11

Physics investigation of financial markets

R. N. Mantegna

*Istituto Nazionale di Fisica della Materia, Unità di Palermo
and Dipartimento di Energetica ed Applicazioni di Fisica, Università di Palermo
Palermo, I-90128, Italy*

H. E. Stanley

*Center for Polymer Studies and Department of Physics, Boston University
Boston, MA 02215, USA*

"È importante, quindi, che i principi della meccanica quantistica abbiano portato a riconoscere (....) il carattere statistico delle leggi ultime dei processi elementari. Questa conclusione ha reso sostanziale l'analogia fra fisica e scienze sociali, tra le quali è risultata un'identità di valore e di metodo."
— Ettore Majorana [1]

1. – Introduction

During the last thirty years, physicists have achieved important results in the fields of phase transitions, statistical mechanics, nonlinear dynamics, disordered and self-organized systems. New paradigms have been developed and a range of complex systems have been carefully investigated and described. This description has sometime also been performed in the presence of noise or quenched randomness. With this, relatively recent, background the study of economics systems performed by physicists may produce results relevant for both physics and economics.

Economic systems, strictly regulated and very frequently monitored, are ideal for a study performed using tools and paradigms developed to describe physical systems. Due to strict regulation, such systems suffer only slightly from various modifications of the rules underlying the process during the time window of the investigation of the process. Moreover, due to continuous monitoring the amount of data describing the phenomenon is usually sufficient for a detailed statistical analysis.

The time evolution of price of goods or assets (or indices) in financial markets belong to the above category. Financial markets are almost continuously monitored (high frequency data with recording time interval as short as few seconds exist) and the rules governing these institutions are rather stable.

In this lecture, we first discuss the empirical results obtained by analyzing the time evolution of the Standard & Poor's 500 index of the New York Stock Exchange with high temporal resolution. We then introduce and discuss a simple stochastic model, the truncated Lévy flight (TLF). This model is able to describe several of the major features observed in empirical data. We also discuss observations that are not explained by the TLF model. We end by considering similarity and differences between the price dynamics in a financial market and the dynamics of the velocity of a 3-dimensional turbulent fluid.

2. – Scaling and its breakdown in the Standard & Poor's 500 index

Stock exchange time series have been modelled as stochastic processes since the seminal study of Bachelier published at the beginning of this century [2]. Several stochastic models have been proposed and tested in the economics [3-10] and physics [11-15] literature. Alternative approaches based on the paradigm of chaotic dynamics have been also proposed [16-18]. The most widely accepted models state that the variations of share price is a random process. For the distribution of variations of the logarithm of asset prices several proposal have been published. These include i) a normal distribution [2], ii) a Lévy stable distribution [4], iii) leptokurtik distributions generated by a mixture of normal distributions [7] and iv) ARCH/GARCH models [8,9].

The proposals of: i) a normal distribution [2], and ii) a Lévy stable distribution [4] obey, respectively, the central-limit theorem [19] or a generalized version of it [20]. The most obvious difference between these two stochastic processes involves the wings of the distributions. Distinguishing between the two processes i) and ii) by comparing the distribution wings can be quite difficult because data sets are unavoidably limited. To maximize the amount of data to be analyzed in a limited time interval (limited to avoid that underlying rules or deep economics changes could happen inside the investigated period) we chose to investigate high-frequency data.

Data, kindly provided by the Chicago Mercantile Exchange, consist of *all* 1 447 514 records of the S & P 500 cash index recorded during the 6-year period 1/84–12/89. The time intervals between successive records are not fixed: the average value between successive records is close to 1 min during 1984 and 1985 and close to 15 s during 1986–1989. We define the trading time as a continuous time starting from the opening of the day until the closing, and then continuing with the opening of the next trading day. From this data base, we select the complete set of non-overlapping records separated by a time interval $\Delta t \pm \varepsilon \Delta t$ (where ε is the tolerance, always less than 0.035).

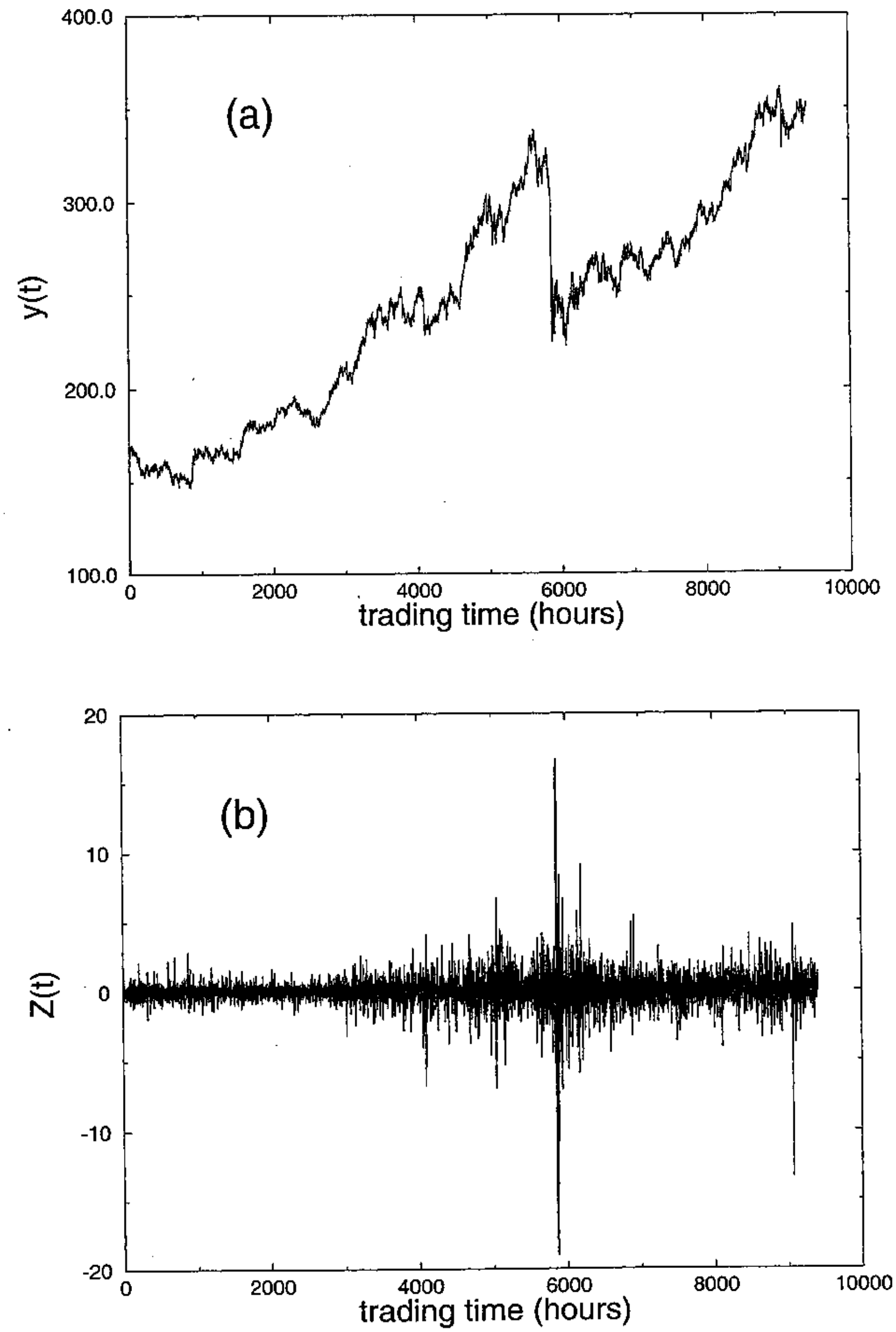

Fig. 1. – (a) Time evolution of the S & P 500 sampled with a time resolution $\Delta t = 1$ h in the period January 1984–December 1989. (b) Hourly variations of the S & P 500 Index in the period January 1984–December 1989.

We denote the value of the S & P 500 as $y(t)$. Figure 1a shows $y(t)$ as a function of the time in the period 1984–1989. The time interval between two successive points in this graph is $\Delta t = 60$ min. In fig. 1b the successive variations of the S & P Index

$$z(t) \equiv y(t) - y(t - \Delta t) \tag{1a}$$

are shown for the same time interval. The intermittent behavior observed in the time evolution of index variations (fig. 1b) will be discussed in the following sections.

In our high-frequency analysis we investigate both price difference and logarithmic difference (this last is the variable more commonly analyzed in economic studies). In this lecture, we present results about price difference. Our choice is motivated by the fact we wish to avoid nonlinear transformations of the investigated stochastic variable (in this case the logarithmic transformation). In fact it is known that nonlinear transformations alter the conditional probabilities of the analyzed stochastic process.

However, in the high-frequency regime, the two stochastic processes $z(t)$ (price differences) and

$$r(t) \equiv \ln(y(t + \Delta t)) - \ln(y(t)), \tag{1b}$$

logarithmic differences have similar statistical properties. This is due to the fact that in the regime $z(t) \ll y(t)$, $r(t) = \ln[1 + z(t)/y(t)]$ is bounded by

$$\frac{z(t)/y(t)}{1 + z(t)/y(t)} < r(t) < \frac{z(t)}{y(t)} \tag{2}$$

and $z(t)$ is a "fast" variable, whereas $y(t)$ is a "slow" variable being the integral of $z(t)$.

To quantitatively characterize the experimentally observed process, we determine [15] the probability distribution $P(z)$ of index variations for different values of Δt. We select Δt values that are logarithmically equally spaced ranging from 1 to 1000 min. The number of data in each set is decreasing from the maximum value of 493,545 ($\Delta t = 1$ min) to the minimum value of 562 ($\Delta t = 1000$ min). As expected for a random process, the distributions are roughly symmetrical and are spreading when Δt increases [15]. We also note that the distributions are leptokurtic, *i.e.* they have wings larger than expected for a normal process. A determination of the parameters characterizing the distributions is difficult if one uses methods that mainly investigate the wings of distributions, especially because larger values of Δt imply a reduced number of data.

Therefore, we use a different approach: we study the «probability of return to the origin» $P(z = 0)$ as a function of Δt. With this choice we investigate the point of each probability distribution that is *least* affected by the noise introduced by the finiteness of the experimental data set. Our investigation of $P(0)$ *vs.* Δt in a log-log plot [15] shows that the data are well fit by a straight line characterized by the slope -0.712 ± 0.025. We observe a non-normal scaling behavior (slope $\neq -0.5$) in an interval of trading time ranging from 1 to 1000 min.

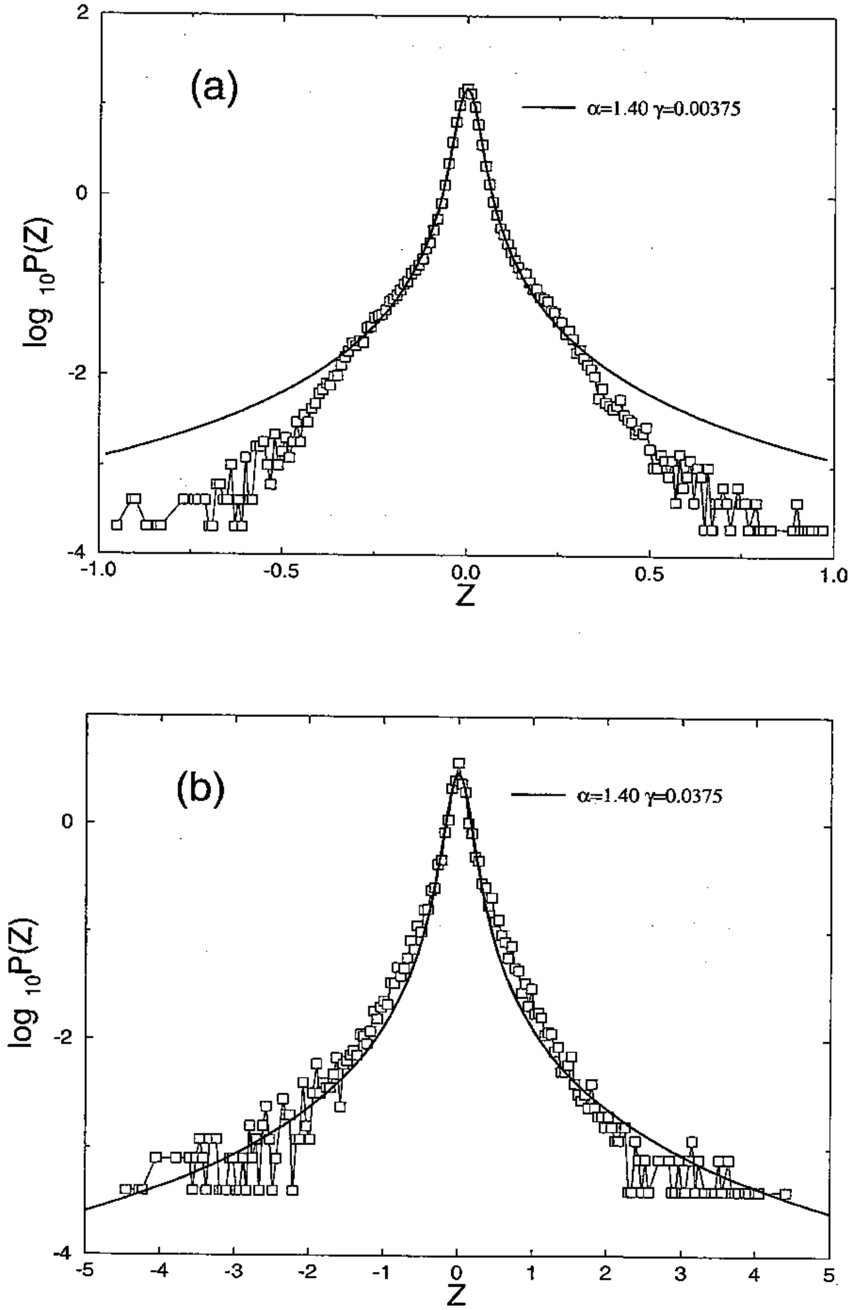

Fig. 2. – Comparison of the (a) $\Delta t = 1$ min, (b) $\Delta t = 10$ min, (c) $\Delta t = 100$ min, and (d) $\Delta t = 1000$ min probability distributions with the symmetrical Lévy stable distributions (solid lines) of index $\alpha = 1.40$ and scale factor $\gamma = \gamma_1 \Delta t$ ($\gamma_1 = 0.00375$). Approximately exponential deviations from the Lévy stable profile are observed for $\Delta t < 10$ minutes ((a) panel).

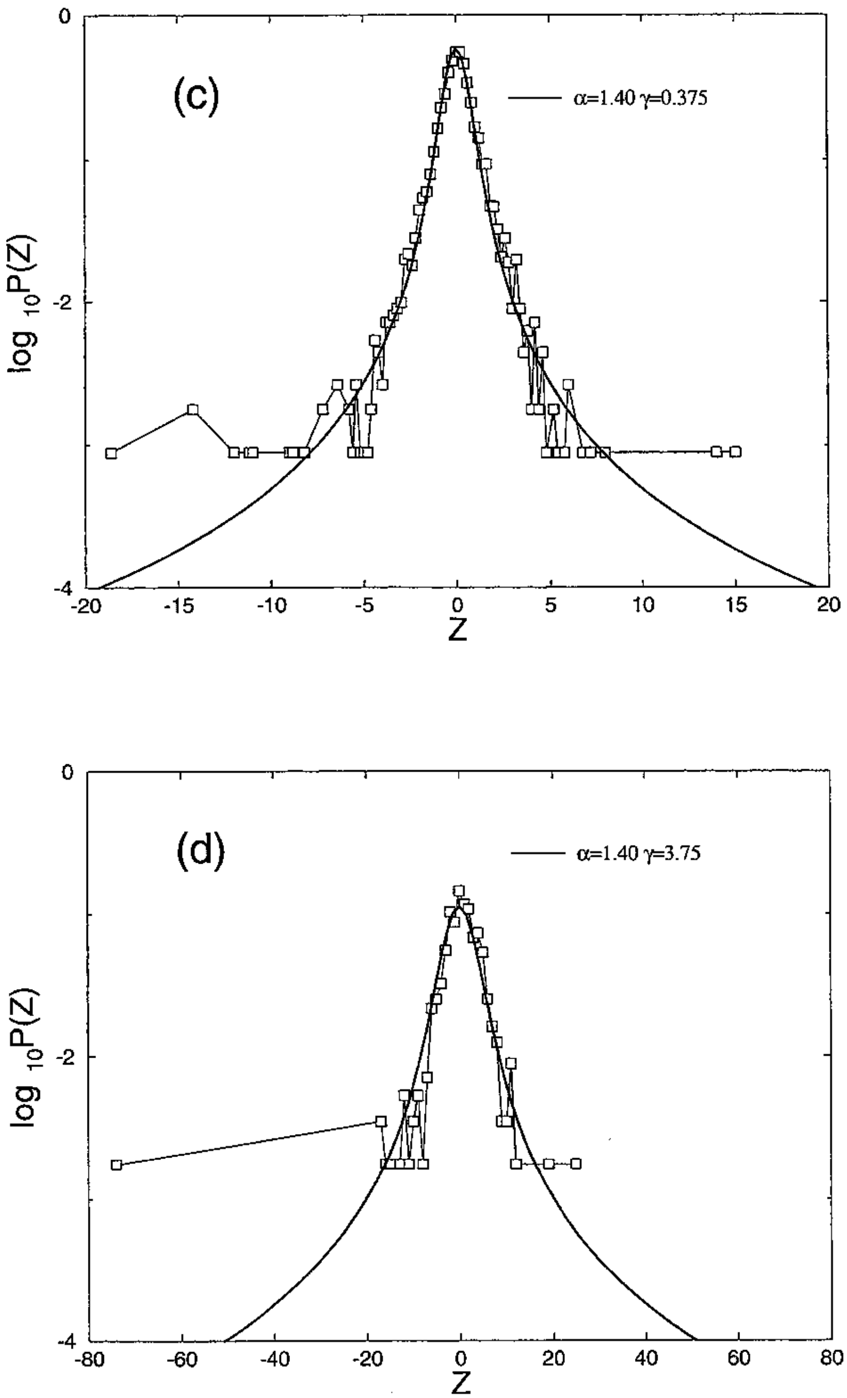

Fig. 2 (*continued*).

This empirical finding agrees with the theoretical model of a Lévy walk or Lévy flight [21, 22]. In fact, if the central region of the distribution is well described by a Lévy stable symmetrical distribution [23],

$$L_\alpha(z, \Delta t) \equiv \frac{1}{\pi} \int_0^\infty \exp[-\gamma \Delta t q^\alpha] \cos(qz)\, \mathrm{d}q \,, \tag{3}$$

of index α and scale factor γ at $\Delta t = 1$, then the probability of return is given by

$$P(0) \equiv L_\alpha(0, \Delta t) = \frac{\Gamma(1/\alpha)}{\pi \alpha (\gamma \Delta t)^{1/\alpha}} \,. \tag{4}$$

By using the value -0.712 from the analysis of the probability of return, one obtains the index $\alpha = 1.40 \pm 0.05$ [15]. We also check if the scaling extends over the entire probability distribution as well as $z = 0$. Figure 2a-d show the $P(Z)$ distributions measured for $\Delta t = 1, 10, 100$, and 1000, respectively. In each figure, we also show the Lévy stable distribution of index $\alpha = 1.4$ (the value of the index was obtained from the analysis of the probability of return to the origin) and $\gamma = \gamma_1 \Delta t$ (where γ_1 is the scale factor of the $\Delta t = 1$ min distribution obtained from the measured value of the probability of return to the origin of the $\Delta t = 1$ minute distribution under the assumption that $\alpha = 1.4$ [15]).

All the distributions agree well with a Lévy stable distribution. The distributions obtained with the highest temporal resolution ($\Delta t < 10$) (fig. 2a) show that in addition to the good agreement with the Lévy (non-Gaussian) profile observed for almost three orders of magnitude an approximately exponential fall-off is present. The clear deviation of the tails of the distribution from the Lévy profile shows us that the experimental tails are less fat than expected for a Lévy distribution. The deviation from the Lévy distribution is not observable for $\Delta t \geqslant 10$, due to the limited number of records used to obtain these distributions.

The Lévy distribution has an infinite second moment (if $\alpha < 2$) [23]. However, our empirical finding of an exponential (or stretched exponential) fall-off implies that the second moment is finite, thereby resolving the question about the finiteness of the variance of the price change distribution [24]. This conclusion might at first sight seem to contradict our observation of Lévy scaling of the central part of the price difference distribution over fully three orders of magnitude. However, there is no contradiction since the above findings might be interpreted in terms of a simple stochastic process, the truncated Lévy flight [25].

3. – The truncated Lévy flight

The truncated Lévy flight (TLF) has been introduced by Mantegna and Stanley in ref. [25]. A TLF is defined as a stochastic process $\{x\}$ characterized by the following probability density function

$$T(x) \equiv \begin{cases} 0\,, & x > l\,, \\ c_1 L(x) & -l \leqslant x \leqslant l\,, \\ 0\,, & x < -l\,, \end{cases} \tag{5}$$

where

$$L(x) \equiv \frac{1}{\pi} \int_0^{+\infty} \exp[-\gamma q^\alpha] \cos(qx)\, \mathrm{d}q \tag{6}$$

is the symmetrical Lévy stable distribution of index α $(0 < \alpha \leqslant 2)$ and scale factor γ $(\gamma > 0)$, c_1 is a normalizing constant and l is the cut-off length. For the sake of simplicity, we set $\gamma = 1$.

The central limit theorem (CLT) is fundamental to statistical mechanics. It states that when $n \to \infty$, the sum

$$z_n \equiv \sum_{i=1}^{n} x_i \tag{7}$$

of n stochastic variables $\{x\}$ that are statistically independent, identically distributed and with a finite variance, converges to a normal (Gaussian) stochastic process. Generally, $n \approx 10$ is sufficient to ensure convergence. In a dynamical system, eq. (9) defines a random walk if the variable x is the jump size performed after a time interval Δt and n is the number of time intervals. In this lecture, the «number of variables» n and the «time» $t = n\Delta t$ can be interchanged everywhere.

We investigate the probability distribution $P(z_n)$ of the stochastic process of eq. (9) when $\{x\}$ is a TLF, *i.e.*, a stochastic process with probability distribution given by eq. (7). We monitor the degree of convergence of the TLF to the asymptotic normal process by investigating the probability of return to the origin of the process $P(z_n = 0)$. The reason for this choice is twofold, first this will give us a concrete parallel to what we investigated in the previous section, and second the point $z_n = 0$ of the distribution $P(z)$ is the last point to converge to the asymptotic normal process for symmetrical stochastic processes.

For low values of n, $P(z_n = 0)$ takes a value very close to the one expected for a Lévy stable process

$$P(z_n = 0) \simeq L(z_n = 0) = \frac{\Gamma(1/\alpha)}{\pi\alpha n^{1/\alpha}}\,. \tag{8}$$

For large values of n, $P(z_n = 0)$ assumes the value predicted for a normal process,

$$P(z_n = 0) \simeq N(z_n = 0) = \frac{1}{\sqrt{2\pi}\,\sigma_0(\alpha, l)\, n^{1/2}}\,, \tag{9}$$

where $\sigma_0(\alpha, l)$ is the standard deviation of the TLF stochastic process $\{x\}$.

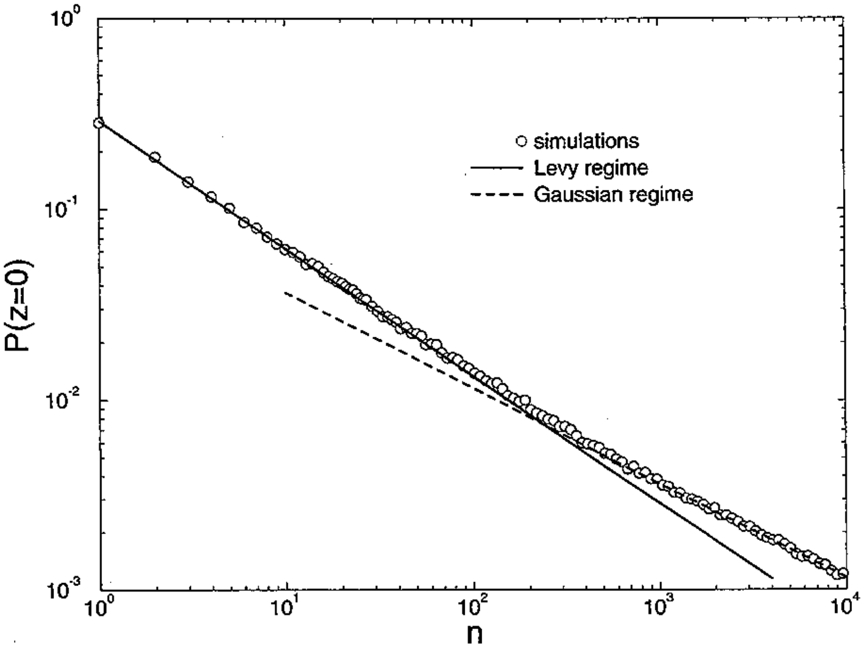

Fig. 3. – Probability of return to the origin of z_n as a function of n for $\alpha = 1.5$ and $l = 100$. The simulations (circles obtained with $5 \cdot 10^4$ realizations) are compared with the asymptotic Lévy regime (solid line) and the asymptotic Gaussian regime calculated for $l = 100$ (dotted lines).

In the interval $1 \leqslant \alpha < 2$, the crossover between the two regimes has been determined in ref. [25] as

$$n_\times \simeq A l^\alpha , \tag{10}$$

where

$$A = \left[\frac{\pi\alpha}{2\Gamma(1/\alpha)[\Gamma(1+\alpha)\sin(\pi\alpha/2)/(2-\alpha)]^{1/2}} \right]^{2\alpha/(\alpha-2)} . \tag{11}$$

The description of the convergence process is not crucially depending on the exact shape of the cut-off [26] and some results of ref. [25] have been confirmed analytically for an exponential cut-off in ref. [27].

By performing numerical simulations, it is possible to investigate the process of convergence of the TLF to its asymptotic Gaussian process. To generate a Lévy stable stochastic process of index α and scale factor $\gamma = 1$, we use the algorithm of ref. [28]. 6.9
Other algorithms can be found in the mathematical literature [29]. In fig. 3 we show the probability of return obtained by simulating the z_n process when $\alpha = 1.5$ and $l = 100$. We also show the asymptotic behaviors predicted by using eq. (7) (solid line) and eq. (8) (dashed lines). We clearly see the crossover between the two regimes. For the

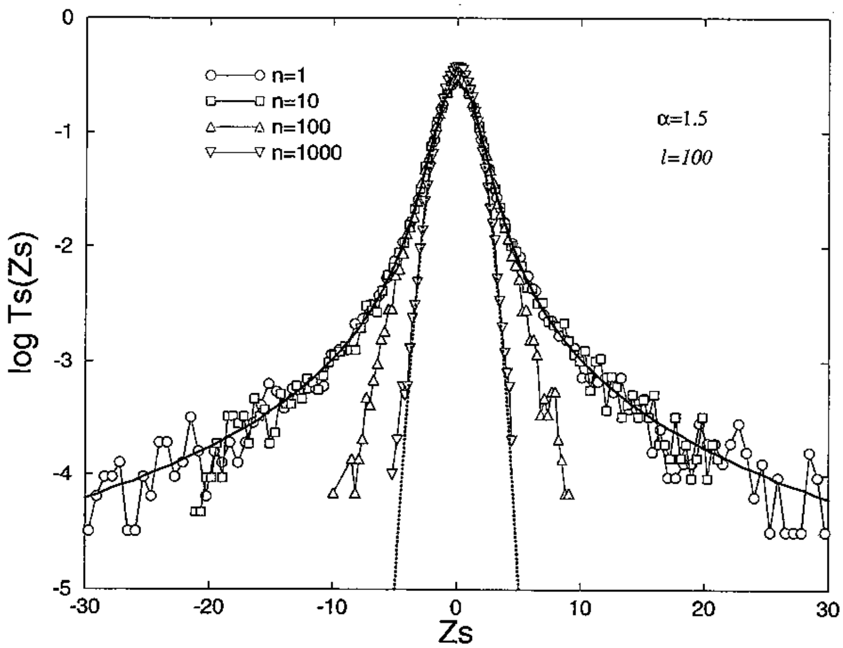

Fig. 4. – Semilogarithmic scaled plot of the probability distributions of the TLF process characterized by $\alpha = 1.5$ and $l = 100$ measured for $n = 1, 10, 100$ and 1000. For low values of n ($n = 1, 10$) the central part of the distributions is well described by the Lévy stable symmetrical profile associated with $\alpha = 1.5$ and $\gamma = 1$ (solid line). For high values of n ($n = 1000$) the TLF process has already reached the Gaussian regime and the distribution is essentially Gaussian (dotted line).

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selected control parameters ($\alpha = 1.5$ and $l = 100$) the crossover $n_\times$ is observed for $n_\times \approx 260$ (where the units are minutes). For the same control parameters we also investigate the distribution $P(z_n)$ at different values of n. In fig. 4 we show the distributions obtained by simulating a TLF for $n = 1, 10, 100$, and 1000. The distributions are drawn in scaled units

$$T(z_S) \equiv \frac{P(z)}{n^{-1/\alpha}}\,, \tag{12}$$

and

$$z_S \equiv \frac{z}{n^{1/\alpha}}\,. \tag{13}$$

We use scaled units to be able to compare the shape of the distributions at very different values of n. From fig. 4 it is clear that the TLF distribution is changing shape as a function of n. For low values of n ($n = 1, 10$), we observe a good agreement with a Lévy profile (solid curve in fig. 4), while for high values of n ($n = 1000$) the distribution is well approximated by the asymptotic Gaussian profile (dotted curve in fig. 4). By comparing the results of figs. 3 and 4 we note that the probability of return to the origin indicates with high accuracy the degree of convergence of the process to one of the two asymptotic regimes. For example, when $n = 1$ and 10 the probability of return is clearly in the Lévy regime (fig. 3) and the central part of the TLF distribution is well described by a Lévy distribution (fig. 4). Conversely, for $n = 1000$ the probability of return to the origin is in the Gaussian regime (fig. 3), and the distribution is almost coincident with the Gaussian distribution characterized by the appropriate standard deviation (fig. 4).

To summarize, our study shows that by investigating the probability of return to the origin of an originally quasi-stable non-normal stochastic process with finite variance a clear crossover between Lévy and Gaussian regimes is observable. Hence a Lévy-like probability distribution can be experimentally observed for a long (but finite) interval of time (or number of variables) even in the presence of stochastic processes characterized by a *finite* variance.

4. – Failures of the description of some aspects of the S&P 500 dynamics in terms of the TLF model

The TLF is then compatible with the experimental observation of non-Gaussian profile of the central part of price change distribution and with the finiteness of the variance of this distribution. Can the TLF also describe entirely the intermittent behavior observed in fig. 1b? To answer to this question we need to study the time dependence of the parameters α and γ characterizing the price change distribution. We now discuss this point.

To investigate if a time dependence is observed in the parameters α and γ, we analyze monthly subintervals of our 6-year data set. Specifically we measure for each of the 72 months the probability of return to the origin within the time interval ($\Delta t =$ 1, …, 32 minutes). Then we determine the logarithm of the probability of return as a function of the logarithm of Δt and we fit with a straight line the experimental points. From the slope of the fitting we determine the index α of the distribution while from the intercept of the straight line at $\Delta t = 1$ we determine the scale factor γ.

The results of our investigations are shown in figs. 5 and 6. Figure 5 shows the time dependence of α. The index α is roughly constant ($\alpha = 1.38 \pm 0.14$) and fluctuating around its average value. The time dependence of γ, shown in fig. 6, is much more pronounced so that this behavior cannot be taken into account by the simple TLF model; moreover, bursts of activity localized in specific months (such as April 87 and October 87 are) are observed.

The pronounced fluctuations of the parameter γ reflects the known observation that «volatility» is time-dependent in financial market [30]. The time dependence of γ (under the assumption that the distribution of $P(\gamma)$ is not power-law) should not affect the stable description of the process because the sum of independent stochastic

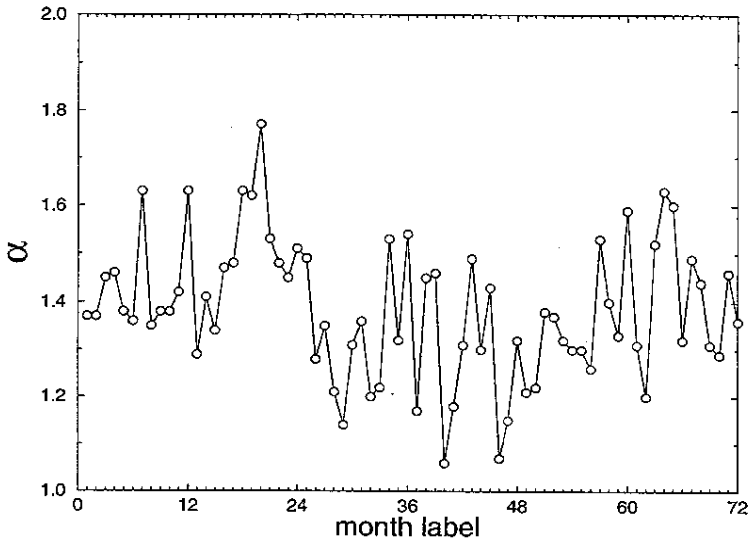

Fig. 5. – Time dependence of the index α of the S&P 500 changes distribution $P(z)$ measured each month of the investigated period. The results are obtained starting from the measured probability of return to the origin of the stochastic process (see text). The index α is roughly constant $\alpha = 1.38 \pm 0.14$ over the investigated period.

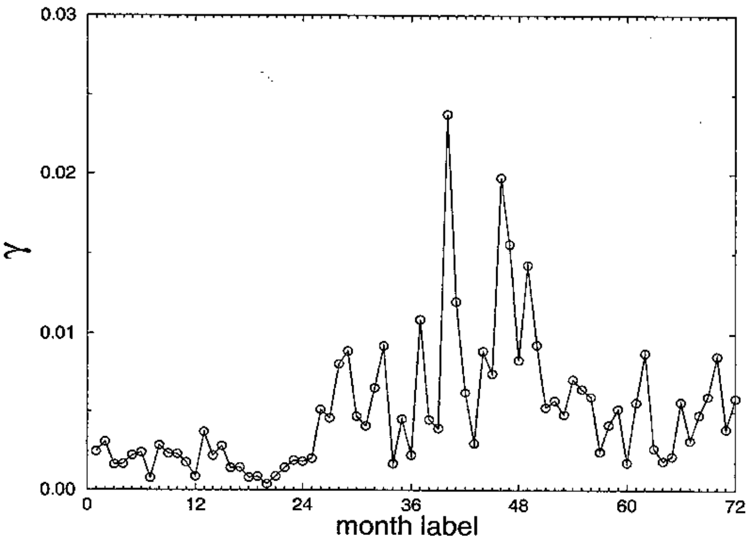

Fig. 6. – Time dependence of the scale factor γ of the S&P 500 changes distribution $P(z)$ measured each month of the investigated period. The results are obtained starting from the measured probability of return to the origin of the stochastic process (see text). The parameter γ (related to the «volatility» of the market) is time dependent. Bursts of significantly higher γ values are observed in specific months as April 87, October 87 and subsequent months.

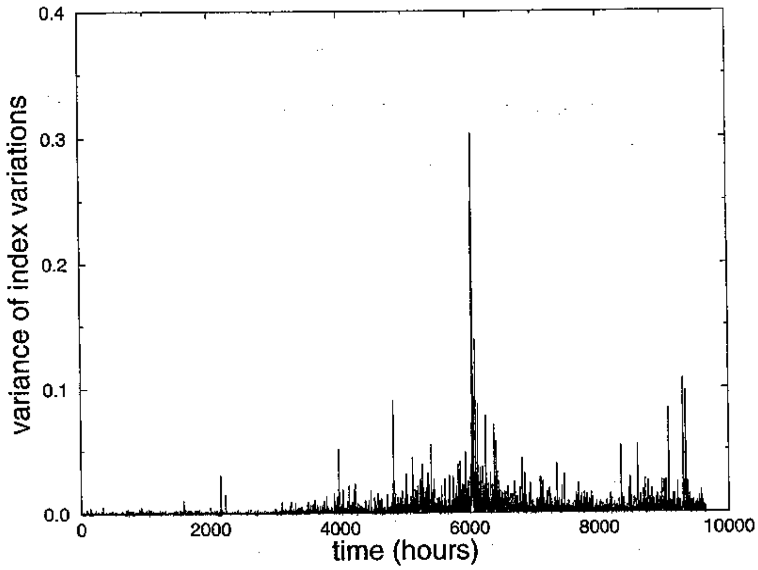

Fig. 7. – One min variance of the variations $z(t)$ of the S & P 500 index measured in a one-hour time interval. A strong intermittent behavior, analogous to the energy dissipation rate of fully-developed turbulence (see fig. 1 of [37]) is observed. The large one-day drop on 19 October 1987 is evident ($t \approx 6000$ h).

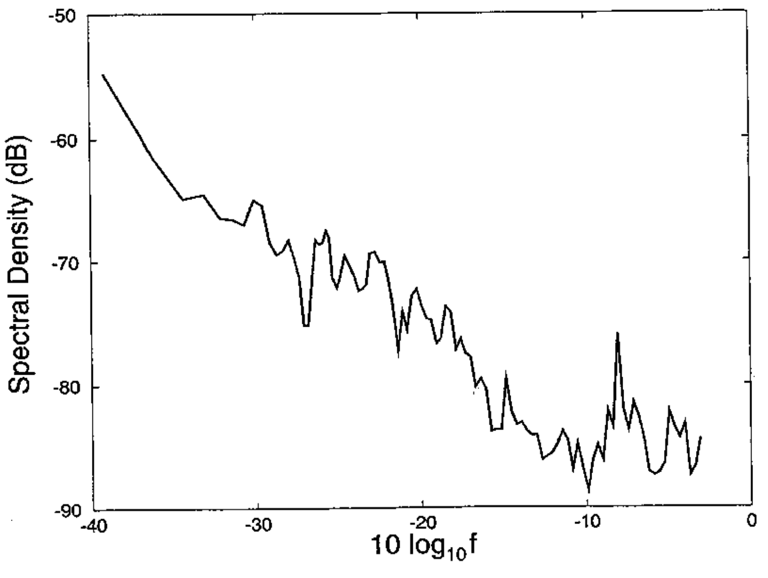

Fig. 8. – Spectral density of time evolution of the variance of the variations $z(t)$ of the S & P 500 index measured in a one hour time interval. At low frequencies an $1/f$-like behavior is observed. The peak observed at high frequency represents an intraday modulation of the variance.

variables of the same index α but different scale factor γ is still a stable variable characterized by same index α.

Another question concerns the time evolution of the variance of index changes. In fig. 7 we show the variance of the 1 minute index changes of the S&P 500 measured each market hour from January 1984 to December 1989. The profile of the index change variance (related to what is called «volatility» in the economics literature) is strongly intermittent. Moreover a Fourier analysis of this time behavior shows a spectral density which may be roughly approximated by a $(1/f)$-like behavior at lower frequency (fig. 8). The presence of the peak observed at high frequency in fig. 8 corresponds to the known intraday variation of volatility observed in financial market [31].

The time dependence of γ and of the variance of the index changes are not explainable in terms of the TLF model of sect. 2. A generalization of this model is then needed to also catch these important features observed in the dynamics of the S&P 500 index.

Of course, other models might also be considered to fully describe the stock market data. For example, by using a rather different approach, an alternative possible physical phenomenon that it is worthwhile to investigate is turbulence [32]. The goal is to see if turbulence might be used as a paradigm to describe some of the phenomena empirically observed in the analysis of data of the S&P 500 dynamics. The rationale for this choice is that it is known that intermittency of the dissipation rate and non-Gaussian profile of the probability density function of velocity changes are observed in the time evolution of a fully turbulent fluid moving in a 3-dimensional space. This research has been performed independently by different groups [33-35].

5. – Analogies and differences with turbulence

To investigate analogies and differences between the quantitative measures of fluctuations in an economic index and the fluctuations in velocity of a fluid in a fully turbulent state we have systematically compared [34] the statistical properties of the S&P 500 cash index with the statistical properties of the velocity of turbulent air.

The turbulence data were kindly provided by Prof. K. R. Sreenivasan. Measurements were made [36] in the atmospheric surface layer about 6 m above a wheat canopy in the Connecticut Agricultural research station. The Taylor microscale Reynolds number R_λ was of the order of 1500. Velocity fluctuations were measured using the standard hot-wire velocimeter operated in the constant temperature mode on a DISA 55M01 anemometer. The file consists of 130000 velocity records $v(t)$ digitized and linearized before processing. The associated velocity differences is defined as $U_{\Delta t}(t) \equiv v(t) - v(t - \Delta t)$.

The variance of index variations (fig. 7) shows a time dependence approximately similar to the time evolution of a representative component of the rate of dissipation of the kinetic energy $\varepsilon(t)$ namely $\varepsilon' \approx (\mathrm{d}v/\mathrm{d}t)^2$ (See for example fig. 1 of refence [37].) Considerable experimental evidence that the rate of dissipation of fully developed turbulence is multifractal has been obtained [37,38], but a more detailed study of stock exchange data is needed before drawing conclusions concerning the usefulness of a multifractal model in the time evolution of the index variance.

Quantitative parallel analysis have been performed [34] by measuring the time dependence of the standard deviations $\sigma_Z(\Delta t)$ and $\sigma_U(\Delta t)$ of $P(Z)$ and $P(U)$, we find that: i) In the case of the S&P 500 index variations the time dependence of the standard deviation, when $\Delta t \geqslant 15$ minutes fits well the behavior

$$\sigma_Z(\Delta t) \propto (\Delta t)^{0.53}. \tag{14}$$

The exponent is close to the typical value of 0.5 observed in random processes with independent increments. ii) The velocity difference of the fully turbulent fluid shows a time dependence of the standard deviation, fitting the behavior

$$\sigma_U(\Delta t) \propto (\Delta t)^{0.33}, \tag{15}$$

which is observed in short-time anticorrelated random processes.

Equivalent conclusions are reached if we measure the spectral density of the time series $y(t)$ and $v(t)$. Economic data have the spectral density typical of a Brownian motion, $S(f) \propto f^{-2}$ [19], while for turbulence data the spectral density shows a wide inertial range $S(f) \propto f^{-5/3}$ (see, *e.g.*, [32]).

Another difference between the two processes is observed by investigating the probability of return to the origin $P(U=0)$ as functions of the time interval Δt between successive observations. The deviation from a Gaussian process is measured by comparing $P(U=0)$ with the value of $P_g(0)$. $P_g(0)$ is determined starting from the measured values of $\sigma(\Delta t)$ by using the equation

$$P_g(0) = \frac{1}{\sqrt{2\pi}\sigma(\Delta t)} \tag{16}$$

valid for a Gaussian process.

We observe [34] a clear difference between $P(0)$ and $P_g(0)$. The difference observed shows that both PDFs have a non-Gaussian distribution, but the detailed shape and the scaling properties of the two PDFs are different.

In a previous section we reported that a scaling compatible with a Lévy stable process is observed for economic data and indeed a Lévy distribution reproduces quite well the central part of the distribution of the S&P 500 index variations. A similar scaling *does not exist* for turbulence data over a wide time interval [34]. Moreover, by using the measured values of $P(0)$ and σ_u and hypothesizing a stretched exponential PDF [36], we can describe quite well the experimental PDF of the velocity difference with a stretched exponential distribution

$$P(U) = \frac{m}{2l^{1/m}\Gamma(1/m)} \exp\left[-\frac{|U|^m}{l}\right] \tag{17}$$

characterized by a (time dependent) stretching exponent m and a scale factor l.

The parallel analysis of the statistical properties of an economic index and the velocity of a turbulent fluid in a 3-dimensional space shows that the two processes are quantitatively different. The absence of an inertial range associated with the economic time series and the differences observed in the scaling properties of the «probability of return» to the origin clearly rule out the possibility that a Navier-Stokes type of equation might describe the dynamics of the index in a 3-dimensional space. However, for d-dimensional turbulence with non-integral d [39], it is possible to select a non-integral dimension (≈ 2.05) at which the spectral density of the turbulent fluid shows the same behavior observed in uncorrelated stochastic processes. Thus our results cannot rule out the possibility that stock indices are controlled by a Navier-Stokes type of equation in an abstract space of non-integral dimension.

6. – Discussion

In this lecture, we have tried to give a concrete example on how and why physicists may consider economics systems (and in particular financial systems) as very interesting *«complex systems»*. We believe we have shown as the investigation of such systems is not so exotic as it may appear at first sight.

It is true that, at the moment, it may seem to be an unusual challenge for a physicist to investigate *economic* systems by using tools and paradigms developed to describe *physical* phenomena. There are certainly rational reasons for this view. For example, physical and economic systems are, of course, rather different and economic systems are usually quite "complex." Moreover, the lack of conservation principles for economic systems implies that economic systems are often in a non-equilibrium regime.

On the other hand, there are opposite arguments that favor the involvement of physicists in the study of economic systems. The theoretical, numerical and experimental tools allowing to investigate non-equilibrium disordered systems (included non-ergodic systems) are steady increasing in this period. The interaction between the two disciplines might then be crucial for the development of new theoretical models and paradigms.

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