

Shao, Havlin, and Stanley Reply: A recent Letter [1] introduces a nonconsensus opinion (NCO) model in which two clusters holding two different opinions coexist. The NCO model in complex networks reveals a percolationlike phase transition in the process of opinion formation. A more recent report [2] claims that the linkage with invasion percolation with trapping (IPT) in Ref. [1] is based on simulations utilizing inadequate statistics and that the NCO model in 2D lattices belongs to the class of regular percolation. We argue below that this claim lacks solid evidence.

Reference [2] applies the NCO model to a larger square lattice system and finds a different τ for the probability density function (PDF) of the cluster sizes at criticality. Based on this finding, Ref. [2] claims the NCO belongs to the same universality class as regular percolation, and not to IPT. However, an excess of large clusters exists in the PDF of cluster sizes (note the bump in the right of their figure), which differs from the PDF of cluster sizes in regular percolation and also from our simulation of the NCO on a similar-sized square lattice system (see the inset of Fig. 1). The cutoff of the power law region in our simulation (at $S \sim 10^5$) also differs from that of Fig. 1 in Ref. [2] (at S well above 10^6). These differences indicate strong discrepancies between either the model or the network system used in Ref. [2] from the results shown in Ref. [1] and in Fig. 1. Note that in Ref. [1] as well as in this Letter we use an open boundary condition. In Ref. [2], however, the critical point is found in a lattice with open boundary conditions and the PDF of cluster sizes is obtained on a network with helical boundary conditions. Without maintaining consistent simulation conditions, the conclusion in Ref. [2] cannot be compared to that of Ref. [1].

Reference [3] reported recently that when the average degree of the network increases, the second-order phase transition becomes first-order. This is also mentioned in Ref. [2]. This interesting property contradicts the assertion of Ref. [2] that the NCO is the same as regular percolation.

Furthermore, Fig. 1 shows in simulations of a similar order of Ref. [2] that the power law exponent τ in $P(S) \sim S^{-\tau} \approx 1.955$ is significantly smaller than the value 2.055 reported in Ref. [2]. This discrepancy indicates that percolation in the NCO model differs from that of random percolation.

The fractal dimensions of the clusters at criticality, which are measured by the power law exponent of cluster sizes as a function of the radius of gyration (d_f) and the average hopping distance (d_l), are alternative good indicators of the universality class of the system [1] but not given in Ref. [2].

Although Ref. [2] claims the results shown in Ref. [1] violate the hyperscaling relation ($\tau = d/d_f + 1$), the

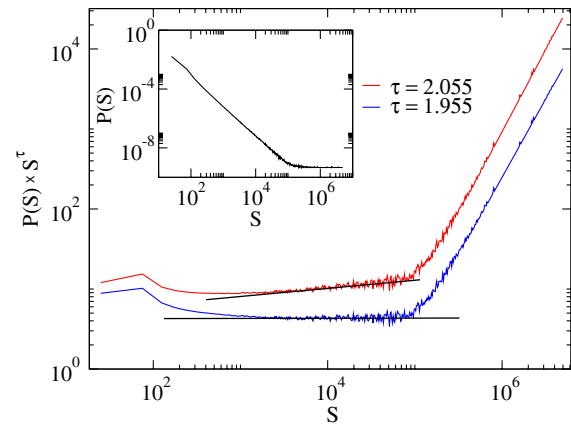


FIG. 1 (color online). The inset shows the PDF of the cluster sizes, $P(S)$, as a function of the cluster size S for square lattice of the size of 16×10^6 nodes (4000×4000) at criticality $f_c = 0.506$. The plot of $P(S)S^\tau$ as a function of the cluster size S for the same lattice at criticality is also shown for two different values of τ . Compared with $\tau = 2.055$, $\tau = 1.955$ is a better fit for the power law function $P(S) \sim S^{-\tau}$. Note that there is no bump of $P(S)$ at large S .

validity of the hyperscaling relation for the NCO and IPT models is greatly questionable. As shown in Ref. [1], the PDF of the trapped clusters in IPT is roughly 1.89, which is smaller than 2. If the hyperscaling relation holds, it implies the unrealistic result that $d_f > 2$.

In summary, the finding of Ref. [2] verifies differences between the NCO model and regular percolation but, taking into account the differences between our simulations and those of Ref. [2], the conclusion of Ref. [2] is based on insufficient evidence. Further analysis is thus needed to clarify these issues.

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Received 8 June 2012; published 16 August 2012

DOI: [10.1103/PhysRevLett.109.079802](https://doi.org/10.1103/PhysRevLett.109.079802)

PACS numbers: 89.75.Hc, 64.60.ah, 87.23.Ge

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