# Econophysics: Can Physicists Contribute to the Science of Economics?

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THE FUNDAMENTAL PRINCIPLES GOVERNING THE COMPLEX SYSTEM CALLED ECONOMICS ARE NOT COMPLETELY UNCOVERED. THIS OBSERVATION SEEMS TO BE ALMOST GENERALLY ACCEPTED; FOR EXAMPLE, THE 23 AUGUST 1997 ISSUE OF *THE ECONOMIST* FEATURED THE COVER ARTICLE, "THE PUZZLING FAILURE OF ECONOMICS."

Then how can computational physicists contribute to the search for solutions to the puzzles posed by modern economics that economists themselves cannot solve? An approach—not very commonly used in economics—is to begin empirically, with real data that you can analyze in some detail, but without prior models. In economics, a great deal of real data is available. If you, moreover, have at your disposal the tools of computational physics and the computing power to carry out any number of approaches, this abundance of data is a great advantage. Thus, for physicists, studying the economy means studying a wealth of data on a well-defined complex system. Indeed, physicists in increasing numbers are finding problems posed by economics sufficiently challenging to engage their attention, independent of any personal profit that might be made. Various terms have been applied to this new interdisciplinary subfield of physics. Some French physicists prefer the term *phynance*, while others prefer other terms. In an analogy with the terms *biophysics*, *geophysics*, and *astrophysics*, in 1994 or 1995 I introduced the term *econophysics* to attempt to legitimize why physics graduate students should be allowed to work on problems originating in economics.

If we physicists have any prior bias, it might be the lesson learned years ago when many of us worked on critical phenomena: Everything depends on everything else. A careful analysis of any system involves studying the propagation of correlations from one unit of the system to the next. We learned that these correlations propagate both directly and indirectly. At one time, it was imagined that "scale-free" phenomena are relevant to only a fairly narrow slice of physical phenomena. However, the range of systems that apparently display power-law and hence scale-invariant correlations has increased dramatically in recent years. Such systems range from base-pair correlations in noncoding DNA, lung inflation, and interbeat intervals of the human heart, to complex systems involving large numbers of interacting subunits that display "free will," such as animal behavior[1] and even human behavior.[2] In particular, economic time series—for example, stock market indices or currency exchange rates—depend on the evolution of a large number of strongly interacting systems far from equilibrium, and belong to the class of complex evolving systems. Thus, the statistical properties of economic time series have attracted the interests of many physicists. Space limitations motivate me to focus mainly on the Boston group's results; the work of other research groups is described elsewhere.

## Economic time series: correlations or the lack thereof

The recent availability of "high-frequency" data lets us study economic time series on a wide range of time scales varying from seconds up to years. Consequently, researchers have applied a large number of methods known from statistical physics to characterize the time evolution of stock prices and foreign exchange rates.

Much of our recent work is based on analysis of the S&P 500 index, an index of the New York Stock Exchange (NYSE) that consists of the 500 largest companies in the US. It is a market-value weighted index (stock price times the number of shares outstanding), with each stock's weight in the index proportionate to its market value.[3–5] The S&P 500 index is one of the most widely used benchmarks of US equity performance. Data typically cover a long period, such as 13 years (from January 1984 to December 1996), with a recording frequency of one minute or shorter. The total number of data points in this 13-year period exceeds one million, three orders of magnitude greater than Benoit Mandelbrot's classic analysis of cotton price fluctuations.[6]

The S&P 500 index $Z(t)$ from 1984 to 1996 tends to drift constantly upward on a semilog graph—except during crashes, such as in October 1987 and May 1990. We analyze the difference of the logarithm of the index values $G(t) \equiv \log_e Z(t + \Delta t) - \log_e Z(t)$, where $\Delta t$ is the time lag. We count only the number of minutes during the stock market's opening hours and remove the nights, weekends, and holidays from the data set. That is, the market's closing and next opening is continuous.

The distributions of the increments of economic time series, both in stock market indices and foreign currency exchange rates, turn out to be nearly symmetric and have very fat tails (strong leptokurtic wings). Index increments as a function of time show exponentially decaying correlations that are at noise level after a few minutes. This makes these increments fundamentally different from many well-studied examples of complex dynamical systems in physics. One such example is turbulent flow, which commonly displays power-law correlations on long time scales.[7,8]

The situation is different for the volatility, which is calculated, for example, averaging market fluctuations over a suitable time interval. The volatility has long time persistence—much larger than the correlation time for price changes.[4,5] Quantifying the volatility's dynamics is important. Volatility is the key input of virtually all option pricing models, including the classic Black and Scholes model, which is based on estimates of the asset's volatility over the option's remaining life.

Specifically, using both traditional power-spectrum methods as well as a new method—*detrended fluctuation analysis* (DFA)[9]—Yanhui Liu and his coworkers[4,5] detect long-range volatility correlations embedded in a nonstationary time series. This new method avoids the spurious detection of apparent long-range correlations that are an artifact of nonstationarities.

Both methods show the existence of two distinct regions of power-law behavior for the autocorrelation function of volatility, with the exponents $\alpha_1 = 0.66$ and $\alpha_2 = 0.95$ for $t$ less than or greater than a characteristic time scale $t_\times$ on the order of one day. It is as if the information used in a single day to make trading decisions differs from the long-term information used.

To test whether this correlation is a spurious artifact of the distribution function, which might have long tails, Liu and his colleagues shuffle each point of the volatility time series. The random shuffling keeps the volatility distribution unchanged but totally kills any correlations in the time series. DFA analysis of this randomly shuffled data does not show any correlations and gives the exponent $\alpha = 0.50$. This tells us that the long-range correlations are due to the economic system's dynamics and not simply due to the fat-tailed distribution, because the distribution does not change when the data are shuffled.

## Histograms of price changes

Although the correlations in the price change $G(t)$ are not particularly novel, the histograms of price changes are. Because economic systems consist of a large number of interacting units, it is plausible that they might be amenable to scaling analysis. Mandelbrot in 1963 demonstrated that the histogram of fluctuations in cotton prices obeys a scaling distribution, the *Lévy distribution*.[6] A recent study determined that the high-frequency fluctuations in the S&P 500 index also exhibit scaling behavior.[3] Analyzing almost one million records at one-minute intervals over six years of trading, Rosario N. Mantegna and I determined that fluctuations on a one-minute time interval were reflected in 10-minute, 100-minute, and 1,000-minute intervals.[3] The distribution of index returns fits a Lévy distribution with a sharp drop-off in the tails. These scaling properties mean that viewing stock market returns at one-minute intervals provides insight on the behavior at 1,000-minute intervals.

Thus, the Lévy part of the S&P 500 distribution agrees with Mandelbrot's 1963 cotton-price results, but the tail truncation does not (presumably because the tail statistics in the low-frequency results are not above the noise level). Recently, Parameswaran Gopikrishnan and his colleagues asked whether this discrepancy could be because the S&P is an average over many firms.[10] To this end, they analyze a database documenting every trade in the three major US stock markets—the NYSE, the American Stock Exchange (AMEX), and the National Association of Securities Dealers Automated Quotation (NASDAQ)—for the entire two-year period of January 1994 to December 1995. They thereby extract a sample of approximately 40 million data points, which is much larger than the one million data points analyzed by Liu and his colleagues[4,5] and the approximately 1,000 data points studied by Mandelbrot. Gopikrishnan and his colleagues find, remarkably, an asymptotic power-law behavior, with an exponent $\alpha \approx 3$ for the cumulative distribution (see Figure 1) that is well outside the Lévy regime ($0 < \alpha < 2$).[10]

In summary, previous proposals for the histogram of index changes have included a Gaussian distribution, a Lévy distribution,[6] and a truncated Lévy distribution, where the tails become "approximately exponential."[3] The inverse cubic result differs from all three proposals. Unlike the Gaussian distribution and the truncated Lévy distribution, it has diverging higher moments, and unlike the Gaussian distribution and the Lévy distribution, it is not a stable distribution.

## Economic organizations

Economics is of course much broader than just analyzing economic time series. Many physicists imagine they can add new ideas on how to analyze a time series, but what about general questions in social science, which concerns itself with the organization of individuals—each with free will? Taking the same "empirical" approach, the Boston group has also studied a range of data on economic organizations—viewing the data through the special eyeglasses of critical phenomena (imagining that "everything depends on everything else"). Specifically, in collaboration with a card-carrying economist, Michael A. Salinger, we studied the possibility that all the companies in a given economy might interact, more or less, like an Edwards-Anderson spin glass.[11] In that spin glass, each spin interacts with every other spin—but not with the same coupling and not even with the same sign.

For example, a 10% decrease in the sales of a given business firm will have repercussions in the economy. Some of the repercussions will be favorable—firm B, which

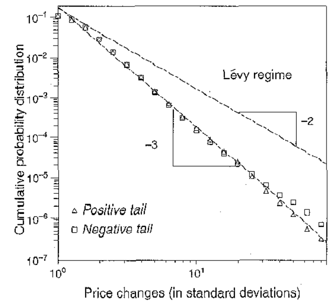


**Figure 1. A log-log plot of the cumulative probability distribution $P(g)$ of the normalized price increments where $g$ is calculated in units of a standard deviation. The lines are power-law fits to the data over the range from two to 100 standard deviations. The regression lines yield $\alpha = 2.93$ and $\alpha = 3.02$ for the positive and negative tails. (Figure courtesy of Parameswaran Gopikrishnan, Luis A.N. Amaral, and Martin Meyer.)**

competes with A, might experience an increase in market share. Others will be negative—service industries that provide personal services for firm A employees might experience a drop-off in sales because employee salaries will surely decline. Almost any economic change has positive and negative correlations. Can we view the economy as a complicated spin glass?

To approach this interesting bit of statistical "poetry" and make sense of it, Michael H.R. Stanley and Michael A. Salinger first located and secured a database—called Compustat—that lists the annual size of every US firm. With this information, they and their colleagues calculated histograms of how firm sizes change from one year to the next.[11] They then made 15 histograms for each of 15 bins of firm sizes. The largest firms have very narrow growth-rate distributions—plausible because the percentage of size change from year to year for the largest firms cannot be that great. On the other hand, a tiny firm or a garage-based start-up can radically increase (or decrease) in size from year to year. Thus, these 15 histograms have widths that depend on the firm size. When this width is plotted on the $y$-axis of log-log paper as a function of firm size on the $x$-axis, the data are approximately linear over eight orders of magnitude, from the tiniest firms in the database to the largest. The width scales with the firm size to an exponent $\beta$, with $\beta \approx 1/6$.[11] We can therefore normalize the growth rate and show that all the data collapse on a single curve—demonstrating the scaling of this measure of firm size.

Why does this data collapse occur? Researchers are working on that. Sergey V. Buldyrev models this firm structure as an approximate Cayley tree, in which each subunit of a firm reacts to its directives from above with a certain probability distribution.[12] More recently, Luis A.N. Amaral and his colleagues have proposed a microscopic model that reproduces both the exponent and the distribution function.[13] Hideki Takayasu and Kenji Okuyama extended the empirical results to a wide range of countries and developed still another model.[14]

It is not impossible to imagine some very general principles of complex organizations are at work here, because similar empirical laws appear to hold for data on a range of systems that at first sight might not seem to be so closely related. For example, instead of studying the growth rate of firms, you can study the growth rates of countries by analyzing the ratio of a country's GDP (gross domestic product) in one year compared to its value in the previous year. The histograms of country GDP sizes appear to behave the same way as the histograms of firm sizes[15] (see Figure 2), even with the same exponent value $\beta \approx 1/6$. Very recently, Vasiliki Plerou and her colleagues analyzed in the same way a database comprising research budgets of 719 US universities and found similar qualitative results, but a slightly larger exponent value, $\beta \approx 1/4$.

Instead of a firm's size at time $t$ (or the size of a GDP or a university budget), you might analyze the population $N_s(t)$ of a species $s$ in successive years. Such data exist for a 30-year period for every species sighted in North America. Very recently, Timothy H. Keitt and I have analyzed this database using the same sort of techniques used to describe long-term data sets on economics and finance.[2] We find statistical properties that are remarkably similar, and consistent with the idea that "every bird species interacts with every other bird species," just as the economic analysis supports the notion that "every economic entity interacts with every other economic entity."

These empirical results are not without interest, because they cast doubt on models of economic systems—and bird populations—that partition the entire data set into strongly interacting and weakly interacting subsets and then oversimplify or ignore the interactions in the weakly interacting subset.

What can we say so far, other than just that apparently a number of natural questions in economics can be investigated quantitatively, using empirical analysis methods not unlike those used in the study of critical phenomena? And that the quantitative behavior of these complex economic systems—comprising many animate subunits—is not unlike that found in interacting systems comprising many inanimate subunits. Can we understand why methods developed in, say, critical phenomena to quantify systems comprising inanimate subunits should apparently apply to complex systems comprising animate subunits? Indeed, the conceptual framework of critical phenomena is increasingly finding application in other fields, ranging from chemistry and biology to econophysics and even liquid water. Why is this? One possible answer concerns the way in which correlations spread throughout a system comprising subunits in which "everything depends on everything else."

The paradox is simply stated: our intuition suggests that the correlation $C(r)$ between subunits separated by a distance $r$ should decay exponentially with $r$—for the same reason the value of money stored in your mattress decays exponentially with time (each year it loses a constant fraction of its worth). Thus, we might expect that $C(r) \sim e^{-r/\xi}$, where $\xi$, the correlation length, is the characteristic length scale above which the correlation function is negligibly small. Experiments and calculations on mathematical models confirm that correlations usually do decay exponentially. But, if the system is at its critical point, the rapid exponential decay magically turns into a long-range power-law decay: magically $\xi \to \infty$.

So then how can correlations actually propagate an infinite distance, without re-

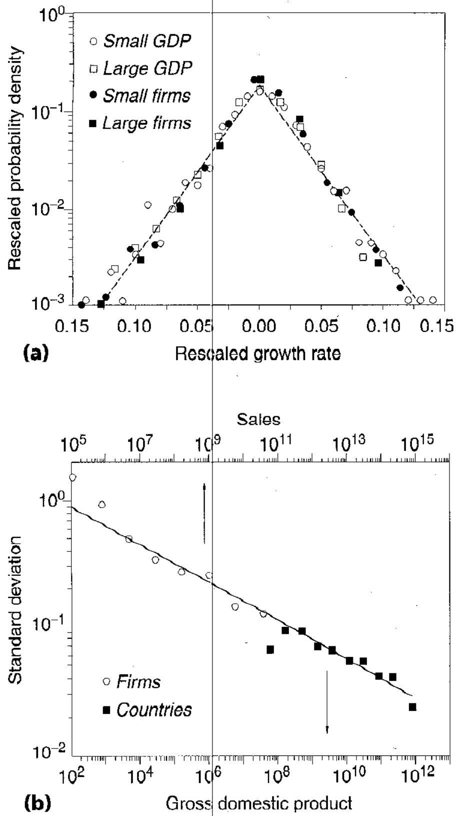


**Figure 2. A test of the similarity of the results for the growth of firms and countries: (a) The conditional probability density of annual growth rates for countries and firms. All rescaled data collapse onto a single curve, showing that the distributions have indeed the same functional form. (b) Standard deviation of the distribution of annual growth rates. The standard deviations decay with size with the same exponent for both firms and countries. The size is measured in sales for the firms and in GDP for the countries. The firm data include all 4,000 publicly-traded manufacturing firms from the 19-year period 1974–1993,[11] while the GDP data include 152 countries for the 43-year period 1950–1992.[15] (Courtesy of Youngki Lee, Luis A. N. Amaral, David Canning, and Martin Meyer.)**

quiring a series of amplification stations all along the way? We can understand such "infinite-range propagation" as arising from the huge multiplicity of interaction paths that connect two spins. The correlation between two spins along each of the interaction paths that connect them decreases exponentially with the path's length. On the other hand, the number of such interaction paths increases exponentially, with a characteristic length that is temperature-independent, depending primarily on the lattice dimension. This exponential increase is multiplied by a "gently decaying" power law that is negligible except for one special circumstance—the critical point. Right at the critical point, the gently decaying power-law correction factor in the number of interaction paths, normally negligible, emerges as the victor in this standoff between the two warring exponential effects. So, two spins are well-correlated even at an arbitrarily large separation.

Will the power laws found empirically to describe complex economic systems ever be understood in analogous terms? We will see—as the flip saying goes, "stay tuned."


## Acknowledgments

I thank those under whose tutelage I learned what little I understand of this subject. These include the students, postdocs, and faculty visitors to our research group with whom I have enjoyed the pleasure of scientific collaboration, especially those whose research provided the basis of this short report: Luis A.N. Amaral, Sergey V. Buldyrev, David Canning, Pierre Cizeau, Parameswaran Gopikrishnan, Shlomo Havlin, Timothy H. Keitt, Youngki Lee, Heiko Leschhorn, Yanhui Liu, Philipp Maass, Rosario N. Mantegna, Martin Meyer, Chung-Kang Peng, Vasiliki Plerou, Jeffrey Sachs, Michael A. Salinger, and Michael H.R. Stanley. I also thank Luis A.N. Amaral and Parameswaran Gopikrishnan for helpful comments on the early drafts of this manuscript.


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