Experimental Evidence for Multifractality

The diffusion-limited aggregation (DLA) model has proven useful in describing a wide range of physical phenomena. Recently, considerable attention has focused on the question of how a DLA aggregate grows. Such growth phenomena are characterized by the assignment to each perimeter site i the number p_i, the probability that site i is the next to grow.[1] Recent theoretical arguments suggest that the numbers p_i form a multifractal set: This set cannot be characterized by a single exponent (as in the case of the DLA aggregate itself), but rather an infinite hierarchy of exponents is required.[1] The physical basis for this fact is that the "hottest" tips of a DLA aggregate grow much faster than the deep "fjords"; hence the rate of change of the p_i differs greatly when i is a tip perimeter site than when i is a fjord perimeter site.

Although there have been theoretical calculations[1] of the multifractality of DLA, this is the first experimental test. We will focus upon two-dimensional fractal viscous fingers. We first calculate (Fig. 1) the distribution function $n(p)$, where $n(p)dp$ is the number of perimeter sites with p_i in the range $[p,p+dp]$. This curve has a long tail extending to the extremely small values of p_i for perimeter sites deep inside fjords. We next form the moments $Z_q = \sum (p_i)^q$, which are characterized by the hierarchy of exponents τ_q defined through $Z_q = L^{-\tau_q}$ (L is a characteristic linear dimension). Our results [Fig. 2(a)] show that when q is large τ_q is linear in q, but for q small there is downward curvature in τ_q, showing that the fjords have different growth rates than the tips. We also calculate the Legendre transform with respect to q of τ_q: $-f(\alpha) = \tau(q) - q\alpha$, where $\alpha = d\tau/dq$. Downward curvature in $\tau(q)$ corresponds to upward curvature in $-f(\alpha)$ [Fig. 2(b)]. We find good agreement between all three of the experimental functions and the corresponding functions calculated for DLA.[1]

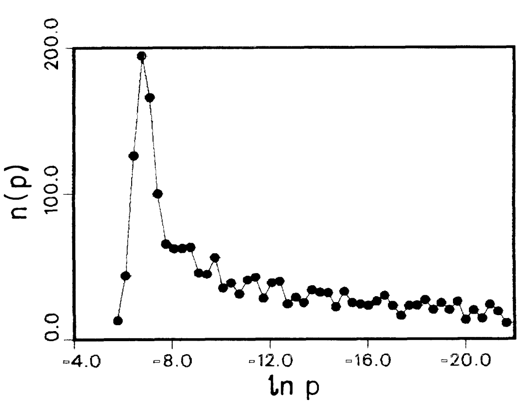

FIG. 1. The distribution function $n(p)dp$ giving the number of perimeter sites with p_i in the interval $[p,p+dp]$.

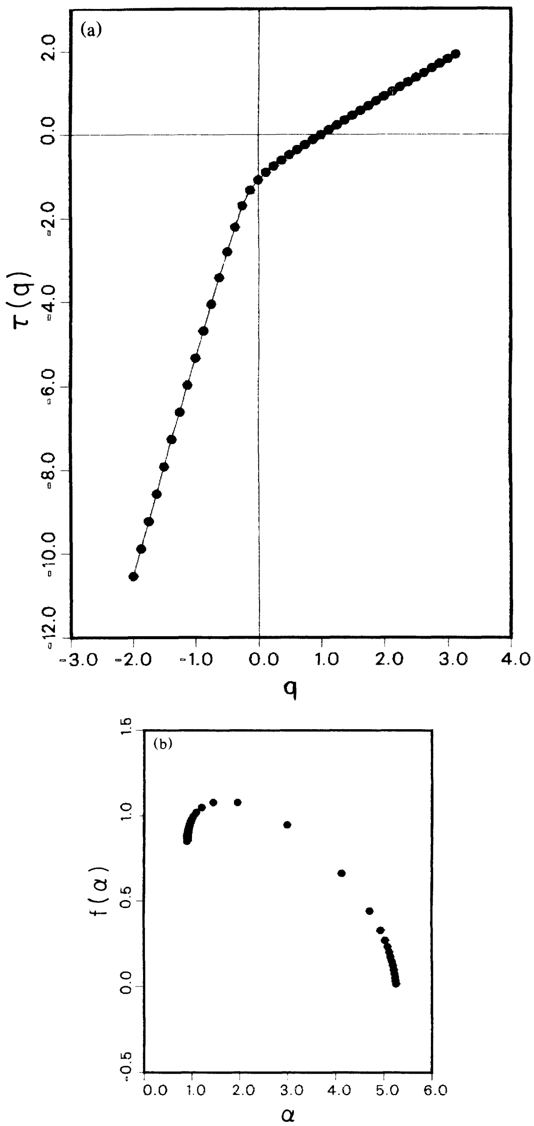

FIG. 2. (a) The hierarchy of exponents τ_q. (b) The function $f(\alpha)$.

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[1]C. Amitrano, A. Coniglio, and F. di Liberto, Phys. Rev. Lett. 57, 1016 (1986).