance of a market, previous discussions either paid less attention or made improper choices. For instance, in the studies of Minority game, the efficiency was measured by the variance of the attendance [27,28]. This choice of criterion came from an intuition instead of a firm economic base. As argued by Yi-Cheng Zhang in other literature, it must be the economic efficiency that can evaluate the final outcome of markets performance [29,30].

In PEA, the economic efficiency is measured by market surplus, which is the sum of consumer surplus and producer surplus. The amount by which the value of purchases to consumers exceeds the amount paid is called consumer surplus. Producer surplus refers to the amount by which producers' receipts from sale of the goods exceed the total private cost of production. When the market is in equilibrium, the sum of economic surplus is maximized. Setting the price at a state other than equilibrium one, we can see a loss in total social benefits which is called deadweight loss in economics textbooks. This analysis on the relation between market equilibrium and economic efficiency has been shown to be very powerful when being applied to examine how the welfare of market participants changes with government policy. It is also expected to be a useful tool to evaluate performance of financial markets [31].

The purpose of this work is to bridge the gap between current modeling of markets and traditional PEA. Our main goal is to present a restatement of PEA in a statistical way and provide a framework for theoretical study of markets, especially for agent-based market modeling.

2. Willingness price, supply and demand

In contrast to the general equilibrium analysis, which concerns about all markets that comprising the whole economy, PEA has been used to examine only a single commodity market, in which the commodity exchanged by buyers and sellers might be one of consumption goods, inputs of production or financial securities and so on. In this market, buyers and sellers have their own willingness price before trading, which is usually called "reservation price" or "cost" in economic literature [32]. In our work, the willingness price of a buyer is defined as the maximum amount that he is willing to pay for one unit of the good. On the other hand, the willingness price of a seller is defined as the minimum amount that he is willing to sell one unit of the good.

It follows from these definitions that the willingness price plays a crucial role in the exchange between the traders of a market. As the market price of the commodity is given, the rational traders will make a decision of whether or not to reach a settlement at this price by comparing the actual price with their own willingness price. In each transaction, given the actual market price p, if a buyer's willingness price, υ^d, is not less than p, i.e.,

$$\upsilon^d \geq p, \tag{1}$$

then he will buy one unit of the good. Otherwise, he will give up his purchase. On the other hand, if a seller's willingness price, υ^s, is not greater than p, i.e.,

$$\upsilon^s \leq p, \tag{2}$$

then he will sell one unit of the good. Otherwise, he will withdraw his offer. Thus the necessary condition for closing a bargain is that the buyer's willingness price must be greater than or equal to the seller's willingness price, and the closing price must lie between the buyer's willingness price and the seller's one.

We now consider a market where a group of sellers and buyers trade for one commodity. Each trader is endowed with a specific willingness price which will not change as the market price varies. Each trader only trade one unit of the commodity in each transaction. It is reasonable to assume that willingness price spreads over the domain of $(0, \infty)$. These spreads can be characterized by probability density functions $f_s(\upsilon)$, $f_d(\upsilon)$ for sellers and buyers respectively. Suppose the numbers of sellers and buyers are Q_{st} and Q_{dt}, and denote $F_s(\upsilon) = Q_{st} * f_s(\upsilon)$ and $F_d(\upsilon) = Q_{dt} * f_d(\upsilon)$ correspondingly, according to the normalizations of corresponding probability functions, we then can get the integrals of $F_s(\upsilon)$ and $F_d(\upsilon)$ over the whole region of willingness price as the following respectively,

$$\int_0^\infty F_s(\upsilon)\mathrm{d}\upsilon = Q_{st}, \tag{3}$$

$$\int_0^\infty F_d(\upsilon)\mathrm{d}\upsilon = Q_{dt}. \tag{4}$$

In general, we can regard Q_{st} as the total potential quantity supplied by the sellers, and Q_{dt} as the total potential quantity demanded by the buyers. They actually reflect the scale of the market and the values of them are positive and finite.

We further assume that the price of the good is set exogenously and all the traders are price takers. So given the market price p, from the necessary conditions specified by Eqs. (1) and (2), we know that only the sellers whose willingness prices are not greater than the actual price are willing to sell the good, and only the buyers whose willingness prices are not less than the price are willing to purchase the good. Thus the supply and demand functions can be written respectively as follows:

$$Q_s(p) = \int_0^p F_s(\upsilon)\mathrm{d}\upsilon, \tag{5}$$

$$Q_d(p) = \int_p^\infty F_d(\upsilon)\mathrm{d}\upsilon. \tag{6}$$

These statistical expressions show that the quantity supplied is a portion of Q_{st}, while the quantity demanded is a part of Q_{dt}.

One important inference of supply and demand formula of (5) and (6) is the relation between quantity supplied or demanded and the market price. It can be seen from the first derivatives of supply and demand functions with respect to the price, which are given by

$$\frac{\mathrm{d}Q_s}{\mathrm{d}p} = F_s(p), \tag{7}$$

$$\frac{\mathrm{d}Q_d}{\mathrm{d}p} = -F_d(p). \tag{8}$$

Due to the non-negativity of $F_s(p)$ and $F_d(p)$, we have the following properties immediately,

$$\frac{\mathrm{d}Q_s}{\mathrm{d}p} \geqslant 0, \tag{9}$$

$$\frac{\mathrm{d}Q_d}{\mathrm{d}p} \leqslant 0. \tag{10}$$

The results state that when market price gets higher, less is demanded and more is supplied. These properties are called laws of supply and demand in economics, which are very important bases of the building of microeconomic theory [33]. When economists claim the law of demand or that of supply, they always add a confining term, such as "other things being equal" or "all the other variables that determine quantity supplied or demanded are held constant" [33]. This is identical to the assumption that the willingness prices keep unchanged when the market price varies.

Many economists have been putting their efforts into the derivation of these laws. They have derived the law for individual supply from the diminishing marginal return law, and the law for individual demand from the diminishing marginal utility law [34]. However, difficulties arise when turning to the aggregate level of a market to see whether the laws are still valid [35]. Eqs. (9) and (10) indicate obviously that it is true for the collective supply and demand as long as the willingness prices of buyers and sellers are all given initially and keep unchanged during the process of transactions.

Another inference of Eqs. (5) and (6) is the impacts of relevant factors on the supply and demand. The statistical expressions of supply and demand indicate that all the relevant factors are incorporated into the functions of $F_s(\upsilon)$ and $F_d(\upsilon)$. Generally, the main channels through which any change of relevant factors takes its effect can be categorized into two ways. One is to change the scale of the market, the other is to change the willingness prices of traders. When any one factor changes, the corresponding potential total quantity or the value of willingness price or both of them will change as a response and then result in a new supply or demand.

As the economic environment of a market always changes over time, the supply and demand of the market should not be fixed. For any two relevant markets, their supply and demand are dependent with each other, since the willingness prices of one good are associated with the price of the other one. In some cases, the willingness prices of traders for one good may even depend on its actual market price. To address these complicated interactions, we must resort to the general equilibrium analysis, which considers many markets together. In contrast, PEA leaves all these effects out of consideration by regarding the market as isolated one from others. This is to say that PEA aims to a single market with fixed forms of $F_s(\upsilon)$ and $F_d(\upsilon)$.

3. Equilibrium and non-equilibrium

Supply and demand are two indispensable parts of the market, which are derived from the willingness of sellers and buyers respectively. The change in anyone of them will have impacts on the outcomes of transactions in the market. In order to study the impacts, it is convenient to examine the excess demand function $E(p)$, which is defined as quantity demanded minus quantity supplied, i.e.,

$$E(p) = Q_d(p) - Q_s(p). \tag{11}$$

According to the sign of excess demand, the state of a market can be sorted into three cases as follows. First, when quantity demanded is equal to quantity supplied, namely $E(p) = 0$, the market is at equilibrium. In this case, the market price is called equilibrium price or market-clearing price. Second, when quantity demanded is greater than quantity supplied, namely $E(p) > 0$, the market now is in shortage. Third, when quantity demanded is less than quantity supplied, namely $E(p) < 0$, the market is in surplus.

From Eq. (11) we know that the sign of excess demand function is only governed by the price. When $p = 0$, we have

$$E(0) = Q_{dt}. \tag{12}$$

As price goes to infinity, we can get

$$E(\infty) = -Q_{st}. \tag{13}$$

So the excess demand changes from a positive value to a negative one as the price increases. This property can also be validated by the sign of price derivative of excess demand function, which can be obtained from Eqs. (7), (8) and (11),

$$E'(p) = -F_d(p) - F_s(p) \leq 0. \tag{14}$$

All these facts indicate that the necessary condition of existence of equilibrium is the consistency of the excess demand function. Besides, if the function is monotone, then there will be only one market-clearing price. When the functions $F_d(\upsilon)$ and $F_s(\upsilon)$ are not zero for the price p around the equilibrium point, we can have

$$E'(p) < 0, \tag{15}$$

which guarantees the monotone property of excess demand. Then there must be one equilibrium price $p*$ which satisfies $E(p*) = 0$.

For the market price is exogenous, it is not always the same as market-clearing price. When the market price p is lower than $p*$, a shortage will be brought about. On the other hand, if market price p is higher than $p*$, a surplus will result. Thus, the realized quantity of transaction depends on the price. When a market is at equilibrium, it is equal to quantity supplied or quantity demanded. But if the quantity supplied is not the same as quantity demanded, the realized quantity is determined by the minimum of them, namely,

$$T(p) = Minimize\{Q_s(p), Q_d(p)\}. \tag{16}$$

This rule is often called 'short-side principle', where the traders with more willing exchange are at the long-side and those with less are at the short-side.

This principle means that the traders at the short-side are able to make out all they want, but those at the long-side can realize only a part of their willing exchange. Now we define rationing rate as the actual exchange quantity divided by the willing exchange quantity, which are written for sellers and buyers respectively as follows,

$$G_s = \frac{T}{Q_s}, \tag{17}$$

$$G_d = \frac{T}{Q_d}. \tag{18}$$

From these definitions, it is obvious that the rationing rate spreads over the domain of $[0, 1]$. In a perfect market, the rationing rate of the traders at the short-side is one, and that of traders at the long-side should be less than 1. However in a real market the rationing rate of both sides may be less than 1 due to the imperfection of markets.

This overall rationing rate can also be described at individual level by taking its dependence on the willingness price into account. We can express the rationing rates in terms of willingness price for sellers and buyers respectively as follows:

$$G_s(\upsilon) = \frac{T_s(\upsilon)\mathrm{d}\upsilon}{F_s(\upsilon)\mathrm{d}\upsilon}, \tag{19}$$

$$G_d(\upsilon) = \frac{T_d(\upsilon)\mathrm{d}\upsilon}{F_d(\upsilon)\mathrm{d}\upsilon}, \tag{20}$$

where $T(\upsilon)\mathrm{d}\upsilon$ denotes the actual exchange of those traders whose willingness prices are between υ and $\upsilon + \mathrm{d}\upsilon$, and the subscripts s and d correspond to the sellers and buyers respectively.

Given the market price p, the total actual exchange can be obtained by summing up all the realized willing exchange of the corresponding traders. Thus the realized quantities of supply and demand can be given by

$$Q_{sr}(p) = \int_0^p T_s(\upsilon)\mathrm{d}\upsilon, \tag{21}$$

$$Q_{dr}(p) = \int_p^\infty T_d(\upsilon)\mathrm{d}\upsilon. \tag{22}$$

From Eqs. (19) and (20), they can be rewritten as

$$Q_{sr}(p) = \int_0^p F_s(\upsilon)G_s(\upsilon)\mathrm{d}\upsilon, \tag{23}$$

$$Q_{dr}(p) = \int_p^\infty F_d(\upsilon)G_d(\upsilon)\mathrm{d}\upsilon. \tag{24}$$

Because of the identity of transactions in a closed market, the amount that buyers actually have purchased must be equal to the amount that sellers have sold out. So the total actual exchange quantity can be got by integrating the individual realized quantity of supply and demand sides separately, i.e.,

$$T(p) = \int_0^p T_s(\upsilon)\mathrm{d}\upsilon = \int_p^\infty T_d(\upsilon)\mathrm{d}\upsilon. \tag{25}$$

This says that no matter what the price is, the total realized quantity of supply must be equal to that of demand. Then we have the following identity equation:

$$\int_0^p F_s(\upsilon)G_s(\upsilon)\mathrm{d}\upsilon \equiv \int_p^\infty F_d(\upsilon)G_d(\upsilon)\mathrm{d}\upsilon. \tag{26}$$

The rationing rate also changes with market price. When the price goes up, the quantity supplied will increase and the quantity demanded will decrease. So the overall rationing rate of buyers would become higher, while that of sellers would become lower. Then we have

$$\frac{\mathrm{d}G_s}{\mathrm{d}p} \leqslant 0, \tag{27}$$

$$\frac{\mathrm{d}G_d}{\mathrm{d}p} \geqslant 0. \tag{28}$$

Now we turn to see how individual rationing rates respond to the change in price. When market price $p = p*$, namely the market is in equilibrium, every willing exchange will be carried out. Then we get

$$G_s(\upsilon) = G_d(\upsilon) = 1. \tag{29}$$

In contrast, when market price $p < p*$, namely the market is in shortage, the sellers would sell all their goods, but the buyers could not get all of their willing purchases. Thus we have

$$G_s(\upsilon) = 1; \qquad G_d(\upsilon) < 1. \tag{30}$$

In this case, as the market price increases, the possibility that buyers get their rations becomes higher with the quantity supplied increasing and quantity demand decreasing. Meanwhile the rate of sellers will not change. It is reasonable to assume that the relation between rationing rate of buyers and market price is the same as that of overall rationing rate described by (28). We then have

$$\frac{\mathrm{d}G_s(\upsilon)}{\mathrm{d}p} = 0; \qquad \frac{\mathrm{d}G_d(\upsilon)}{\mathrm{d}p} \geqslant 0. \tag{31}$$

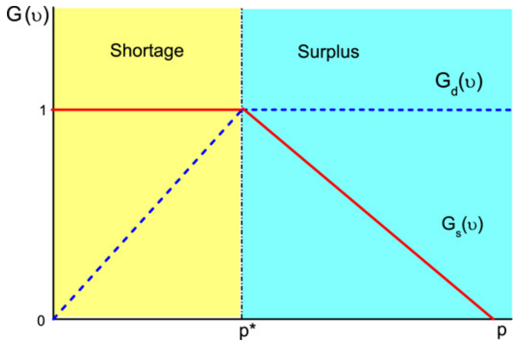

Fig. 1. Illustrative diagram of dependence of rationing rate on market price for sellers and buyers respectively.

For the opposite case: $p > p*$, the market is in surplus, according to the short-side principle the buyers could purchase all they want, but some sellers would fail to sell out even though they have willingness to do so. Then

$$G_s(\upsilon) < 1; \qquad G_d(\upsilon) = 1. \tag{32}$$

In this case, as the market price increases, the possibility that sellers sell their goods out becomes lower with the quantity supplied increasing and quantity demanded decreasing. Meanwhile the rationing rate of buyers will not change. Likewise, we can assume that the relation between rationing rate of sellers and market price can be formulated as (27). We also have

$$\frac{\mathrm{d}G_s(\upsilon)}{\mathrm{d}p} \leqslant 0; \qquad \frac{\mathrm{d}G_d(\upsilon)}{\mathrm{d}p} = 0. \tag{33}$$

As an illustration, the dependence of rationing rates on the market price is shown in Fig. 1. From this figure we can see that the equilibrium price $p*$ is a critical point. At this point, the rationing rates for both sides are 1. From the left to the right, the rationing rate of the sellers begins to decrease and that of the buyers is approaching 1. The change of rationing rates for both sides at this point is discontinuous.

4. Market surplus and efficiency

Trade makes everyone better-off. Economists usually employ market surplus to measure the benefits that the traders have gained through exchanges in markets. The surplus of an individual transaction is the difference between the buyer's willingness price and the seller's one. When an individual transaction is successfully made, according to the necessary conditions described by Eqs. (1) and (2), the closing price must lie within the willingness prices of trade pairs. Thus the magnitude of surplus resulted from this transaction must be non-negative and independent on the market price. The impact that the price has is only on the distribution of the surplus between the buyer and the seller. The part above the price is the buyer's surplus. The part below is the seller's surplus.

Given the market price p of the good, the total seller's surplus and buyer's surplus of the market can be calculated by adding up all the corresponding traders' surplus, which can be expressed for the sellers and buyers respectively as follows:

$$Z_{se}(p) = \int_0^p F_s(\upsilon)(p-\upsilon)\mathrm{d}\upsilon, \tag{34}$$

$$Z_{de}(p) = \int_p^\infty F_d(\upsilon)(\upsilon-p)\mathrm{d}\upsilon. \tag{35}$$

And the total surplus of the market is the sum of them, which is given by

$$Z_e(p) = \int_0^p F_s(\upsilon)(p-\upsilon)\mathrm{d}\upsilon + \int_p^\infty F_d(\upsilon)(\upsilon-p)\mathrm{d}\upsilon. \tag{36}$$

Actually, at a given market price, not all of the above surplus can be realized in transactions for the market is more likely to be at a non-equilibrium state. So we add a subscript e to all the surplus Z's to indicate they are just expected by the corresponding traders. The actual surplus results from the actual exchange. Like actual exchange, it can be formulated similarly by adding the rationing rates of traders. Thus, the realized surplus of the market for sellers and buyers respectively can be written as

$$Z_{sr}(p) = \int_0^p F_s(\upsilon)(p-\upsilon)G_s(\upsilon)\mathrm{d}\upsilon, \tag{37}$$

$$Z_{dr}(p) = \int_p^\infty F_d(\upsilon)(\upsilon-p)G_d(\upsilon)\mathrm{d}\upsilon. \tag{38}$$

Likewise, the total realized market surplus takes the following form,

$$Z_r(p) = \int_0^p F_s(\upsilon)(p-\upsilon)G_s(\upsilon)\mathrm{d}\upsilon + \int_p^\infty F_d(\upsilon)(\upsilon-p)G_d(\upsilon)\mathrm{d}\upsilon. \tag{39}$$

When the market is in equilibrium, both rationing rates of sellers and buyers are equal to one. Under this condition, the realized market surplus is equal to the expected one. But for other cases, due to presence of unrealized exchange, the realized market surplus must be less than the expected one.

From Eq. (39) we know that the variable that governs the market surplus is mainly the market price. The relation between them can be analyzed by looking into the price derivative of realized market surplus, which is given by

$$\begin{aligned}\frac{\mathrm{d}Z_r(p)}{\mathrm{d}p} &= \int_0^p F_s(\upsilon)G_s(\upsilon)\mathrm{d}\upsilon - \int_p^\infty F_d(\upsilon)G_d(\upsilon)\mathrm{d}\upsilon \\ &\quad + \int_0^p F_s(\upsilon)(p-\upsilon)\frac{\partial G_s(\upsilon)}{\partial p}\mathrm{d}\upsilon + \int_p^\infty F_d(\upsilon)(\upsilon-p)\frac{\partial G_d(\upsilon)}{\partial p}\mathrm{d}\upsilon. \end{aligned}\tag{40}$$

Substituting the identical Eq. (26) into (40), it turns into a simplified form

$$\frac{\mathrm{d}Z_r(p)}{\mathrm{d}p} = \int_0^p F_s(\upsilon)(p-\upsilon)\frac{\partial G_s(\upsilon)}{\partial p}\mathrm{d}\upsilon + \int_p^\infty F_d(\upsilon)(\upsilon-p)\frac{\partial G_d(\upsilon)}{\partial p}\mathrm{d}\upsilon. \tag{41}$$

Since the change of rationing rate at the equilibrium point is discontinuous as shown in Fig. 1, we examine how the realized surplus depends on the price in the two domains separately. When the market is in shortage, i.e., $p < p*$, substituting Eq. (31) into Eq. (41), we can obtain

$$\frac{\mathrm{d}Z_r(p)}{\mathrm{d}p} \geq 0. \tag{42}$$

In the other region of surplus, i.e, $p > p*$, substituting Eq. (33) into Eq. (41), we can get

$$\frac{\mathrm{d}Z_r(p)}{\mathrm{d}p} \leq 0. \tag{43}$$

These results indicate that on the left of $p*$ the realized surplus is monotone-increasing, but on the right of $p*$ it becomes monotone-decreasing. It follows immediately that the realized total market surplus reaches its maximum value when the market is in the equilibrium. In other words, the market is most efficient at the equilibrium point $p = p*$.

5. Conclusion and discussion

Partial equilibrium analysis is the foundation of market theory. In our paper, we rebuild the theoretical system of PEA in a statistical way. Using this method to proceed on reconstruction of PEA, the expressions of supply and demand as well as the market surplus can be presented. Based on these statistical expressions, the proposition that the market surplus will attain its maximum only at equilibrium point can be proved in a strict mathematical way.

The individual agents that compose a market are sorted into two broad groups: sellers and buyers. Their action in the market is to make exchange with each other. The most important factor affecting their choices in exchange is willingness price. A buyer's willingness price is defined as the highest price that he wants to pay. A seller's willingness price is defined as the lowest price that he would sell. Then the traders will make decisions according to their willingness prices of the good and the market price. Considering the distinctions of traders' willingness prices in a market, we introduce a kind of distribution function of willing exchange over the willingness price to express the differentia for each side of the market. By employing this function, we can get the statistical expression of supply and demand and concisely prove the laws of them.

The market is in equilibrium when quantity supplied is equal to quantity demanded. By analyzing the excess demand function, the necessary condition of the existent and exclusive equilibrium price is specified. The non-equilibrium of markets have two sorts: shortage and surplus. By introducing rationing rate, we can present the expression of actual exchange quantity when markets fall in non-equilibrium states. How the rationing rate changes with price in different conditions is also derived.

The market surplus is the difference between buyer's and seller's willingness prices. When market price is given, the traders' expected surplus is the one that corresponding to their willing exchange and the realized surplus is the one that corresponding to the actual exchange. Only when the market is in equilibrium, can all the expected surplus be realized. In other cases, the realized surplus must be less than the expected one. We used statistical function to express these two kinds of surplus and found that when market is in equilibrium the realized surplus gets to its maximum, namely, the market is most efficient at this state.

Analyzing markets in this way, we find the main link between the market analysis and individual analysis is willingness price. This concept facilitates understanding and generalizing the common properties and behaviors of the economic entities in markets. The willingness price can be refined with the individual optimization analysis. However, setting the willingness prices exogenously, markets can be studied separately. Consequently, the PEA can be carried out without considering the individual choices. This means the theory of PEA becomes more independent and self consistent.

In many works aiming to mimic financial markets in econophysics, the force that drives the markets to evolve is also deemed to be the change of supply and demand [17,24]. In fact, any change of them could be specified as a variation of willingness price of traders. The so-called strategy to buy or sell of the traders in many models actually corresponds to the formation of new willingness price as a response to the changing situation. Therefore, an outstanding model must be based on a penetrative understanding of the formation of willingness price.

This approach to market equilibrium and efficiency analysis not only simply reproduces the predictions of PEA with intensifying its logistical strictness, but also lets us know deeply some important aspects of real markets. For instance, this approach was applied to investigate the impact of asymmetric information on the market evolution and how reverse selection takes its effect [36]. The economic efficiency of financial markets was also discussed by specifying the roles of producers and speculators [31]. It is expected that this approach can be applied more extensively as an efficient tool in market analysis.

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