Correlated randomness: Some examples of exotic statistical physics

H EUGENE STANLEY

Department of Physics, Center for Polymer Studies, Boston University, Boston,
MA 02215, USA
E-mail: hes@bu.edu

Abstract. One challenge of biology, medicine, and economics is that the systems treated by these sciences have no perfect metronome in time and no perfect spatial architecture – crystalline or otherwise. Nonetheless, as if by magic, out of nothing but randomness one finds remarkably fine-tuned processes in time and remarkably fine-tuned structures in space. To understand this 'miracle', one might consider placing aside the human tendency to see the universe as a machine. Instead, one might address the challenge of uncovering how, through randomness (albeit, as we shall see, strongly correlated randomness), one can arrive at many spatial and temporal patterns in biology, medicine, and economics. Inspired by principles developed by statistical physics over the past 50 years – scale invariance and universality – we review some recent applications of correlated randomness to fields that might startle Boltzmann if he were alive today.

1. Introduction

The title I have given to this talk, 'Correlated randomness', I owe in part to interactions with biological and medical researchers. They think that randomness means uncorrelated randomness. They learn that statistical physics deals solely with random phenomena, so they imagine that our field cannot possibly yield any insights into the real world as they correctly know that no system in which they are interested corresponds to simple uncorrelated randomness. Hence we found using the adjective 'correlated' helped persuade our collaborators that what we do may possibly be applicable to systems in which they are interested.

To help educate our collaborators, as well as ourselves, we have learned to present simple visual examples of the concept of correlated randomness. One example we found useful was comparing a simple, unbiased random walk in two dimensions (uncorrelated randomness) and a simple, self-avoiding random walk in two dimensions (correlated randomness). In the case of the uncorrelated walk, the spread of a 10^4
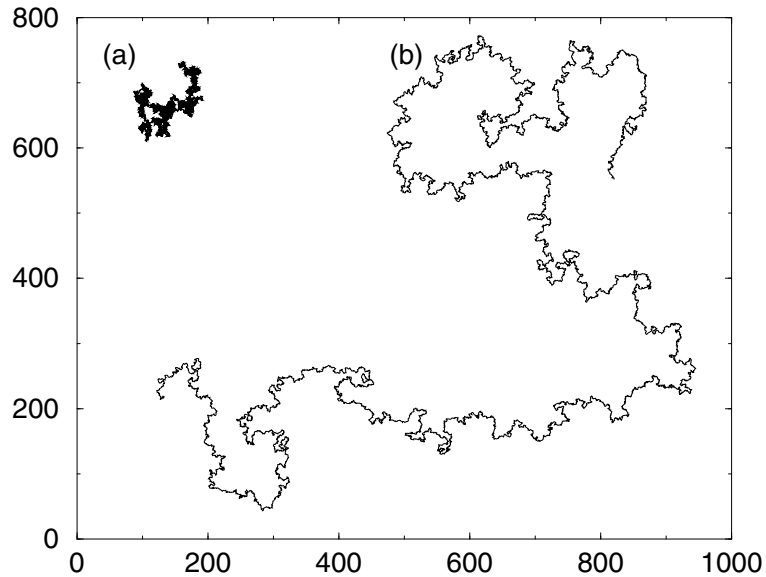
Figure 1. (**a**) The trail of a random walk of 10^4 steps, compared with (**b**) the trail of a self-avoiding random walk of the same number of steps. The 'correlated randomness' of the latter results in drastically different behavior. Specifically, the characteristic diameter jumps by a factor of 10, from approximately $(10^4)^{1/2} = 100$ to approximately $(10^4)^{3/4} = 1000$, where we have used the fact that the fractal dimensions (defined as the exponent to which the length is raised to obtain the mass) are 2 and 4/3 respectively. This figure is courtesy of S V Buldyrev.

step path is 10^2. In the correlated random walk, the spread of a 10^4 step path is on the order of 10^3 steps, an order of magnitude larger (figure 1).

A second simple example of correlated randomness that people from other fields can appreciate is critical opalescence, first discovered and interpreted – in terms of correlated randomness – by Andrews in 1869 [1]. This can occur in two-component fluids but also in one-component fluids – the so-called liquid–liquid phase transition [2]. In the more traditional two-component fluid, the concentrations of the two components and the temperature have been adjusted so that the system is near its consolute point. The correlated fluctuations observed at that consolute point are so strong that their length scale has become comparable to the wavelength of visible light and one sees a scattering of that visible light in the form of an opalescent glow.

In this talk, I will discuss recent applications of correlated randomness to three areas of science for which statistical physics is essential: liquid water, economics, and Alzheimer's disease. I organize each of these seemingly unrelated topics around the same three questions: (i) what is the question or problem that has emerged from the area of inquiry? (ii) why should we (practically and scientifically) care about this question or problem? and (iii) what have we actually done in response to the question or problem? The 'we' in each case involves a sizable subset of

collaborators who have both (i) made our fruitful scientific results possible and (ii) reinforced my changed attitude toward 'working well with others'. The list of collaborators in each area appears in the abstract book, and at the end of this document.

Our overall 'take-home' message today sounds pretty general. In general, systems that display correlated randomness cannot be solved exactly. Not even the simple self-avoiding random walk can be solved! Nonetheless, there are two unifying principles that have organized many of the results we will be presenting today – scale invariance and universality. The key idea is that scale invariance is a statement not about algebraic equations of the form $x^{-3} = 1/8$ with a numerical solution (2) but about functional equations of the form $f(\lambda x) \sim \lambda^p f(x)$ and its relevant generalizations. These functional equations have as their solutions functional forms, and the solution to this homogeneous functional equation is a power-law form.

2. Water

2.1 *What is the phenomenon?*

We start with three thermodynamic functions. The first is the compressibility – the response of the volume to an infinitesimal change in pressure. In a typical liquid, this response function decreases when we lower the temperature. I understand this decrease via statistical physics. This thermodynamic response function is proportional to the thermal average of all the fluctuations in specific volume in the system. As we lower the temperature, we imagine that fluctuations of necessity decrease, thus the compressibility decreases.

Water is unusual in three respects. First, the average compressibility of water is twice as large as what one would expect were water a typical fluid and were one to plug all the prefactors into the formulas that give compressibility in terms of volume fluctuations. Second, the magnitude of that factor of two actually increases as one lowers the temperature. That being the case, there is ultimately a minimum – which occurs at 46°C. Below that temperature, the compressibility increases dramatically. At the lowest attainable temperature (-40°C) the compressibility takes on a value that is twice of that at the minimum. This is not a tiny effect; it is huge (figure 2).

The second thermodynamic function is the specific heat, and we observe three similar anomalies: it is twice as large as that of a typical liquid, the discrepancy gets bigger as the temperature is lowered, and a minimum occurs at 35°C.

The third thermodynamic function is the coefficient of thermal expansion, the response of the volume to an infinitesimal change in temperature. This quantity we assume to always be positive because if there is a local region of the liquid in which the specific volume is larger than the average, then there will be more arrangements of the molecules and hence the entropy will be larger than the average. This is true of almost all liquids, but the magnitude of this cross-fluctuation of volume and entropy in water is approximately three times smaller than we would expect, and at 4°C the coefficient of thermal expansion passes through zero and actually becomes negative.
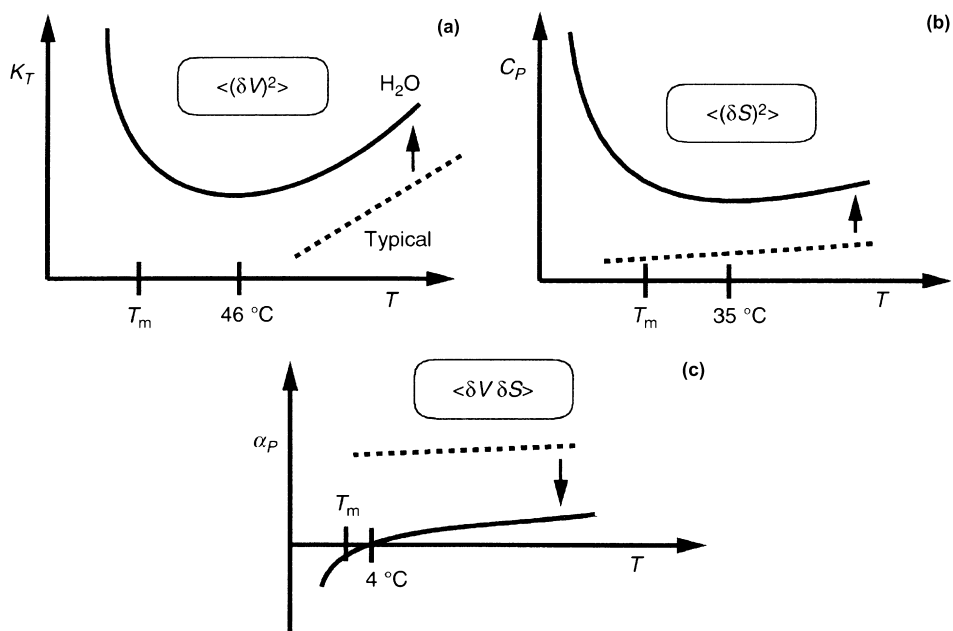
Figure 2. Schematic dependence on temperature of (**a**) the isothermal compressibility K_T, (**b**) the constant-pressure specific heat C_P, and (**c**) the thermal expansivity α_P. The behavior of a typical liquid is indicated by the dashed line, which, very roughly, is an extrapolation of the high-temperature behavior of liquid water. Note that while the anomalies displayed by liquid water are apparent above the melting temperature T_m, they become more striking as one supercools below T_m.

2.2 *Why do we care about this anomalous behavior?*

To begin with, if we do not understand water we will never understand biology. That is a major reason to care. Scientifically, water is the prototype complex fluid. It is not a simple, 'bag-of-marbles' liquid, but a 'bag of tetrahedra'. These tetrahedra are not only irregularly shaped, but are charged. Two of the arms are positively-charged, corresponding to the protons on each water molecule, and two are negatively-charged, corresponding to the lone pairs. In addition to short-range forces, these 'charged tetrahedra' interact with long-range Coulomb forces.

2.3 *What do we do?*

Our approach is based on the fact that water has a tetrahedral local geometry. In this sense water shares features with other liquids such as silicon (studied in Bangalore by Prof. Srikanth Sastry). Because water is both tetrahedral and charged, a simple Lennard–Jones potential is not sufficient to describe its complexity. One way to modify the Lennard–Jones potential to provide at least a simplified description
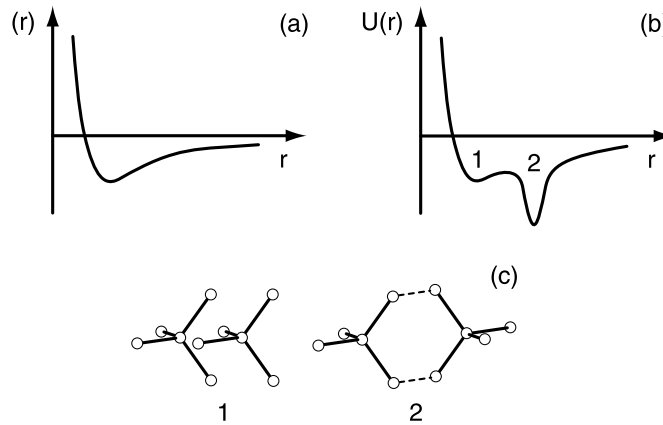
Figure 3. Physical arguments relating to the plausibility of the existence of the known liquid–gas critical point C and the hypothesized LDL-HDL critical point C′. (**a**) Idealized system characterized by a pair interaction potential with a single attractive well. At low enough T ($T < T_c$) and high enough P ($P > P_c$), the system condenses into the 'liquid' well shown. (**b**) Idealized system characterized by a pair interaction potential whose attractive well has two sub-wells, the outer of which is deeper and narrower. For low enough T ($T < T_c'$) and low enough P ($P < P_c'$), the one-phase liquid can 'condense' into the narrow outer 'LDL' sub-well, thereby giving rise to a LDL phase, and leaving behind the high-density liquid phase occupying predominantly the inner subwell. (**c**) Two idealized interaction clusters of water molecules ('Walrafen pentamers') in configurations that may correspond to the two sub-wells of (b). This figure is courtesy of O Mishima.

is to bifurcate the single minimum into two minima. The first minimum, at a closer distance, corresponds to two pentamers (a water molecule and its four neighbors) of water interacting with each other in a rotated configuration. The second minimum, at a greater distance, occurs in the unrotated position. This second position is a deeper minimum because although the pentamers are farther apart there is the potential for hydrogen bonding between the molecules and we can see the beginnings of an ice-like hexagonal structure (figure 3).

The important point is that there are two minima with the outer one corresponding to a larger specific volume – because the distance is larger – and a lower entropy. The possibility is that liquid water could at low temperatures condense not into a single phase – as we anticipate when a gas with a simple interaction like a Lennard–Jones potential condenses into a fluid – but into two different phases. This possibility was first raised by Takahashi 60 years ago and various elaborations of this model have been made by a number of people since then, including seminal work of Hemmer and Stell in 1971 [3–5]. The implication of this is the possibility of two different liquid phases contributing to an increase in these fluctuations in specific volume and a negative contribution to the cross-fluctuations, negative because the deeper well has a larger volume and a lower entropy. The consequences of this fact qualitatively explain the phenomenon we were describing at the beginning – volume fluctuations are increased, entropy fluctuations are increased, and cross-fluctuations
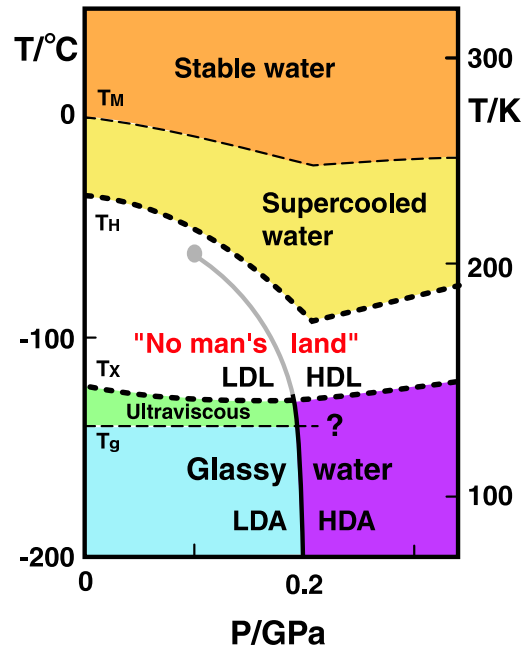
Figure 4. Schematic illustration indicating the various phases of liquid water (color-coded). This figure is courtesy of O Mishima.

of volume and entropy are decreased. This picture further predicts the possibility that at low temperatures there will be a genuine phase transition in which the single component liquid separates into two different phases. The implications of this when applied to real water molecules produce a phase diagram of liquid water (figure 4). This was first uncovered by Poole *et al* [6]. At one atmosphere (the left axis) we see the melting temperature and the limit of supercooling around −40°C. At a very low temperature we see the presence of the glassy phase, not unlike that of any other liquid except that at high pressure this glassy phase shifts from a low-density form to a high-density form. In typical liquids we do not find two different glassy phases. These two forms correspond to the two different local arrangements characteristic of water tetrahedra. The order parameter jump between these two phases is not a trivial amount, but on the order of 30% [7].

Between the liquid and glassy phases of water we have a region in which water does not exist as a liquid. I like to call this a 'No Man's Land'. The hypothesis that follows from the reasoning we have just described is that this first-order phase transition line known to separate the two amorphous forms of solid water extends into this No Man's Land and ultimately terminates at a critical point. Just as the glassy water first-order transition line separates a low-density amorphous from a high-density amorphous phase of water, so also this extension of the line into the liquid region separates a low-density liquid from a high-density liquid. The power-law behavior uncovered over the years by Angell, Anisimov and collaborators corresponds to the fact that the extension of this first-order line beyond the critical

point – the 'Widom line' – has the effect where any experiment approaching that line looks as though it is going to diverge with critical exponents but does not.

This phase diagram is hypothesized, but it has not been proved. What has been proved is that computer simulations using tried and tested models of liquid water confirm the broad features of this phase diagram (see ref. [8] and references therein). But computer models of water (like computer models of anything) are subject to the charge 'garbage-in, garbage-out' – you get out what you put in. All computer models of complex systems such as liquid water are of necessity simplifications.

Current experiments on this problem are of two sorts. The first is a set of experiments inspired by Mishima that involves probing the No Man's Land by studying the metastable extensions of the melting lines of the various high-pressure polymorphs of ice: ice III, ice V, ice IV, and ice XII [9,10]. Two of these lines clearly display 'kinks'. Since the slope of any melting line is the difference of the volume change divided by the entropy change of the two phases that coexist at that line, if there is a change in slope there must be a change in these quantities. Since there is no change in the crystal part, there must be a change in the liquid part. This means the liquid must undergo a jump in either its volume or its entropy or both. That is the definition of a first-order phase transition.

Very recently an article appeared in *Phys. Rev. Lett.* describing a testing and probing of this hypothesis that does not have to be carried out in the No Man's Land using the tricks of Mishima. Engemann, Reichert and collaborators at Grenoble [11] discovered the high-density liquid by taking a silicon substrate, placing an amorphous 2.1 Å silicon layer on top of it, placing a 1.7 Å layer that is liquid water but below the melting temperature on top of the other two layers, and measuring the properties of this quasi-liquid layer – interpreting the properties in terms of this liquid–liquid phase transition hypothesis (figure 5).
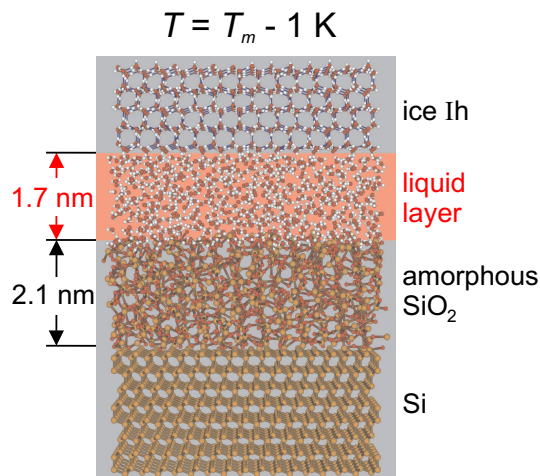

Figure 5. Schematic of the experiments of Reichert and collaborators, in which a thin layer of liquid water appears at the interface between ordinary hexagonal ice and an amorphous silica substrate. This figure is courtesy of H Reichert.

3. Statistical physics and economics

3.1 *What is the phenomenon?*

One quarter of any newspaper with a financial section is filled with economic fluc-
tuation data. Most economic graphs look approximately like the one we get when
we plot the S&P 500 stock index as a function of time over 40 years (figure 6).
We can compare this empirical data with that generated by a simple uncorrelated
biased random walk, a model first used over 100 years ago by Bachelier. At first
it seems that there is little difference, but looking more closely we see events in
the real data that do not have counterparts in the random walk. Black Mon-
day in October 1987 is reflected in the real data, which shows a loss of 30% of
the total value of the market in just one day. In the random walk we do not
see fluctuations anywhere near this magnitude because the probability of taking n
steps in the same direction of a random walk is $(1/2)^n$ – it decreases exponentially
with n.

Economists nevertheless have traditionally used this uncorrelated biased
Gaussian random walk to describe real economic data, relegating events such as
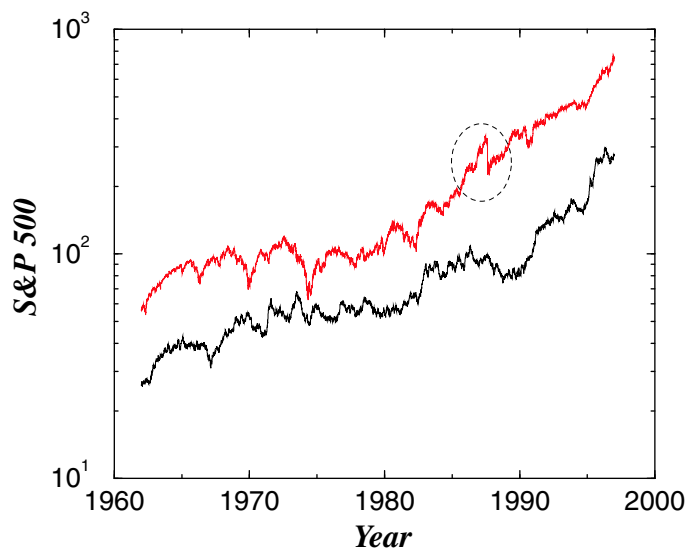Black Monday to the dustbin category of 'outliers' [12–14].

Figure 6. The S&P 500 index is the sum of the market capitalizations
of 500 companies. The sharp jump seen in 1987 is the market crash of
October 19. Comparison of the time evolution of the S&P 500 for the
35-year period 1962–96 (top line) and a biased Gaussian random walk (bot-
tom line). The random walk has the same bias as the S&P 500 – approx-
imately 7% per year for the period considered. This figure is courtesy of
P Gopikrishnan.

3.2 *Why do we care?*

We physicists do not like to do things this way. We do not take Newton's law seriously part of the time, and then – if we suddenly see an example of what appears to be levitation – simply call it an 'outlier'. We like to find laws that describe all examples of a phenomenon. Economists themselves, in a journal called *The Economist*, have admitted failure. This is a strong motivation for we physicists to step in and try our hand – we smell a delicious scientific challenge. Also, practically speaking, catastrophic economic events such as Black Monday have extreme societal impacts; widespread suffering is the usual outcome, especially among the poor. The ability to predict economic crashes (and other large-scale risks) would have an obvious utility.

3.3 *What do we do?*

We return to our two graphs, the S&P 500 stock index as a function of time over 40 years and the simple uncorrelated biased random walk, and plot not the absolute value of the index but instead the *change* in the index (the numerical derivative, the 'return'). We normalize that by the standard deviation. We look over a 13-year period rather than our original 40-year period (figure 7) and see, e.g., that on Black
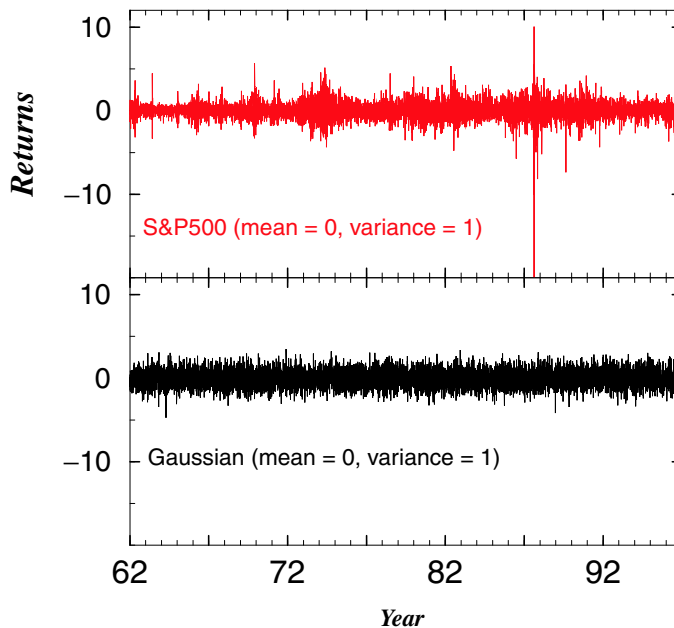
Figure 7. Sequence of 10-min returns for the S&P 500, normalized to unit variance, compared with sequence of i.i.d. Gaussian random variables with unit variance, which was proposed by Bachelier as a model for stock returns. Note that, in contrast to the top curve, there are no 'extreme' events in the bottom curve. This figure is courtesy of P Gopikrishnan.

Monday the fluctuations were more than 30 standard deviations (both positive and negative) for the day, and we also see a very noisy signal. The striking thing is to look at the other curve, the uncorrelated random walk, and see the Gaussian distribution for the fluctuations – which rarely display fluctuations greater than five standard deviations. The 'outliers' that the economists are content to live with are any fluctuations of the actual data that are greater than five standard deviations. In this 13-year period there are exactly 64, i.e., 2^6. If we count only those fluctuations of the actual data that are greater than ten standard deviations, we get exactly 8, i.e., 2^3. If we count only those that are greater than 20, we get one, i.e., 2^0: Black Monday. Each time we double the x-axis we change the y-axis by a power of 2^3. This should ring a bell. At the top of this presentation we made reference to a power law of the form $f(x) = x^{-3}$, which corresponds to a functional equation, a scaling equation, with $p = -3$. The possibility that these economic data obey scaling was pointed out in 1963 by Mandelbrot [15] in his study of cotton price fluctuations.

If we replace our visual examination of these two graphs with a close computer analysis of not just the S&P 500 stock index but every stock transaction over an extended time period (approximately 1 GB of data), we find [16–18] that the actual graph giving the number of times a fluctuation exceeds a given amount as a function of that amount is perfectly straight on log–log paper out to 100 standard deviations (figure 8). The slope of the line, α, is indistinguishable from the value $\alpha = 3$ that we deduced from visual inspection. Note that this slope is significantly larger (by almost a factor of two) than the slope found by Mandelbrot in his research on cotton prices. Note also that our slope is outside the Lévy stable regime [19].
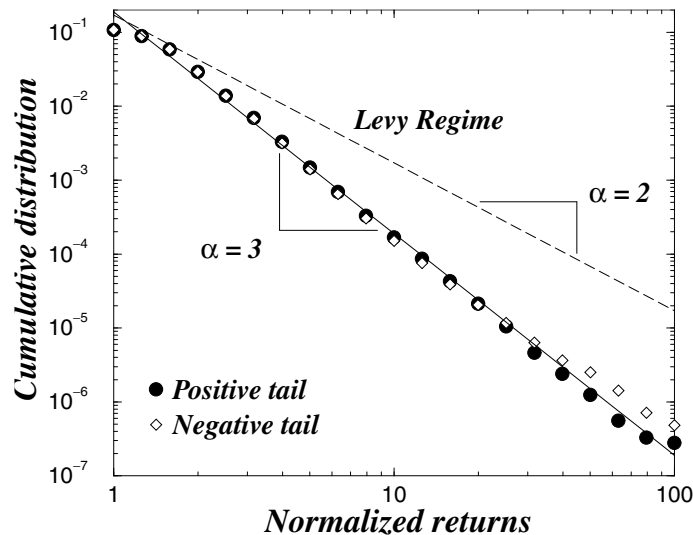

Figure 8. Cumulative distributions of the positive and negative tails of the normalized returns of the 1000 largest companies in the TAQ database for the 2-year period 1994–1995. The solid line is a power-law regression fit in the region $2 \leq x \leq 80$. This figure is courtesy of V Plerou.

This is how we find laws in statistical physics, but finding them is only the first part – the empirical part – of our task. The second part – the theoretical part – is understanding them.

When we studied critical phenomena, the empirical part was a very important contributor toward our ultimate understanding of phase transitions and critical phenomena. The amassing of empirical facts led to the recognition of regularities to which certain approaches could be applied, e.g., the scaling hypothesis – first formulated by Ben Widom and others – and the Wilson renormalization group. So also in economics we can perhaps first discover empirical regularities – e.g., the inverse cubic law – that will prove useful in ultimately understanding the economy. I wish I could say that we already have an explanation for this inverse cubic law, but I cannot. We have the beginnings of an explanation, but it is only the beginning since the current theory explains the inverse cubic law of price changes, as well as the 'half cubic law' of trade volume [20,21] but does not explain the strange nature of the temporal correlations. The autocorrelation function of price changes decays exponentially in time so rapidly that after 20 min it is in the level of 'noise' (figure 9). However the autocorrelation function of changes in the absolute value of the price (called the 'volatility') decays with a power law of exponent approximately 0.3 (figure 10).

One reason the economy is of interest to statistical physicists is that, like an Ising model, it is a system made up of many subunits. The subunits in an Ising model are called spins, and the subunits in the economy are buyers and sellers. During
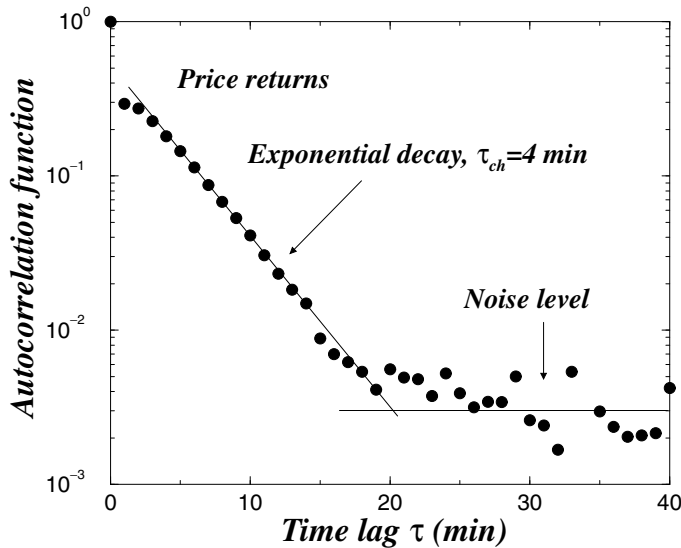

Figure 9. Semilog plot of the autocorrelation function for the S&P 500 returns $G_{\Delta t}(t)$ sampled at a $\Delta t = 1\,\text{min}$ time-scale, $C_{\Delta t}(\tau) \equiv [\langle G_{\Delta t}(t)\,G_{\Delta t}(t+\tau)\rangle - \langle G_{\Delta t}(t)\rangle^2]/[\langle G_{\Delta t}(t)^2\rangle - \langle G_{\Delta t}(t)\rangle^2]$. The straight line corresponds to an exponential decay with a characteristic decay time $\tau_{\text{ch}} = 4\,\text{min}$. Note that after 20 min the correlations are at the noise level. This figure is courtesy of V Plerou.
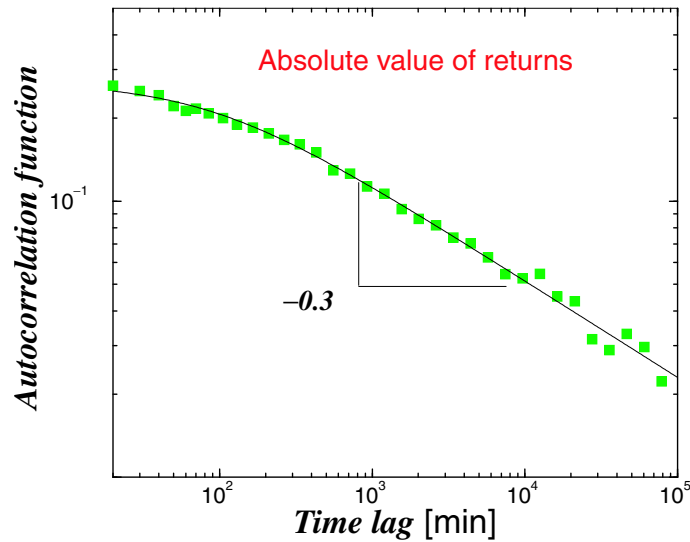
Figure 10. Log–log plot of the autocorrelation function of the absolute returns. The solid line is a power-law regression fit over the entire range, which gives an estimate of the power-law exponent, $\eta \approx 0.3$. Better estimates of the exponent η can be obtained from the power spectrum or from other more sophisticated methods. This figure is courtesy of P Gopikrishnan.

any unit of time these subunits of the economy may be either positive or negative as regards perceived market opportunities. People interact with each other, and this fact often produces what economists call 'the herd effect'. The orientation of whether we buy or sell is influenced not only by our neighbors but also by news. If we hear bad news, we may be tempted to sell. So the state of any subunit is a function of the states of all the other subunits and of a field parameter.

On a qualitative level, economists often describe a price change as a hyperbolic-tangent-like function of the demand. The catch is that 'demand' is not quantified. So one of the first things we had to do was quantify demand [22].

We did this by analyzing huge databases comprising every stock bought or sold – which gives not only the selling price and buying price, but also the asking price and the offer price. If we go to the Bangalore open market to buy presents we will often be given an asking price we are not willing to pay, and we may counter with a much smaller offer. Ultimately when the sale is struck, the price may be above the midpoint between the asking price and the offer – and we assign a variable $a_i = +1$ to the sale – if below the midpoint, $a_i = -1$. If we sum all these indices a_i over a time interval Δt

$$H \equiv \sum_{i-1}^{N} a_i = \begin{cases} + & \text{[Big demand]} \\ - & \text{[Small demand]} \end{cases}$$

$N = N_{\Delta t} =$ Number of sales in Δt,

then we can calculate the analog of a magnetic field, which provides a way of quantifying demand. If most of the a_i are positive, the field will be positive, and vice versa. A hint that this definition of magnetic field makes sense is the fact that a plot of price change as a function of the 'magnetic field' variable defined above remarkably resembles a plot of the magnetization of a magnet as a function of the magnetic field [22]. The implications of the remarkable observation that a plot of price change as a function of the 'magnetic field' resembles a plot of the magnetization in a magnet are not yet clear.

4. Alzheimer's disease

4.1 *What is the problem?*

We have discussed our hypothesis concerning liquid water. I think that, that hypothesis is correct. I know that our empirical results concerning the economy are correct. In contrast, I cannot guarantee any of our work thus far in Alzheimer's disease (AD). Nevertheless, we must start somewhere. At present, no one knows much for certain about AD. We do not even know what it is – we only know its tragic effects. In societies in which lifespans are long it has been estimated that 50% of all newborns will die of this disease, if we do not find a cure, by the time they are 85 years old.

Since we do not even know what AD is, our group's approach is to try to discover what it is. We are not the only ones trying to do this, but our focus is on the 'first three minutes' of the disease, a phrase we like to use because it is the analog for AD of Weinberg's *The First Three Minutes of the Universe*. What initially triggers AD? One of the current hypotheses is that it is triggered by a phase transition. In a liquid-to-crystal phase transition, the liquid nucleates to a lower free-energy state called a crystal. In AD, it looks as if the analogous event is a two-step process – first of protein folding and then of protein aggregation. So one starts with a protein, specifically a peptide, in a correlated random configuration, which first folds itself and then aggregates into a form called an amyloid fibril. These amyloid fibrils aggregate and form microscopic objects in the brain.

4.2 *Why do we care?*

Aggregation diseases appear to be universal. Alzheimer's disease is only one form of aggregation disease. Others may include ALS (amyotrophic lateral sclerosis) and Parkinson's disease.

4.3 *What do we do?*

The protein that folds is actually a peptide, a fragment of a protein, and it comes in two forms: one with 40 amino acids and the other with 42 amino acids. The extra two amino acids are hydrophobic. The 'first three minutes' of Alzheimer's disease involves the aggregation of the entire peptide. The first step seems to

be the formation of what are called paranuclei, the joining together of a small number of – e.g., 6 or 12 – peptides, and then the aggregation of the paranuclei into larger objects. The time-scale here is slow, so the study of this phenomenon by molecular dynamics is problematic. A typical time-scale for a molecular dynamics simulation is on the order of nanoseconds, and here we are talking about minutes. We need to do something to speed up the simulation, and thus we draw on the concept of universality.

One thing we learn from universality is that completely different fluids behave the same way near phase transitions. They even have identical critical exponents, regardless of the details. The critical factor does not seem to reside in the details of the molecule but in the fact that they have an attractive part. Similarly, in the spirit of universality, one finds the same generic collective behavior when one coarse grains the actual peptide by replacing each amino acid group by only four balls: three in the peptide backbone and one representing the side group.

A typical simulation might start with 28 of these coarse-grained peptides at time zero [23,24]. Then using the algorithm [25] that speeds up the simulation by ten orders of magnitude, one can achieve aggregation of these peptides in a reasonable amount of computer time (on the order of days of computer time). The structure of what we found is of considerable interest to those studying Alzheimer's disease. In particular, we find aggregation, with remarkably reproducible microscopic detail concerning exactly what sticks to what (figure 11). This is terribly important
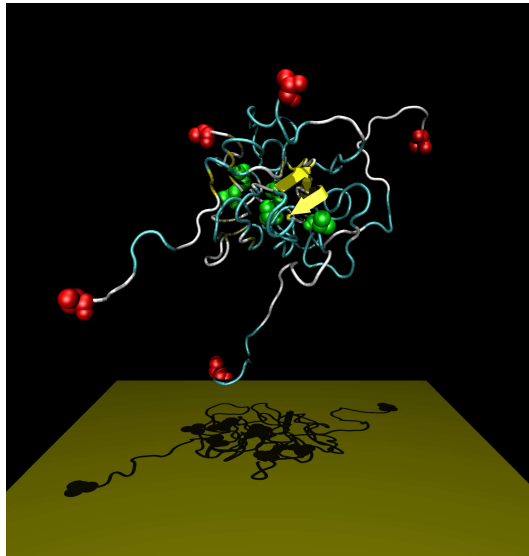

Figure 11. Structure of a typical aggregate of five amyloid beta peptides, each of which has 42 amino acids. The secondary structure of the aggregate is color coded, being shown as a silver tube (random coil structure), light blue tube (turn) and yellow ribbon (β-strand). The five red spheres represent the five N-terminals of the five peptides, while the ten C-terminal amino acids Ile-41 and Ala-42 are shown in green and blue respectively. This figure is courtesy of B Urbanc.

because if we know what sticks to what, we can imagine covering the sticky spots and hence offering some potentially useful hints concerning possible therapies for Alzheimer's disease. That is, if we know what is sticking to what we can begin to think about how to block that sticking process.

5. Concluding remarks

Today I have talked about work that has extended over a period of approximately ten years. This work was not done by me, but by a large number of collaborators. Why so many? One reason is that I love people, and working with people is for me exhilarating. There is a second, more personal reason. When in 1976 I did not receive tenure at MIT, I asked my bosses for the reason. One reason offered was that I worked too much 'all alone'. Obviously I took that advice to heart – and I am grateful that I did. I have enjoyed my many collaborations. The work that this Boltzmann Medal honors – as most of you know – is partly work I did working alone, as a graduate and as a nontenured member of the faculty of MIT. However a large part is work I did with others. I have learned far more from my collaborators than they have learned from me, so if I could divide up this disc-shaped medal like a pie, I would give a piece to each collaborator. I am especially touched that 20 of my present and former collaborators are in this room today: A-L Barabási, R Bansil, M C Barbosa, A Coniglio, G Franzese, S Havlin, H J Herrmann, P Kumar, J Kertész, E La Nave, F Leyvraz, H A Makse, I Ono, P Ray, A Robledo, S Sastry, S Sreenivasan, F W Starr, C Tsallis, and T Vicsek. An additional nine others are also here in the sense that their names are among the co-authors of abstracts of talks presented here: M Barthelémy, S V Buldyrev, X Gabaix, P Gopikrishnan, V Plerou, B Rosenow, F Sciortino, A Vespignani, and G Viswanathan. A complete list of my collaborators appears in *Physica* **314**, 807 (2002).

I conclude with a personal statement. I would like to dedicate this medal to the memory of my wife, who died 9 March 2003 of lung cancer – a disease which even today knows no solution. Idahlia, like me, was a great lover of India and all things Indian: Indian people, Indian art, and Indian food. She would be very happy to be here with all of us today – and she is here in her spirit.

References

[1] T Andrews, *Philos. Trans.* **159**, 575 (1869)
[2] V Brazhkin, S V Buldyrev, V N Ryzhov and H E Stanley (eds), New kinds of phase transitions: Transformations in disordered substances, *Proc. NATO Advanced Research Workshop*, Volga River (Kluwer, Dordrecht, 2002)
[3] P C Hemmer and G Stell, *Phys. Rev. Lett.* **24**, 1284 (1970)
[4] M R Sadr-Lahijany, A Scala, S V Buldyrev and H E Stanley, *Phys. Rev. Lett.* **81**, 4895 (1998)
[5] A Scala, M R Sadr-Lahijany, N Giovambattista, S V Buldyrev and H E Stanley, *Phys. Rev.* **E63**, 041202 (2001)
[6] P H Poole, F Sciortino, U Essmann and H E Stanley, *Nature (London)* **360**, 324 (1992)

[7] O Mishima, *J. Chem. Phys.* **100**, 5910 (1994)

[8] M Yamada, S Mossa, H E Stanley and F Sciortino, *Phys. Rev. Lett.* **88**, 195701 (2002)

[9] O Mishima and H E Stanley, *Nature (London)* **392**, 164 (1998)

[10] O Mishima, *Phys. Rev. Lett.* **85**, 334 (2002)

[11] S Engemann, H Reichert, H Dosch, J Bilgram, V Honkimaki and A Snigirev, *Phys. Rev. Lett.* **92**, 205 701 (2004)

[12] R N Mantegna and H E Stanley, *An introduction to econophysics: Correlations and complexity in finance* (Cambridge University Press, Cambridge, 2000)

[13] J P Bouchaud and M Potters, *Theory of financial risk* (Cambridge University Press, Cambridge, 2000)

[14] H E Stanley, *Physica* **A318**, 279 (2003)

[15] B B Mandelbrot, *J. Business* **36**, 394 (1963)

[16] P Gopikrishnan, M Meyer, L A N Amaral and H E Stanley, *Eur. Phys. J.* **B3**, 139 (1998)

[17] V Plerou, P Gopikrishnan, L A N Amaral, M Meyer and H E Stanley, *Phys. Rev.* **E60**, 6519 (1999)

[18] P Gopikrishnan, V Plerou, L A N Amaral, M Meyer and H E Stanley, *Phys. Rev.* **E60**, 5305 (1999)

[19] K Matia, L A N Amaral, S Goodwin and H E Stanley, *Phys. Rev.* **E66**, 045103 (2002) Rapid Communications

[20] X Gabaix, P Gopikrishnan, V Plerou and H E Stanley, *Nature (London)* **423**, 267 (2003)

[21] X Gabaix, P Gopikrishnan, V Plerou and H E Stanley, A simple theory of asset market fluctuations, motivated by the cubic and half cubic laws of trading activity in the stock market, *Quarterly Journal of Economics* (submitted).

[22] V Plerou, P Gopikrishnan, X Gabaix and H E Stanley, *Phys. Rev.* **E66**, 027104 (2002)

[23] B Urbanc, L Cruz, S Yun, S V Buldyrev, G Bitan, D B Teplow and H E Stanley, *Proc. Natl. Acad. Sci.* **101**, 17345 (2004)

[24] B Urbanc, L Cruz, F Ding, D Sammond, S Khare, S V Buldyrev, H E Stanley and N V Dokholyan, *Biophys. J.* **87**, 2310 (2004)

[25] N Dokholyan, S V Buldyrev, H E Stanley and E I Shakhnovich, *Folding & Design* **3**, 577 (1998)