Experimental Evidence for Multifractality

The diffusion-limited aggregation (DLA) model has proven useful in describing a wide range of physical phenomena. Recently, considerable attention has focused on the question of how a DLA aggregate grows. Such growth phenomena are characterized by the assignment to each perimeter site \(i\) the number \(p_i\), the probability that site \(i\) is the next to grow.\(^1\) Recent theoretical arguments suggest that the numbers \(p_i\) form a multifractal set: This set cannot be characterized by a single exponent (as in the case of the DLA aggregate itself), but rather an infinite hierarchy of exponents is required.\(^1\) The physical basis for this fact is that the "hottest" tips of a DLA aggregate grow much faster than the deep "fjords"; hence the rate of change of the \(p_i\) differs greatly when \(i\) is a tip perimeter site than when \(i\) is a fjord perimeter site.

Although there have been theoretical calculations\(^1\) of the multifractality of DLA, this is the first experimental test. We will focus upon two-dimensional fractal viscous fingers. We first calculate (Fig. 1) the distribution function \(n(p)\), where \(n(p)\,dp\) is the number of perimeter sites with \(p_i\) in the range \([p,p+dp]\). This curve has a long tail extending to the extremely small values of \(p_i\) for perimeter sites deep inside fjords. We next form the moments \(Z_q = \sum p_i^q\), which are characterized by the hierarchy of exponents \(\tau_q\) defined through \(Z_q = L^{-\tau_q}\) (\(L\) is a characteristic linear dimension). Our results [Fig. 2(a)] show that when \(q\) is large \(\tau_q\) is linear in \(q\), but for \(q\) small there is downward curvature in \(\tau_q\), showing that the fjords have different growth rates than the tips. We also calculate the Legendre transform with respect to \(q\) of \(\tau_q\): \(-f(a) = \tau(q) - qa\), where \(a = d\tau/dq\). Downward curvature in \(\tau(q)\) corresponds to upward curvature in \(-f(a)\) [Fig. 2(b)]. We find good agreement between all three of the experimental functions and the corresponding functions calculated for DLA.\(^1\)

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