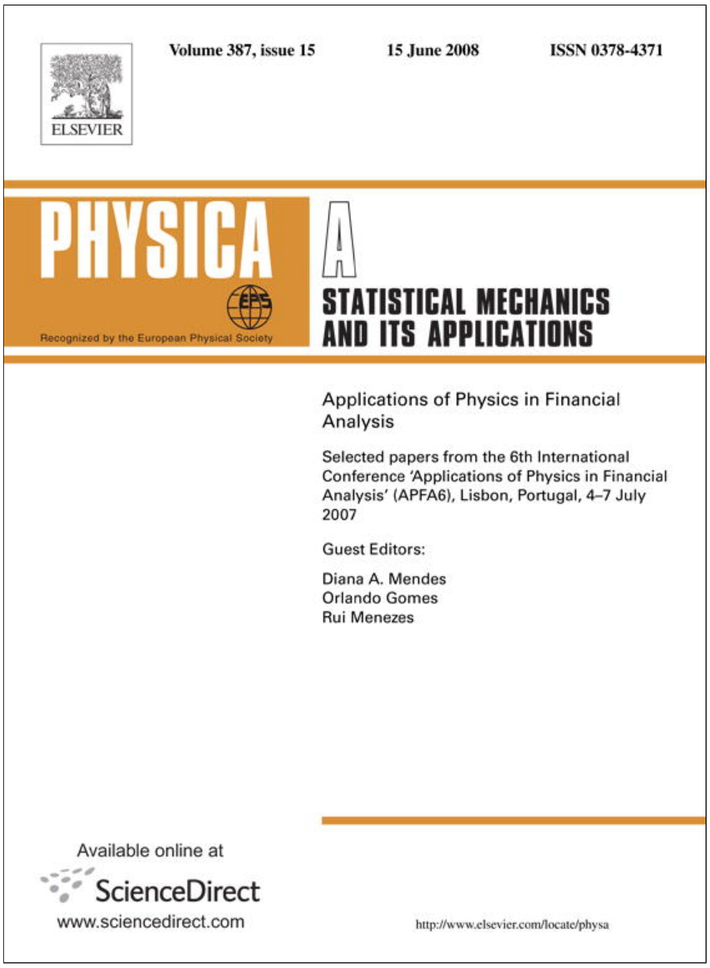

Volume 387, issue 15 15 June 2008 ISSN 0378-4371

ELSEVIER

PHYSICA A

Recognized by the European Physical Society

STATISTICAL MECHANICS AND ITS APPLICATIONS

Applications of Physics in Financial Analysis

Selected papers from the 6th International Conference 'Applications of Physics in Financial Analysis' (APFA6), Lisbon, Portugal, 4–7 July 2007

Guest Editors:

Diana A. Mendes
Orlando Gomes
Rui Menezes

Available online at
ScienceDirect
www.sciencedirect.com

http://www.elsevier.com/locate/physa

Available online at www.sciencedirect.com

ScienceDirect

ELSEVIER

Physica A 387 (2008) 3954–3959

PHYSICA A

www.elsevier.com/locate/physa

Modeling long-range cross-correlations in two-component ARFIMA and FIARCH processes

Boris Podobnik[a,b,*], Davor Horvatic[c], Alfonso Lam Ng[d], H. Eugene Stanley[d], Plamen Ch. Ivanov[d,e,f]

[a] Department of Physics, Faculty of Civil Engineering, University of Rijeka, Rijeka, Croatia
[b] Zagreb School of Economics and Management, Zagreb, Croatia
[c] Department of Physics, Faculty of Science, University of Zagreb, Zagreb, Croatia
[d] Center for Polymer Studies and Department of Physics, Boston University, Boston, MA 02215, United States
[e] Division of Sleep Medicine, Brigham and Woman's Hospital, Harvard Medical School, Boston, MA 02115, United States
[f] Institute of Solid State Physics, Bulgarian Academy of Sciences, Sofia, Bulgaria

Available online 16 January 2008

Abstract

We investigate how simultaneously recorded long-range power-law correlated multivariate signals cross-correlate. To this end we introduce a two-component ARFIMA stochastic process and a two-component FIARCH process to generate coupled fractal signals with long-range power-law correlations which are at the same time long-range cross-correlated. We study how the degree of cross-correlations between these signals depends on the scaling exponents characterizing the fractal correlations in each signal and on the coupling between the signals. Our findings have relevance when studying parallel outputs of multiple component of physical, physiological and social systems.

Many empirical data are characterized by long-range power-law auto-correlations as well as by long-range cross-correlations. Such a scale-invariant organization in both auto-correlations and cross-correlations can be observed either for the data variables or for their absolute values [1–9].

Scale-invariant power-law auto-correlations in stochastic variables can be modeled by the fractionally auto-regressive integrated moving-average process (ARFIMA) [10,11]:

$$x_t = \sum_{n=1}^{\infty} a_n(d) x_{t-n} + \epsilon_t, \tag{1}$$

where $d \in (-0.5, 0.5)$ is a scaling parameter, ϵ_t denotes independent and identically distributed (i.i.d.) Gaussian variables with $\langle \epsilon_t \rangle = 0$ and $\langle \epsilon_t^2 \rangle = 1$, $a_n(d)$ are the weights defined by $a_n(d) = d\,\Gamma(n-d)/(\Gamma(1-d)\Gamma(n+1))$, where Γ denotes the Gamma function and n is the time scale. We denote the auto-correlation function for x_t as $A(x_t, x_{t-n}) \equiv A(n)$. For $d = 0$, the generated variable x_t becomes random.

* Corresponding author at: Department of Physics, Faculty of Civil Engineering, University of Rijeka, Rijeka, Croatia.
E-mail address: bp@sirius.phy.hr (B. Podobnik).

0378-4371/$ - see front matter
doi:10.1016/j.physa.2008.01.062

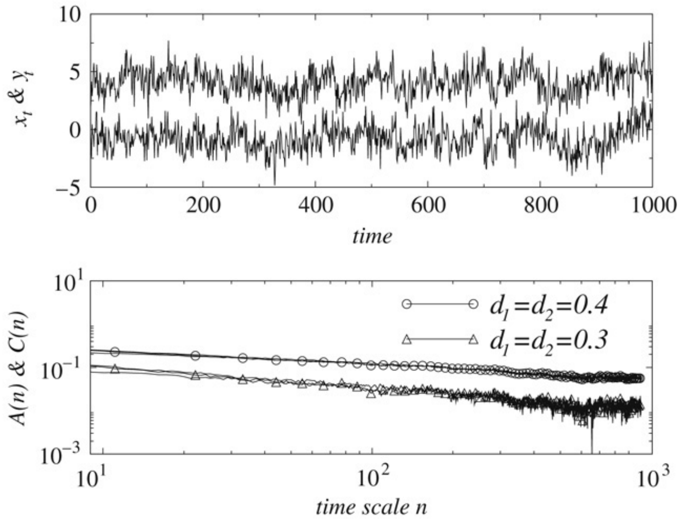

Fig. 1. (a) Time series x_t and y_t for the process defined in Eqs. (2a)–(2d) where $W = 0.8$ and $d_1 = d_2 = 0.4$. The time series x_t is vertically shifted for clarity. Both x_t and y_t exhibit apparent comovement, indicating a high degree of cross-correlation. (b) Log–log plots of the auto-correlation functions $A(n)$ for x_t and y_t, and their cross-correlation function $C(n)$ for the two-component ARFIMA process with $W = 0.8$ and $d_1 = d_2 = 0.4$ (top three curves), and with $W = 0.8$ and $d_1 = d_2 = 0.3$ (bottom three curves). For decreasing values of the scaling parameters d_1 and d_2 both the auto-correlations and cross-correlations decrease, leading to smaller values of $A(n)$ and $C(n)$.

To account for power-law cross-correlations between two variables x_t and y_t, where each variable is itself power-law auto-correlated, we propose a two-component ARFIMA stochastic process defined by two stochastic variables x_t and y_t. Each of these variables at any time depends not only on its own past values but also on past values of the other variable:

$$x_t = [W X_t + (1 - W) Y_t] + \epsilon_t, \tag{2a}$$

$$y_t = [(1 - W) X_t + W Y_t] + \tilde{\epsilon}_t, \tag{2b}$$

$$X_t = \sum_{n=1}^{\infty} a_n(d_1) x_{t-n}, \tag{2c}$$

$$Y_t = \sum_{n=1}^{\infty} a_n(d_2) y_{t-n}, \tag{2d}$$

where ϵ_t and $\tilde{\epsilon}_t$ denote i.i.d. Gaussian variables with $\langle \epsilon_t \rangle = \langle \tilde{\epsilon}_t \rangle = 0$ and $\langle \epsilon_t^2 \rangle = \langle \tilde{\epsilon}_t^2 \rangle = 1$, $a_n(d_1)$ and $a_n(d_2)$ are the weights defined in Eq. (1) through the scaling parameters d_1 and d_2 ($0 \le d_{1,2} < 0.5$), and W is a free parameter controlling the coupling strength between x_t and y_t ($0.5 \le W \le 1$). We denote the cross-correlation function between x_t and y_t as $C(x_t, y_{t-n}) \equiv C(n)$. For different values of W a different degree of cross-correlation between the variables x_t and y_t is observed. For example, for the case when $W = 1$, the process defined in Eqs. (2a)–(2d) reduces to two decoupled ARFIMA processes defined in Eq. (1). Thus, when $W = 1$ the long-range cross-correlations between x_t and y_t vanish, while both x_t and y_t remain long-range power-law auto-correlated.

In Fig. 1(a) we show segments of the time series x_t and y_t generated by the process defined in Eqs. (2a)–(2d) with parameters $W = 0.8$ and $d_1 = d_2 = 0.4$. Both variables exhibit a very similar comovement. In Fig. 1(b) we show the auto-correlation functions $A(n)$ for x_t and y_t, as well as the cross-correlation function $C(x_t, y_{t-n}) \equiv C(n)$. These three curves practically overlap [Fig. 1(b), three top curves]. We also show the same correlation functions for $W = 0.8$ and $d_1 = d_2 = 0.3$ [Fig. 1(b), three bottom curves]. Generally, when the coupling parameter W is kept fixed, the stochastic process we introduce in Eq. (2) generates stronger cross-correlations for larger values of the scaling parameters d_1 and d_2.

Motivated by the fact that for linear processes the auto-correlation function does not change under randomization of the Fourier phase [12,13], we next test how this phase-randomization procedure affects the degree of cross-correlations between x_t and y_t. First, we perform a Fourier transform of the original time series, e.g. x_t, preserving the Fourier amplitudes but randomizing the Fourier phases. Then, we perform an inverse Fourier transform and obtain a surrogate

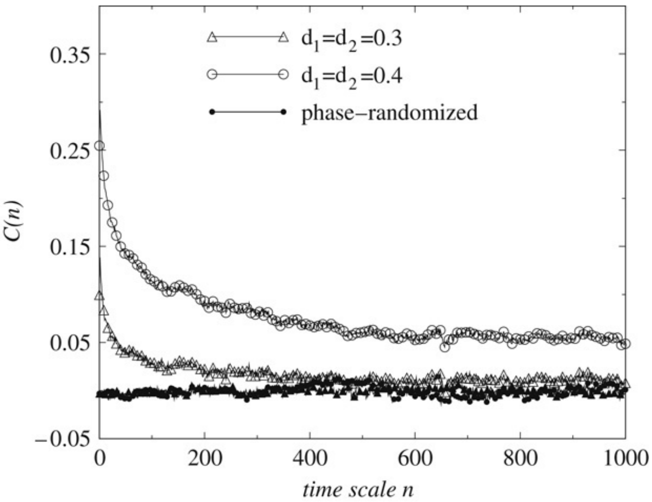

Fig. 2. Cross-correlation function $C(n)$ before Fourier phase-randomization procedure for the time series x_t and y_t shown in Fig. 1 (open symbols). After Fourier phase randomization of x_t and y_t the cross-correlation function virtually disappears (filled symbols) for any value of d_1 and d_2.

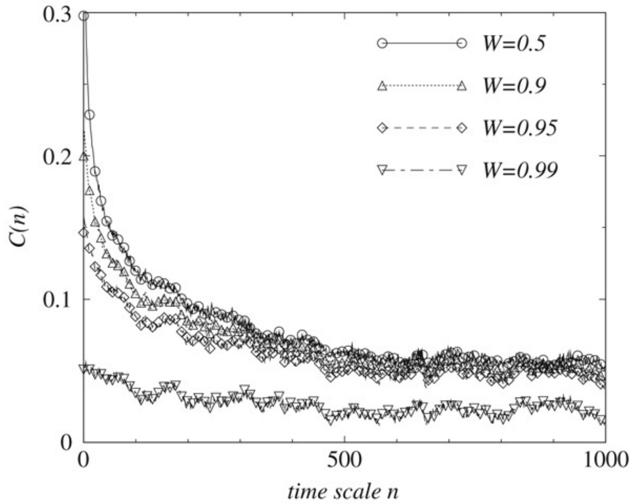

Fig. 3. Cross-correlation function $C(n)$ between time series x_t and y_t generated by the process in Eq. (2) for varying values of W and $d_1 = d_2 = 0.4$. The cross-correlation function has highest values for $W = 0.5$, and tends to zero for W approaching 1. When $W = 1$, x_t and y_t become two decoupled ARFIMA processes.

(linearized) time series $\tilde{x}_t$. Applying this phase-randomization procedure to both time series x_t and y_t generated by the two-component ARFIMA process in Eq. (2), we calculate the two auto-correlation functions for $\tilde{x}_t$ and $\tilde{y}_t$, as well as their cross-correlation function $C(\tilde{x}_t, \tilde{y}_{t-n})$. As expected, the auto-correlation functions remain unchanged after Fourier phase randomization, but the cross-correlation function $C(\tilde{x}_t, \tilde{y}_{t-n})$ completely vanishes [Fig. 2].

Next, we investigate the case when the scaling parameters d_1 and d_2 are fixed, while the coupling parameter W varies. In Fig. 3, we show how the cross-correlation function changes for different values of W and for fixed $d_1 = d_2 = 0.4$. The closer the value of the parameter W to 1, the weaker the cross-correlations ($W = 1$ corresponds to the case of two decoupled ARFIMA processes).

Next we analyze how the degree of power-law auto-correlations changes when varying parameters W, d_1, and d_2 in Eqs. (2a)–(2d). To quantify the auto-correlations we employ the detrended fluctuations analysis (DFA) method. We estimate the rms fluctuation function $F(n)$ for different time scales n [14–18]. A power-law dependence of $F(n)$ on the time scale n – $F(n) \propto n^{\alpha}$, where α is the correlation exponent – indicates presence of power-law auto-correlations. In Fig. 4, we show the DFA scaling curves obtained for x_t and y_t generated by the two-component ARFIMA process in Eqs. (2a)–(2d), where $d_1 = 0.4$ and $d_2 = 0.1$, and the coupling parameter W varies. For $W = 1$ the processes x_t and y_t are decoupled and thus not cross-correlated. In this case, x_t behaves as a power-law

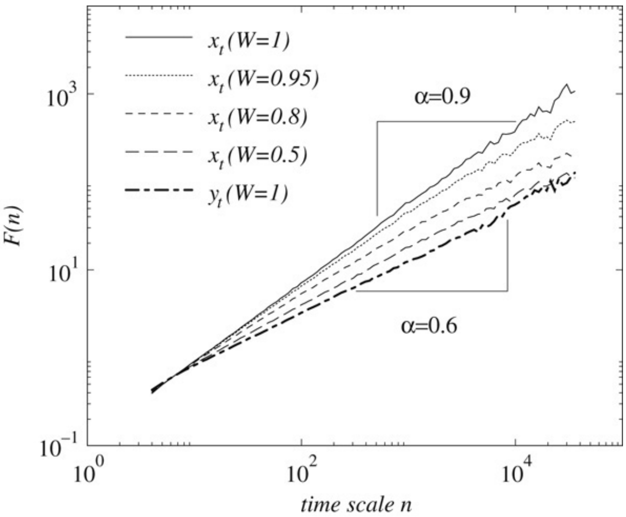

Fig. 4. DFA scaling curves for the time series x_t and y_t generated by the two-component ARFIMA process in Eqs. (2a)–(2d), where $d_1 = 0.4$ and $d_2 = 0.1$. For $W = 1$, x_t and y_t are decoupled and thus not cross-correlated, and x_t behaves as the ARFIMA process in Eq. (1) defined only by the scaling parameter d_1, while y_t becomes a separate ARFIMA process defined only by the scaling parameter d_2. For $W \neq 1$, the scaling properties of x_t depend on both parameters d_1 and d_2. When $W = 0.5$, the DFA correlation exponent α for x_t becomes equal to the DFA correlation exponent for y_t. The DFA exponent for $|y_t|$ does not depend on W.

auto-correlated ARFIMA process controlled by only the scaling parameter d_1, with the DFA correlation exponent equals $\alpha = 0.5 + d_1 = 0.9$. Similarly, y_t becomes a separate ARFIMA process (decoupled from x_t) which is controlled only by the scaling parameter d_2, where $\alpha = 0.5 + d_2 = 0.6$. We find that with decreasing value of W (from 1 to 0.5), x_t becomes a mixture of two ARFIMA processes and the DFA correlation exponent α gradually decreases towards $\alpha = 0.6$ corresponding to the y_t process, controlled by parameter $d_2 = 0.1$. In contrast to x_t, for the process y_t the DFA correlation exponent α virtually does not change with the varying coupling parameter W.

In finance, the efficient market hypothesis (EMH) asserts that prices on traded assets (bonds, stocks) already reflect all information about the assets. In an efficient market, stock prices are commonly tested by employing random walk hypothesis. In Ref. [19] it is found the long-range power-law dependence in the capital markets of six European transition economies. Clearly, one may expect that some of these markets are mutually related and thus a process like the one defined in Eqs. (2a)–(2d) has a potential applicability.

We next consider a separate stochastic process which generates simultaneously two time series with power-law auto-correlated absolute values of their variables and long-range cross-correlations between these absolute values. Power-law auto-correlations in the absolute values of the stochastic variables can be modeled by the Fractionally Integrated ARCH (FIARCH) process [20,21]:

$$x_t = \sigma_t \epsilon_t \tag{3a}$$

$$\sigma_t = \sum_{n=1}^{\infty} a_n(d) \frac{|x_{t-n}|}{\mu_x}, \tag{3b}$$

where ϵ_t denotes an i.i.d. Gaussian variable with $\langle \epsilon_t \rangle = 0$ and $\langle \epsilon_t^2 \rangle = 1$, and $0 < d < 1/2$ and $\mu_x = \langle |x_t| \rangle$. The sum of the weights $a_n(d)$ satisfies $\sum_{n=1}^{\infty} \frac{d\ \Gamma(n-d)}{\Gamma(1-d)\Gamma(n+1)} = 1$, yielding $\langle \sigma_t \rangle = 1$. While for the time series x_t generated by Eq. (1) the auto-correlation function $A(x_t, x_{t-n})$ is zero for all time scales n, for the absolute values $|x_t|$ the auto-correlation function is $A(|x_t|, |x_{t-n}|) = \Gamma(1-d)\Gamma(n+d)/(\Gamma(d)\Gamma(n+1-d))$, which for $n \gg 1$ converges to the power law $A(n) \sim n^{-1+2d}$.

To account for power-law cross-correlations between the absolute values of two variables, where the absolute values of each variable are simultaneously power-law auto-correlated, we have previously introduced [22] a two-component FIARCH process with scaling parameters d_1 and d_2:

$$x_t = [W\sigma_{xt} + (1-W)\sigma_{yt}]\epsilon_t \tag{4a}$$

$$y_t = [(1-W)\sigma_{xt} + W\sigma_{yt}]\tilde{\epsilon}_t \tag{4b}$$

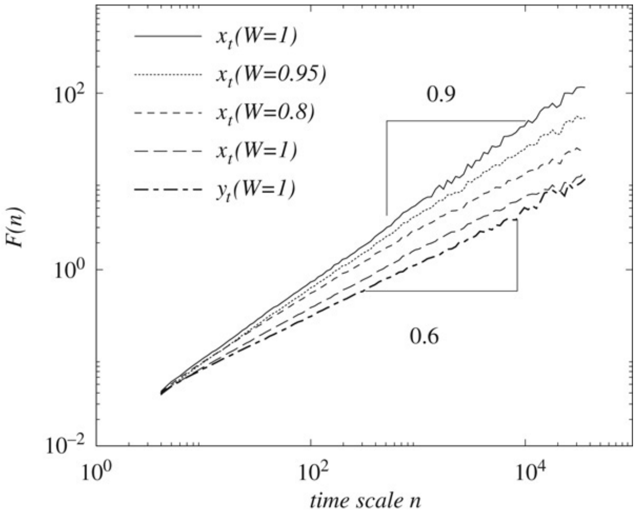

Fig. 5. DFA scaling curves of the time series $|x_t|$ and $|y_t|$ generated by the two-component FIARCH process in Eqs. (4a)–(4d), where $d_1 = 0.4$ and $d_2 = 0.1$. For $W = 1$, $|x_t|$ and $|y_t|$ are decoupled and thus are not cross-correlated. In this case, x_t becomes a separate FIARCH process as defined in Eq. (3), and the auto-correlation properties of x_t depend only on the scaling parameter d_1, while y_t is another FIARCH process with auto-correlation properties depending only on the parameter d_2. For $W \neq 1$, the scaling properties of x_t depend on both parameters d_1 and d_2. When $W = 0.5$, the DFA correlation exponent α for $|x_t|$ becomes equal to the DFA correlation exponent for $|y_t|$. Note that the DFA exponent for $|y_t|$ does not depend on W.

$$\sigma_{xt} = \sum_{n=1}^{\infty} \frac{d_1\,\Gamma(n-d_1)}{\Gamma(1-d_1)\Gamma(n+1)} \frac{|x_{t-n}|}{\mu_x} \tag{4c}$$

$$\sigma_{yt} = \sum_{n=1}^{\infty} \frac{d_2\,\Gamma(n-d_2)}{\Gamma(1-d_2)\Gamma(n+1)} \frac{|y_{t-n}|}{\mu_y}, \tag{4d}$$

where ϵ_t and $\tilde{\epsilon}_t$ are i.i.d. variables with $\langle\epsilon_t\rangle = \langle\tilde{\epsilon}_t\rangle = 0$ and $\langle\tilde{\epsilon}_t^2\rangle = \langle\epsilon_t^2\rangle = 1$, W is the coupling parameter controlling the degree of cross-correlations, and $\mu_x = \langle|x_t|\rangle$ and $\mu_y = \langle|y_t|\rangle$.

Note, that each of the variables is controlled by a composite volatility – e.g. for x_t the composite volatility is $W_1\sigma_{xt} + (1 - W_1)\sigma_{yt}$ [Eq. (4a)] – that is a combination of two FIARCH volatilities σ_{xt} and σ_{xt} [Eq. (3b)]. Stability of the FIARCH process is achieved through the condition $\langle\sigma_t\rangle = 1$. To retain stability for the two-component FIARCH process in Eq. (4), the average values of the composite volatilities $W\sigma_{xt} + (1 - W)\sigma_{yt}$ and $(1 - W)\sigma_{xt} + W\sigma_{yt}$ in Eqs. (4a) and (4b) should be 1. For $W = 1$ the process in Eqs. (4a)–(4d) reduces to two decoupled FIARCH process as defined in Eqs. (3a) and (3b), and thus $|x_t|$ and $|y_t|$ are not cross-correlated.

In Ref. [22] we have analyzed the cross-correlation functions between $|x_t|$ and $|y_t|$ for the process defined in Eqs. (4a)–(4d) for varying values of the parameters W, d_1, and d_2.

Finally, we analyze how the auto-correlations in the absolute values change when varying the parameters W, d_1, and d_2. In Fig. 5, we show the DFA scaling curves for $d_1 = 0.4$ and $d_2 = 0.1$, and for varying W. For $W = 1$, the time series x_t and y_t are decoupled and so not cross-correlated. In this case, x_t is a FIARCH process controlled only by the scaling parameter d_1, and exhibits long-range power-law auto-correlations characterized by a DFA correlation exponent $\alpha = 0.5 + d_1 = 0.9$. Similarly, y_t is another FIARCH process controlled only by d_2, and characterized by $\alpha = 0.5 + d_2 = 0.6$. We find that with decreasing value of W (from 1 to 0.5), x_t is controlled by both parameters d_1 and d_2, and the DFA exponent α gradually decreases towards the value $\alpha = 0.6$. At the same time, the process y_t which is controlled only by the parameter $d_2 = 0.1$ is also characterized by $\alpha = 0.6$, regardless of the values of W.

Besides FIARCH process, we also analyze FIGARCH(1,d,0) process (d is a scaling parameter) defined in Ref. [23]. In Eqs. (4a) and (4d), we replace two FIARCH processes by two FIGARCH processes and find, regarding cross-correlations, the same scaling behavior as found for FIARCH processes.

The presented modeling approach and findings may have relevance when quantifying cross-correlations in simultaneously recorded multivariate time series of fractal nature. This problem is pertinent to multiple component of physical [24–26], physiological, social and financial systems.

Acknowledgements

We thank the Ministry of Science of Croatia, NIH (Grant HL071972) and NSF for financial support.

References

[1] C.C. Ying, Econometrica 34 (1966) 676.
[2] R.L. Crouch, Financ. Anal. J. 26 (1970) 104.
[3] G. Tauchen, M. Pitts, Econometrica 51 (1983) 485.
[4] J. Karpoff, J. Financ. Quant. Anal. 22 (1987) 109.
[5] R. Gallant, P. Rossi, G. Tauchen, Rev. Financ. Stud. 5 (1992) 199.
[6] J. Campbell, A.W. Lo, A. MacKinlay, The Econometrics of Financial Markets, Princeton University Press, Princeton NJ, 1997.
[7] V. Plerou, et al., Quant. Financ. 1 (2001) 262.
[8] P. Gopikrishnan, et al., Phys. Rev. E 62 (2000) 4493.
[9] B. LeBaron, W.B. Arthur, R. Palmer, J. Econom. Dynam. Control 23 (1999) 1487.
[10] C.W.J. Granger, R. Joyeux, J. Time Ser. Anal. 1 (1980) 15.
[11] J. Hosking, Biometrika 68 (1981) 165.
[12] J. Theiler, et al., Physica D 58 (1992) 77.
[13] Y. Ashkenazy, et al., Physica A 323 (2003) 19.
[14] C.-K. Peng, et al., Phys. Rev. E 49 (1994) 1685.
[15] K. Hu, et al., Phys. Rev. E 64 (1) (2001) 011114(19).
[16] Z. Chen, et al., Phys. Rev. E 65 (4) (2002) 041107(15).
[17] Z. Chen, et al., Phys. Rev. E 71 (1) (2005) 011104(11).
[18] L. Xu, et al., Phys. Rev. E 71 (5) (2005) 051101(14).
[19] T. Jagric, B. Podobnik, M. Kolanovic, East. European Econ. 43 (2005) 85.
[20] C.W.J. Granger, Z. Ding, Ann. Econom. Statist. 40 (1995) 67.
[21] B. Podobnik, et al., Phys. Rev. E 72 (2005) 026121.
[22] B. Podobnik, et al., Eur. Phys. J. B 56 (2007) 47.
[23] T. Bollerslev, H.O. Mikkelsen, J. Econometrics 73 (1996) 151.
[24] G. Nugent-Glandorf, et al., Phys. Rev. Lett. 87 (19) (2001) 193002.
[25] O.A. Godin, Phys. Rev. Lett. 97 (2006) 054301.
[26] M. Campilo, A. Paul, Science 299 (2003) 547.