How to Characterize Trend Switching Processes in Financial Markets

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One of the key conceptual elements in modern statistical physics is the concept of scale invariance, codified in the scaling hypothesis that functions obey certain functional equations whose solutions are power laws. The scaling hypothesis has two categories of predictions, both of which have been verified by a wealth of experimental data on diverse systems. The first category is a set of relations, called *scaling laws*, that serve to relate the various critical-point exponents characterizing the singular behavior of functions such as thermodynamic functions. The second category is a sort of data collapse, where under appropriate axis normalization, diverse data "collapse" onto a single curve.

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Econophysics research has been addressing a key question of interest: focusing on the challenge of quantifying the behavior of probability distributions of large fluctuations of relevant variables such as returns, volumes, and the number of transactions. Sampling the fat tails of such distributions require a large amount of data. However, there is a truly gargantuan amount of pre-existing precise financial market data already collected, many orders of magnitude more than for typical complex systems. Accordingly, financial markets are becoming a paradigm of complex systems, and increasing numbers of scientists are analyzing market data [1-7]. Empirical analysis has been focused on quantifying and testing the robustness of power-law distributions that characterize large movements in stock market activity. Using estimators that are designed for serially and cross-sectionally independent data, findings support the hypothesis that the power law exponents that characterize fluctuations in stock price, trading volume, and the number of trades [8, 9] are seemingly "universal" in the sense that they do not change their values significantly for different

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markets, different time periods, or different market conditions.

In contrast to these analysis of averaged distributions we focus on the temporal sequence of fluctuations in volatility, volume, and inter-trade times before and after trend switching points. Our analysis can provide insights into switching processes in complex systems in general and financial systems in particular. The study of dramatic crash events is limited by the fortunately rare number of such events. Increasingly, one seeks to understand the current financial crisis by comparisons with the depression of the 1930's. Here we ask if the smaller financial crises − trend switching processes on all time scales − also provide information of relevance to large crises. If this is so, then the larger abundance of data on smaller crises should provide quantifiable statistical laws for *bubbles on all scales*.

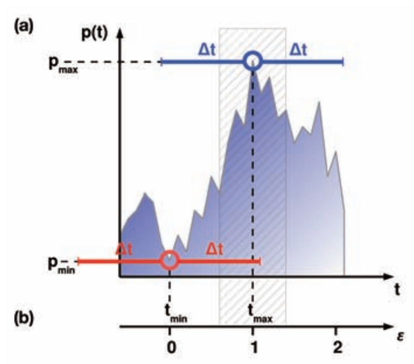

FIG. 1: Visualization of a *microtrend*.
(a) Positive microtrend starting at a local price minimum p_{min} of order Δt and ending at a local price maximum p_{max} of order Δt. The hatched region around p_{max} indicates the interval in which we find scale-free behavior. This behavior is consistent with "self-organized" [10] macroscopic interactions among many traders, not unlike "tension" in a pedestrian crowd [11, 12].
(b) Renormalized time scale ε between successive extrema, where $\varepsilon = 0$ corresponds to the start of a microtrend, and $\varepsilon = 1$ corresponds to the end. The hatched region is surprisingly large, starting at $\varepsilon = 0.6$ and ending at $\varepsilon = 1.4$.

To answer whether smaller financial crises also provide information of relevance to large crises, we perform a parallel analysis of bubble formation and bursting using two different data bases on two quite different time scales: (i) from $\approx 10^1$ ms to $\approx 10^6$ ms, and (ii) from $\approx 10^8$ ms to $\approx 10^{10}$ ms.

For the first analysis, we use a multivariate time series of the German DAX Future contract (FDAX) traded at the European Exchange (Eurex). The time series comprises $T_1 = 13,991,275$ trades of three disjoint three-month periods (16 March 2007 - 15 June 2007, 20 June 2008 - 19 September 2008, and 19 September 2008 - 19 December 2008). The data contains transaction prices, volumes, and corresponding time stamps, with inter-trade times down to 10 ms, which allows us to perform an analysis of microtrends.

For the second analysis, which focuses on macrotrends, we use price time series of daily closing prices of all stocks of the S&P500 index. This index consists of 500 large-cap common stocks which are actively traded in the United States of America. The time series comprises overall $T_2 = 2,592,531$ closing prices of US stocks till 16 June 2009 which were constituent of the S&P500 at this date. Our oldest closing prices date back to 2 January 1962. The data base we analyze contains the daily closing prices and the daily cumulative trading volume. As spot market prices undergo a significant shift by inflation over time periods of more than 40 years we study logarithmized stock prices instead of the raw closing prices. Thus, the results between the two different data bases on two quite different time scales become more comparable.

Less studied than the large fluctuations of major national stock indices such as the S&P500 are the various jagged functions of time characterizing complex financial fluctuations down to time scales as short as a few milliseconds. These functions at first sight are not amenable to mathematical analysis because they are characterized by sudden reversals between up and down microtrends (see Fig. 1 and Fig. 2a) which can also be referred as microscopic bubbles. On these small time scales evidence can be found [2] that the three major financial market quantities of interest − price, volume, and inter-trade times − are connected in a non-trivial way creating complex financial market patterns.

We do not know how to characterize the sudden microtrend reversals. For example, the time derivative of the price $p(t)$ is discontinuous. This behavior is completely different than most real world trajectories, such as a thrown ball for which the time derivative of the height is a smooth continuous function of time. Here we find a way of quantitatively analyzing these sudden microtrend reversals which exhibit a behavior analogous to transitions in systems in nature, and we interpret these transitions in terms of the cooperative interactions of the traders involved. A wide range of examples of transitions exhibiting scale-free behavior ranges from magnetism in statistical physics to

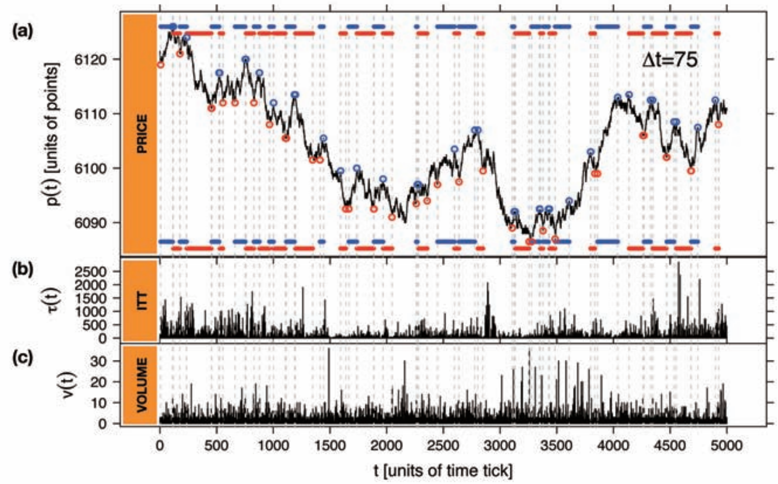

FIG. 2: (a) A small subset comprising 5000 trades (0.04%) of the full trade data set analyzed, extracted from the German DAX future time series. Shown as circles are the extrema of order Δt. We perform our analysis for $\Delta t = 1, 2, ..., 1000$ ticks; in this example, $\Delta t = 75$ ticks. Positive microtrends are indicated by blue bars, which start at a Δt-minimum and end at the next Δt-maximum. A negative microtrend (red bars) starts at a Δt-maximum and ends at the consecutive Δt-minimum.
(b) Time series of corresponding inter-trade times $\tau(t)$ reflecting the natural time between consecutive trades in units of 10 ms.
(c) The volume $v(t)$ of each trade t in units of contracts.

macroscopic social phenomena such as traffic flow (switching from a free traffic phase to a jammed phase) [13].

To focus on switching processes of price movements down to a microscopic time scale, we first propose how a switching process can be analyzed quantitatively. Let $p(t)$ be the transaction price of trade t, which is in the following a discrete variable $t = 1,...,T$. Each transaction price $p(t)$ is defined to be a *local maximum* $p_{max}(\Delta t)$ of order Δt if there is no higher transaction price in the sliding interval $t - \Delta t \leq t \leq t + \Delta t$. Analogously, each transaction price p(t) is defined to be a local minimum $p_{min}(\Delta t)$ of order Δt if there is no lower transaction price in the sliding interval $t - \Delta t \leq t \leq t + \Delta t$. In this sense, the two points in the time series in Fig. 1 marked by blue and red circles are a local minimum and a local maximum, respectively. Figure 2a shows a short subset of the FDAX time series for the case $\Delta t = 75$.

For the analysis of financial market quantities in dependence of trend fraction, we introduce a renormalized time scale ε between successive extrema as follows. Let t_{min} and t_{max} be the time (measured in units of ticks) at which the corresponding transactions take place of a successive pair of $p_{min}(\Delta t)$ and $p_{max}(\Delta t)$ (see Fig. 1). For a positive microtrend, the renormalized time scale is given by

$$\varepsilon(t) \equiv \frac{t - t_{min}}{t_{max} - t_{min}}, \tag{1}$$

with $t_{min} \leq t \leq t_{max} + (t_{max} - t_{min})$, and for a negative microtrend by

$$\varepsilon(t) \equiv \frac{t - t_{max}}{t_{min} - t_{max}}, \tag{2}$$

with $t_{max} \leq t \leq t_{min} + (t_{min} - t_{max})$. Thus, $\varepsilon = 0$ corresponds to the beginning of the microtrend and $\varepsilon = 1$ indicates the end of the microtrend. We analyze a range of ε for the interval $0 \leq \varepsilon \leq 2$, so we can analyze trend switching processes both before as well as after the critical value $\varepsilon = 1$ (Fig. 1). The renormalization is essential to assure that microtrends of various lengths can be aggregated and that all switching points have a common position in the renormalized time scale.

First we analyze the fluctuations $\sigma^2(t)$ during the short time interval of microtrends from one price extremum to the next. The quantity studied is given by squared price differences, $\sigma^2(t) = (p(t) - p(t-1))^2$ for $t > 1$, and can be referred to as local volatility. For the analysis of $\sigma^2(t)$ in dependence of trend fraction, we use the renormalization time scale ε. In Fig. 3a, the color key gives the mean volatility $\langle\sigma^2\rangle(\varepsilon, \Delta t)$ in dependence of ε and Δt normalized by average volatility where the brackets denote the average

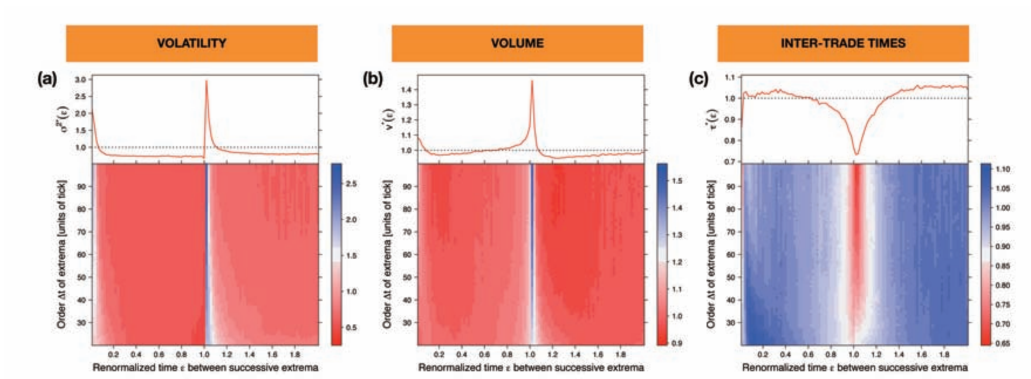

FIG. 3: Renormalization time analysis of volatility σ^2, volumes v, and inter-trade times τ.

(a) The colored volatility profile, averaged over all microtrends and normalized by the average volatility. The color code gives the normalized mean volatility. The color profile exhibits the clear link between mean volatility and price evolution. New extreme values of the price time series are reached with a significant sudden jump of the volatility. Here, $\sigma^{2*}(\varepsilon)$ denotes the average of the volatility profile, averaged only for layers with $50 \leq \Delta t \leq 100$.

(b) The colored volume profile, averaged over all microtrends and normalized by the average volume. The color code gives the normalized mean volume. The volume is connected to the price evolution: new extreme values of the price coincide with peaks in the volume time series, as indicated by the vertical blue regions close to $\varepsilon = 1$. The top panel shows the volume aggregation $v^*(\varepsilon)$, where $v^*(\varepsilon)$ is the average over layers with $50 \leq \Delta t \leq 100$.

(c) The colored inter-trade time profile is performed analogously to our study of volatility and volume. New extreme values of the price time series are reached with a significant decay of the inter-trade times.

over all increasing and decreasing microtrends. In order to remove outliers only those microtrends are collected in which the time intervals between successive trades $\tau(t)$ [14] (Fig. 2b) are not longer than 1 minute, which is roughly 60 times longer than the average inter-trade time ($\approx$ 0.94 s), and in which the volumes are not larger than 100 contracts (the average volume is 2.55 contracts). This condition ensures that time t runs only over the working hours of the exchange---removing overnight gaps, weekends, and national holidays. As expected, the color profiles exhibit a clear link between volatility and price evolution. A new local price maximum is reached with a significant sudden jump of the volatility. This qualitative effect is intuitively understandable and can also be reproduced by a random walk process with Gaussian price changes. The shape of the volatility peak around extrema is characterized by asymmetric tails, which we analyze next. For this analysis, we use the volatility aggregation $\sigma^{2*}(\varepsilon)$, which is the mean volatility $\langle\sigma^2\rangle(\varepsilon, \Delta t)$ averaged for layers from $\Delta t_{cut} = 50$ ticks to $\Delta t_{max} = 1000$ ticks. Figure 4b shows $\sigma^{2*}(\varepsilon)$ on a log-log plot. The evolution of the volatility before and after reaching a maximum shows up as straight lines and thus are consistent with a power law scaling behavior $\sigma^{2*}(|\varepsilon-1|) \sim |\varepsilon-1|^{\beta\sigma^2}$ within the range indicated by the vertical dashed lines. Figure 4c shows $\sigma^{2*}(\varepsilon)$ for macrotrends averaged for layers from $\Delta t_{cut} = 10$ days to $\Delta t_{max} = 100$ days which we obtain by performing a parallel analysis for trends on long time scales using the daily closing price data base of S&P500 stocks.

We perform a parallel analysis of the corresponding volume fluctuations $v(t)$, the numbers of contracts traded in each individual transaction in case of microtrends for the German market and the cumulative number of traded stocks per day in case of macrotrends for the US market. The colored volume profile (see Fig. 3b) exhibits that the volume is clearly connected to the price evolution: new extreme values of the price coincide with peaks in the volume time series, as indicated by the vertical blue regions close to $\varepsilon = 1$. New price extrema are linked with peaks in the volume time series but, surprisingly, we find that the usual cross-correlation function between price changes and volumes vanishes. Thus, one can conjecture that the tendency to increased volumes occurring at the end of positive microtrends is counteracted by the tendency to increased volumes occurring at the end of negative microtrends. The crucial issue is to distinguish between positive and negative trends realized by the renormalization time ε between successive extrema. Figure 4d and 4e show $v^*(\varepsilon)$ versus $|\varepsilon-1|$ as log-log histograms supporting a power law behavior of the form $v^*(|\varepsilon-1|) \sim |\varepsilon-1|^{\beta v}$.

In order to verify a possible universality, we analyze the

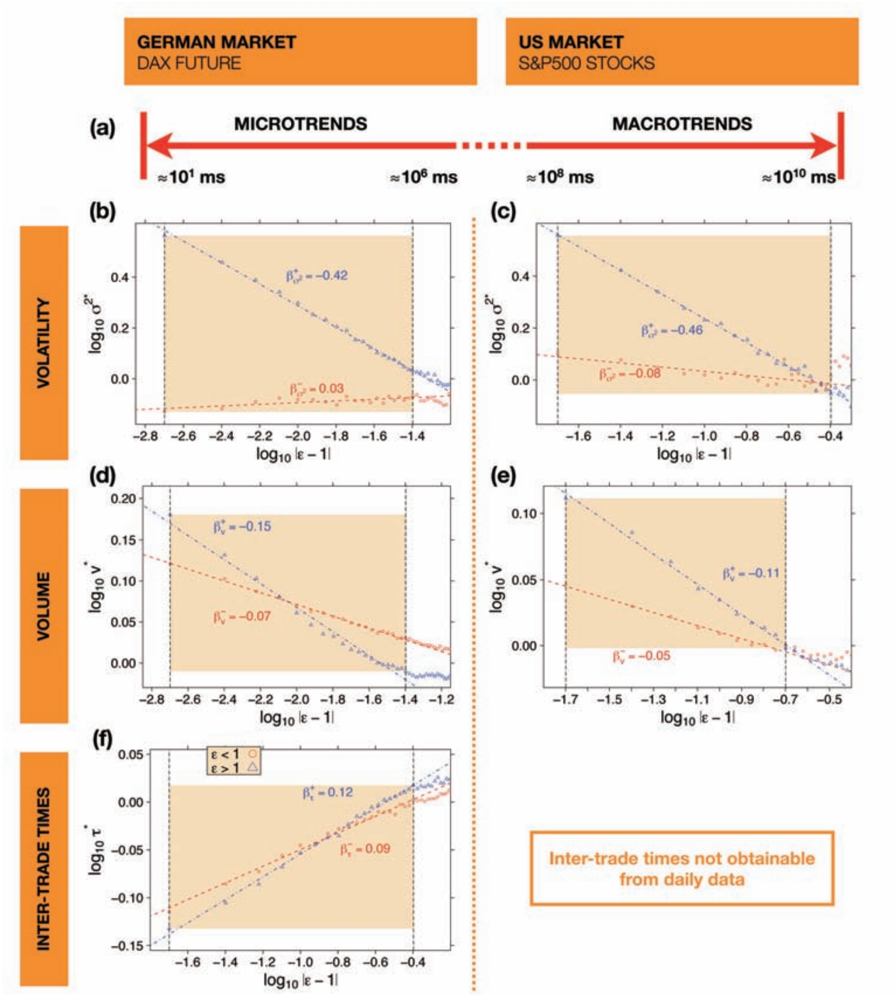

FIG. 4: Overview of time scales studied and log-log plots of quantities with scale-free properties.

(a) Visualization of time scales studied for both the German market and the US market. For the analysis of microtrends, we use the German DAX future data base which enables us to analyze microtrends starting at roughly 105 ms down to the smallest possible time scale of individual transactions measured in multiples of 10 ms. The log-log plots of quantities with scale-free behavior on short time scales are shown in the left column. For the analysis of macrotrends, we use the data base of daily closing prices of all S&P500 stocks which enables us to perform equivalent analysis of macrotrends on long time scales which are shown the right column. Thus, our analysis of switching processes ranges over 9 orders of magnitude from 10 ms to 10^{10} ms.

(b) The volatility (50 ticks $\leq \Delta t \leq$ 1000 ticks) before reaching a new extreme price value ($\varepsilon < 1$, red circles) and after reaching a new extreme price value ($\varepsilon > 1$, blue triangles) aggregated for microtrends. The straight lines correspond to power law scaling with exponents $\beta^{+}_{\sigma^2} = -0.420 \pm 0.01$ and $\beta^{-}_{\sigma^2} = 0.030 \pm 0.01$. The shaded interval marks the region in which this power law behavior is valid. The left border of the shaded region is given by the first measuring point closest to the switching point.

(c) The volatility aggregation of macrotrends determined for the US market on long time scales (10 days $\leq \Delta t \leq$ 100 days). The straight lines correspond to power law scaling with exponents $\beta^{+}_{\sigma^2} = -0.46 \pm 0.01$ and $\beta^{-}_{\sigma^2} = -0.08 \pm 0.02$ which are consistent with the exponents determined for the German market on short time scales.

(d) Log-log plot of the volume aggregation on short time scales (50 ticks $\leq \Delta t \leq$ 1000 ticks) exhibits a power law behavior with exponents $\beta^{+}_{v} = -0.146 \pm 0.005$ and $\beta^{-}_{v} = -0.072 \pm 0.001$.

(e) Log-log plot of the volume aggregation on long time scales (10 days $\leq \Delta t \leq$ 100 days) exhibits a power law behavior with exponents $\beta^{+}_{v} = -0.115 \pm 0.003$ and $\beta^{-}_{v} = -0.050 \pm 0.002$ which are consistent with our results for short time scales.

(f) Log-log plot of the inter-trade time aggregation on short time scales (50 ticks $\leq \Delta t \leq$ 100 ticks) exhibits a power law behavior with exponents $\beta^{+}_{\tau} = 0.120 \pm 0.002$ and $\beta^{-}_{\tau} = 0.087 \pm 0.002$. An equivalent analysis on long time scales is not possible as daily closing prices are recorded with equidistant time steps.

behavior of the inter-trade times $\tau(t)$ of the German market (see Fig. 2b). The linear cross-correlation function between price changes and intertrade times exhibits no significant correlation values as well. Thus, it is of crucial importance to distinguish between positive and negative microtrends realized by the renormalized time ε. In Fig. 3c, the mean inter-trade time is shown mirroring the clear link between inter-trade times and price extrema. Far away from the critical point $\varepsilon = 1$ the mean inter-trade time starts to decrease. After the formation of a new local price maximum the mean inter-trade times increase and return to the average value in a very symmetrical way. Figure 4f shows $\tau^*(\varepsilon)$ versus $|\varepsilon - 1|$ as a log-log histogram supporting a power law behavior of the form $\tau^*(|\varepsilon - 1|) \sim |\varepsilon - 1|^{\beta\tau}$ for microtrends. A log-log histogram of a parallel analysis for the US market on large time scales is not obtainable as the inter-trade times between successive closing prices are constant (exceptions are weekends and general holidays).

The straight lines in Fig. 4 offer insight into financial market fluctuations: (i) a clear connection between volatility, volumes, inter-trade times, and price fluctuations on the path from one extremum to the next extremum, and (ii) the underlying law, which describes the tails of volatility, volumes, and inter-trade times around extrema varying over 9 orders of magnitude starting from the smallest possible time scale, is a power law with a unique exponents which quantitatively characterize the region around the trend switching point. As a direct consequence of the existence of power law tails, the behavior does not depend on the scale. With decreasing Δt, the number of local minima and maxima increases (see Fig. 1), around which we find scale-free behavior, for the same ε interval $0.6 \leq \varepsilon \leq 1.4$.

In summary we have seen that each trend in a financial market starts and ends with a unique switching process, and each extremum shares properties of macroscopic cooperative behavior. We have seen that the mechanism of bubble formation and bubble bursting has no scale for time scales varying over 9 orders of magnitude down to the smallest possible time scale---the scale of single transactions measured in units of 10 ms. On large time scales, histograms of price returns provide the same scale-free behavior. Thus, the formation of positive and negative trends on all scales is a fundamental principle of trading, starting on the smallest possible time scale, which leads to the non-stationary nature of financial markets as well as to crash events on large time scales. Thus, the well-known catastrophic bubbles occurring on large time scales may not be outliers but in fact single dramatic representatives caused by the scale-free behavior of the forming of increasing and decreasing trends on time scales from the very large down to the very small.

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