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Stock market dynamics and turbulence: parallel analysis of fluctuation phenomena

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Abstract

We report analogies and differences between the fluctuations in an economic index and the fluctuations in velocity of a fluid in a fully turbulent state. Specifically, we systematically compare (i) the statistical properties of the S&P 500 cash index recorded during the period January 84–December 89 with (ii) the statistical properties of the velocity of turbulent air measured in the atmospheric surface layer about 6 m above a wheat canopy in the Connecticut Agricultural Research Station. We find non-Gaussian statistics, and intermittency, for both processes (i) and (ii) but the deviation from a Gaussian probability density function are different for stock market dynamics and turbulence.

Stock exchange time series have been modelled as stochastic processes since the seminal study of Bachelier published at the beginning of this century [1]. Several stochastic models have been proposed and tested in the economics [2–9] and physics [10–14] literature. Alternative approaches based on the paradigm of chaotic dynamics have been also proposed [15–17]. Several statistical techniques (e.g. the measure of the probability density function, the measure of the spectral density, etc.) commonly used in the study of the stochastic processes have been used for a long time in turbulence [18,19]. Moreover, in recent years there has been much effort to select stochastic process with statistical properties that are close to the one observed in turbulence [20]. Here we report analogies and differences between the quantitative measures of fluctuations in an economic index (i) and the fluctuations in velocity of a fluid in a fully turbulent state (ii). We observe non-Gaussian statistics, and intermittency, for both processes (i) and (ii) but the time evolution of the second moment and the shape of the probability density functions are different for stock market dynamics and turbulence.

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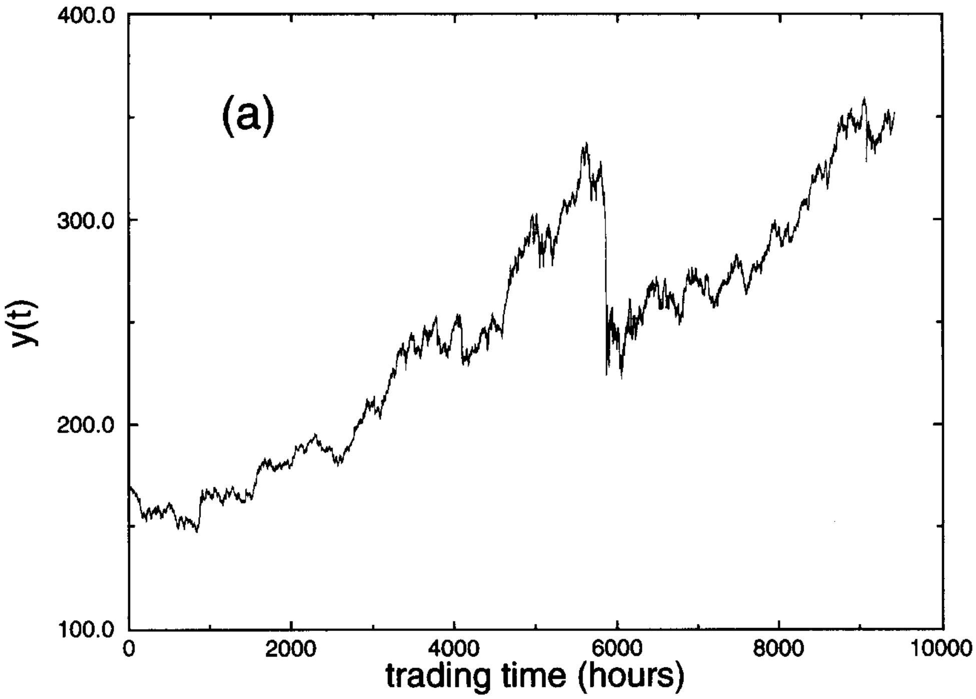

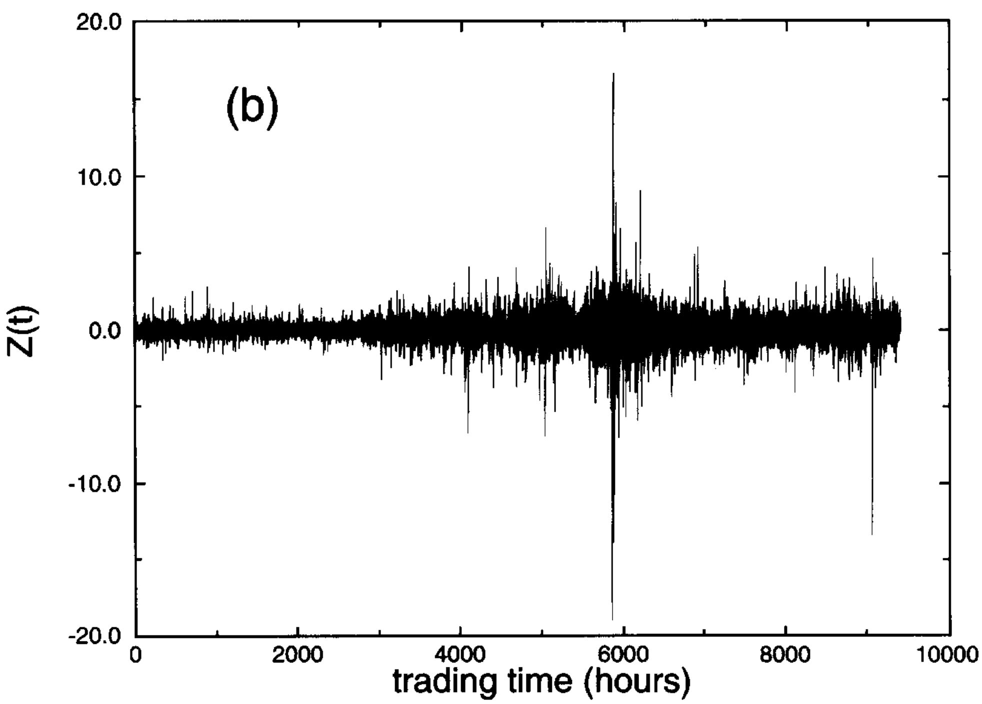

Fig. 1. (a) Time evolution of the S&P 500, sampled with a time resolution $\Delta t = 1$ h, over the period January 1984–December 1989. (b) Hourly variations of the S&P 500 index in the 6-year period January 1984–December 1989. (c) Time evolution of the wind velocity recorded in the atmosphere at very high Reynolds number; the Taylor microscale Reynolds number is of the order of 1500. The time units are given in arbitrary units. (d) Velocity differences of the time series given in (c).

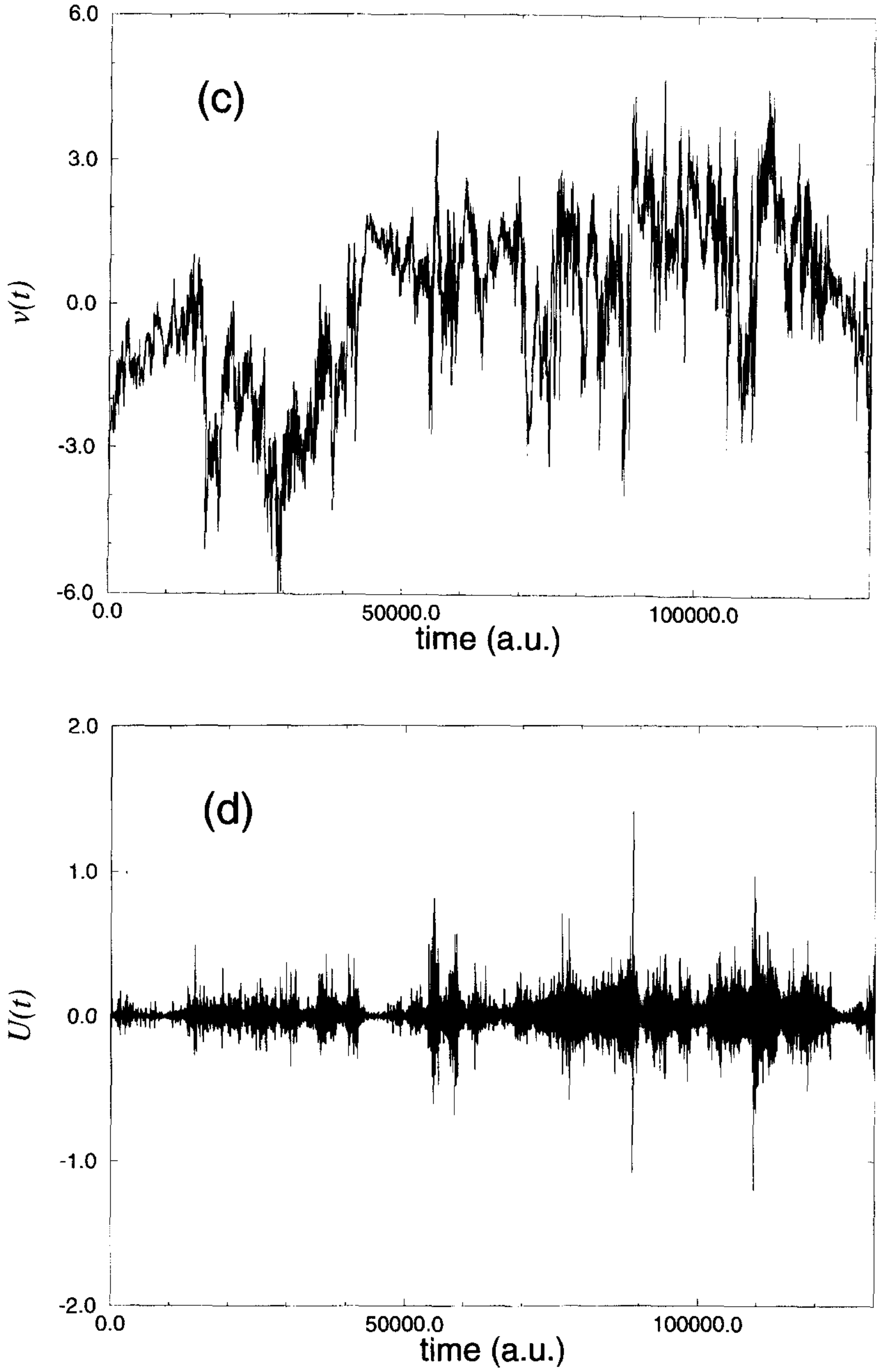

Fig. 1. contd.

The economic data set consists of all 1 447 514 records of the S&P 500 cash index recorded during the period January 84–December 89. In our analysis we define a "trading time" starting from the opening of the day until the closing, and then continuing with the opening of the next trading day. The time intervals between successive records

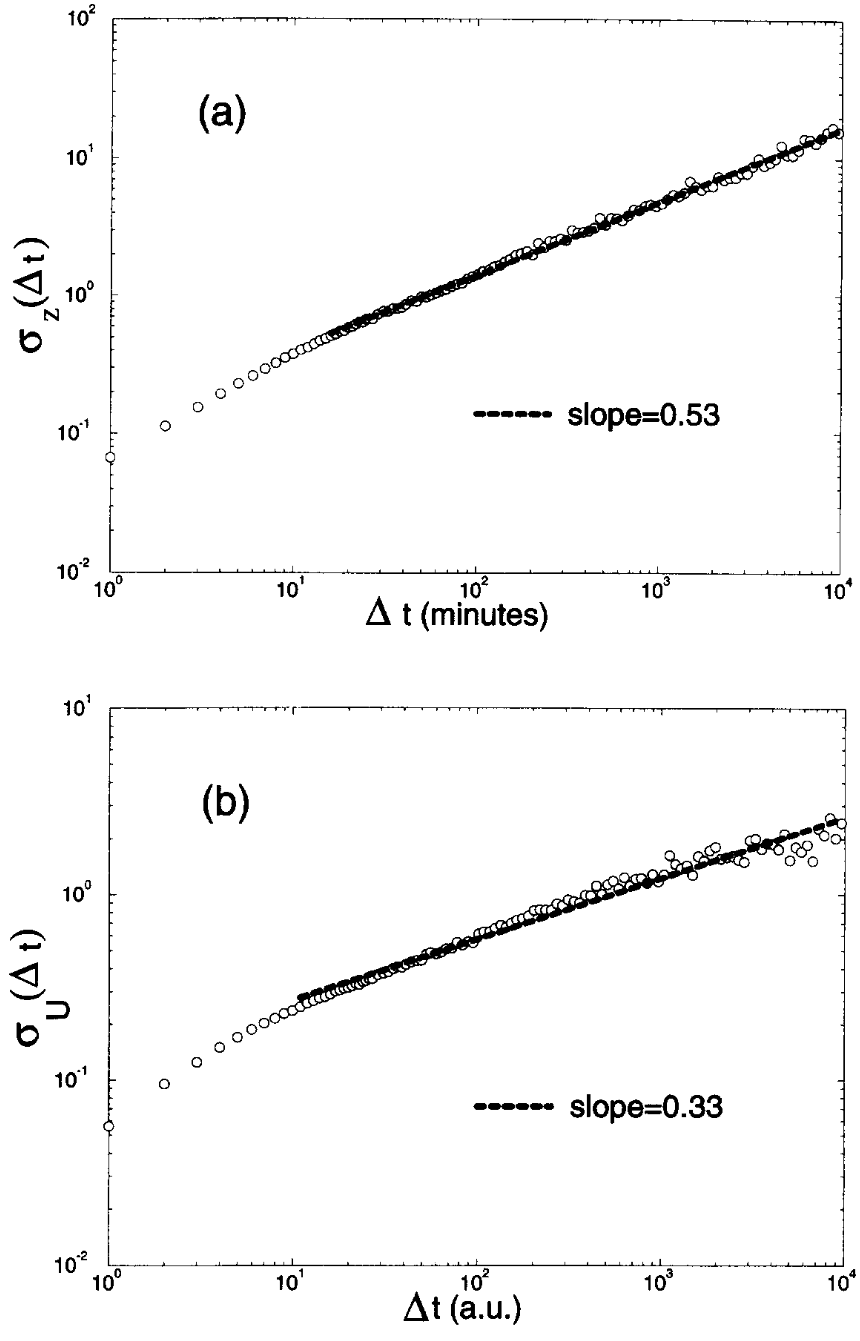

Fig. 2. (a) Standard deviation $\sigma_Z(\Delta t)$ of the probability distribution $P(Z)$ characterizing the increments $Z_{\Delta t}(t)$ plotted double logarithmically as a function of Δt for the S&P 500 time series. After a time interval of superdiffusive behavior ($0 < \Delta t \leqslant 15$ min) a diffusive behavior close to the one expected for a random process with independent identically-distributed increments is observed; the measured diffusion exponent 0.53 is close to the theoretical value $\frac{1}{2}$. (b) Standard deviation $\sigma_U(\Delta t)$ of the probability distribution $P(U)$ characterizing the velocity increments $U_{\Delta t}(t)$ plotted double logarithmically as a function of Δt for the velocity difference time series in turbulence. After a time interval of superdiffusive behavior ($0 < \Delta t \leqslant 10$), a diffusive behavior close to the one expected for a fluid in the inertial range is observed (the measured diffusion exponent 0.33 is close to the theoretical value $\frac{1}{3}$). (c) Spectral density of the S&P 500 time series. The $1/f^2$ power-law behavior expected for a random process with increments independent and identically distributed is observed over a frequency interval of more than 4 orders of magnitude. (d) Spectral density of the velocity time series. The $1/f^{5/3}$ inertial range (low frequency) and the dissipative range (high frequency) are clearly observed.

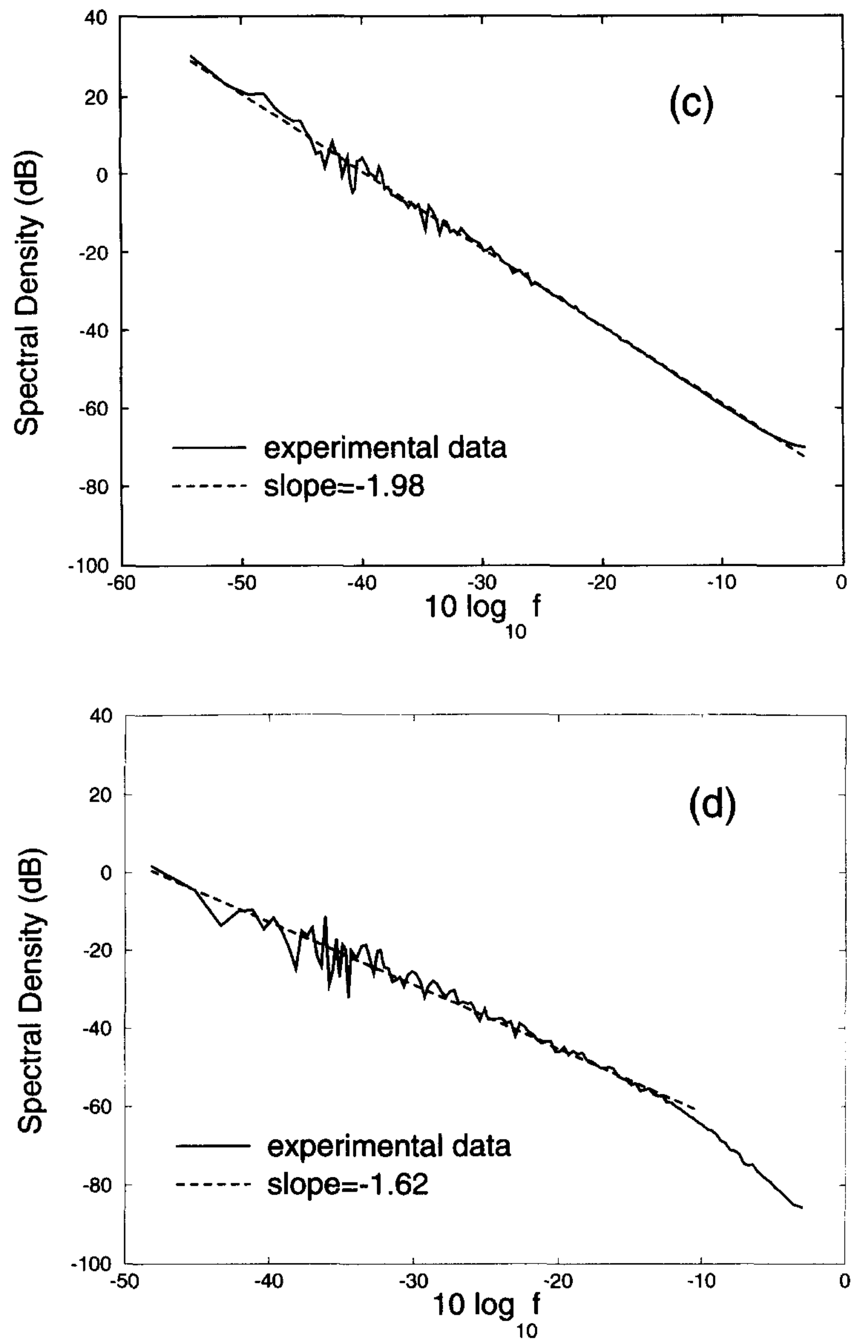

Fig. 2. contd.

are not fixed: the average value between successive records is close to 1 min during 1984 and 1985 and close to 15 s during 1986–1989. From this database, we select the complete set of non-overlapping records separated by a time interval $\Delta t \pm \varepsilon \Delta t$ (where ε is the tolerance, always less than 0.035). We denote the value of the S&P 500 as $y(t)$ (Fig. 1(a)), and the successive variations of the S&P index is $Z_{\Delta t}(t) \equiv y(t) - y(t - \Delta t)$ (Fig. 1(b)).

The turbulence data were kindly provided by Prof. K.R. Sreenivasan. Measurements were made [21] in the atmospheric surface layer about 6 m above a wheat canopy in the Connecticut Agricultural research station. The Taylor microscale Reynolds number R_λ was of the order of 1500. Velocity fluctuations were measured using the standard hot-wire velocimeter operated in the constant temperature mode on a DISA 55M01 anemometer. The file consists of 130 000 velocity records $v(t)$ digitized and linearized before processing (Fig. 1(c)). The associated velocity differences $U_{\Delta t}(t) \equiv v(t) - v(t - \Delta t)$ is shown in Fig. 1(d).

We focus attention on the dynamics of the index variation $Z_{\Delta t}(t)$ and on the dynamics of the velocity difference $U_{\Delta t}(t)$, and denote by $P(Z)$ and $P(U)$ the associated probability density functions (PDFs). We perform several tests to detect the properties of the investigated stochastic process.

By measuring the time dependence of the standard deviations $\sigma_Z(\Delta t)$ and $\sigma_U(\Delta t)$ of $P(Z)$ and $P(U)$, we find that:

(i) In the case of the S&P 500 index variations (Fig. 2(a)) the time dependence of the standard deviation, when $\Delta t \geqslant 15$ min fits well the behavior

$$\sigma_Z(\Delta t) \propto (\Delta t)^{0.53} . \tag{1}$$

The exponent is close to the typical value of 0.5 observed in random processes with independent increments.

(ii) The velocity difference of the fully turbulent fluid shows a time dependence of the standard deviation, fitting the behavior (Fig. 2(b))

$$\sigma_U(\Delta t) \propto (\Delta t)^{0.33} \tag{2}$$

which is observed in short-time anticorrelated random processes.

Similar conclusions are reached if we measure the spectral density of the time series $y(t)$ and $v(t)$. Economic data (Fig. 2(c)) have the spectral density typical of a Brownian motion, $S(f) \propto f^{-2}$ [22]. For turbulence data (Fig. 2(d)) the spectral density shows a wide inertial range (see, e.g., [18]).

Next we present a different kind of analysis of the PDFs $P(Z)$ and $P(U)$ which turns out to be quite powerful for the description of experimental results. Specifically, we study the dependence on Δt of the point of each PDF that is *least* affected by the noise introduced by the finiteness of the data set [23] – $P(0)$, the "probability of return" to the origin. In Figs. 3(a) and (b) we show double logarithmic plots of $P(Z=0)$ and $P(U=0)$ as functions of the time interval Δt between successive observations. The deviation from a Gaussian process is shown by plotting on the same figure the value of $P_g(0)$ determined starting from the measured values of $\sigma(\Delta t)$ under the hypothesis of Gaussian processes – using the equation

$$P_g(0) = \frac{1}{\sqrt{2\pi}\sigma(\Delta t)} . \tag{3}$$

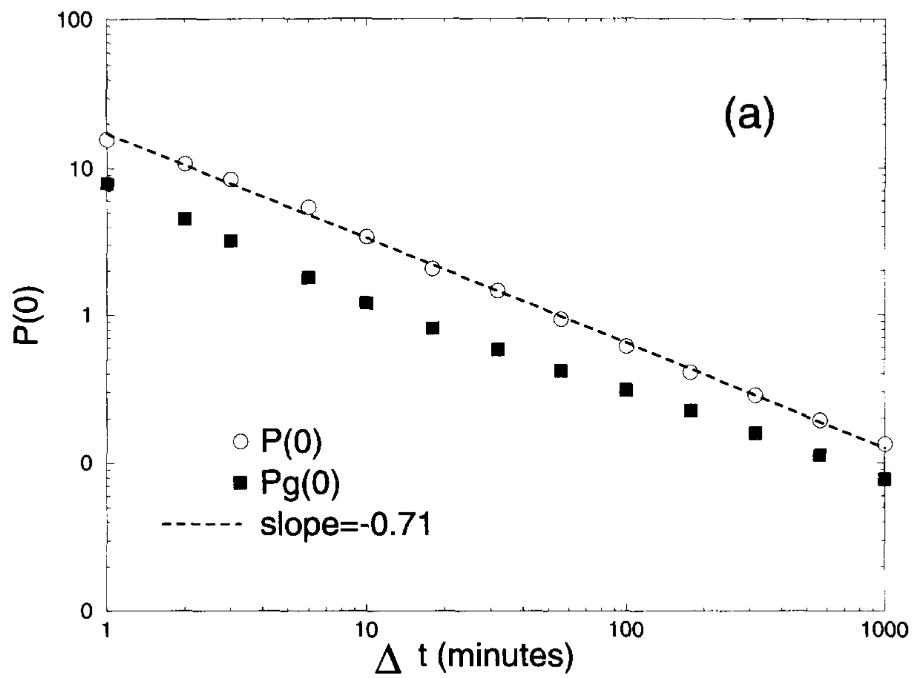

Fig. 3. (a) S&P 500 index. Probability of return to the origin $P(Z=0)$ (○) and $P_g(Z=0)$ (filled squares) (see Eq. (3)) as functions of the time sampling interval Δt. The two measured quantities differ in the full interval implying that the profile of the PDF must be non-Gaussian. A power-law behavior is observed for the entire time interval spanning three orders of magnitude. The slope of the best linear fit is -0.71 ± 0.025. (b) Velocity of the fully turbulent fluid. Probability of return to the origin $P(0)$ (○) and $P_g(0)$ (filled squares) (see Eq. (3)) as functions of the time sampling interval Δt. Again, the two measured quantities differ in the full interval, implying that the profile of the PDF must be non-Gaussian. However in this case, a single scaling power-law behavior does not exist for the entire time interval spanning three orders of magnitude. The slope of the best linear fit (which is of quite poor quality) is -0.59 ± 0.11. (c) Experimental PDF $P(Z)$ of the S&P 500 index variations $Z_{\Delta t}(t)$ observed at time intervals $\Delta t = 1$ min (circles). In the figure we also plot as a solid line the symmetrical Lévy stable distribution of index $\alpha = 1.40$ and scale factor $\gamma = 0.00375$ (solid line). The parameters characterizing the stable distribution are obtained from the analysis of the scaling properties of the experimental data on the probability of return to the origin $P(Z=0)$ [13]. (d) Experimental PDF $P(U)$ of the velocity difference $U_{\Delta t}(t)$ of a fluid in fully developed turbulence observed at the highest temporal resolution available $\Delta t = 1$ (circles). In the figure we also plot as a solid line the symmetrical stretched exponential distribution of index $m = 0.61$ and scale factor $l = 0.0654$ (solid line). The characterizing parameters of the stretched exponential distribution are obtained starting from the experimental value of the probability of return to the origin $P(U=0)$.

The clear difference between $P(0)$ and $P_g(0)$ shows that both PDFs have a non-Gaussian distribution, but the detailed shape and the scaling properties of the two PDFs are different.

A scaling compatible with a Lévy stable process [24–26] is observed for economic data (Fig. 3(a)) and indeed a Lévy distribution reproduces quite well the central part of the distribution of the S&P 500 index variations (Fig. 3(c)). The Lévy stable modeling with an index $\alpha = 1.40$, obtained from the best fit of the probability of return to the origin data, describes the data well over a three order of magnitude time interval (ranging from 1 to 1000 minutes). The tails deviate from the Lévy profile when $Z \geqslant 0.3$, ensuring a finite variance to the stochastic process [14].

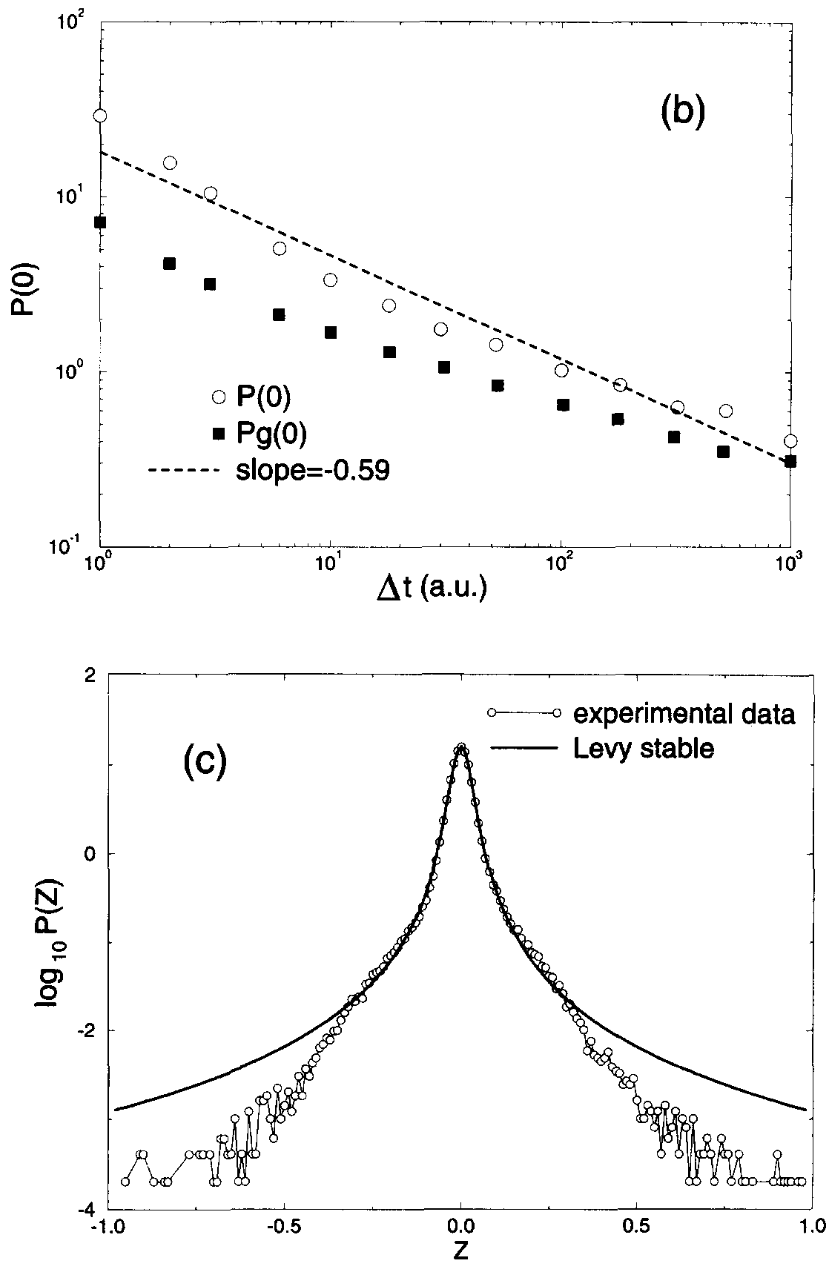

Fig. 3. contd.

A similar scaling *does not exist* for turbulence data over a wide time interval (Fig. 3(b)). By using the measured values of $P(0)$ and σ_U and hypothesizing a stretched exponential PDF[21], it is possible to describe quite well the experimental PDF of the velocity difference with a stretched exponential distribution

$$P(U) = \frac{m}{2l^{1/m}\Gamma(1/m)} \exp\left(-\frac{|U|^m}{l}\right) \tag{4}$$

characterized by a (time-dependent) stretching exponent m and a scale factor l. In Fig. 3(d) we show the experimental probability density function measured for $\Delta t = 1$,

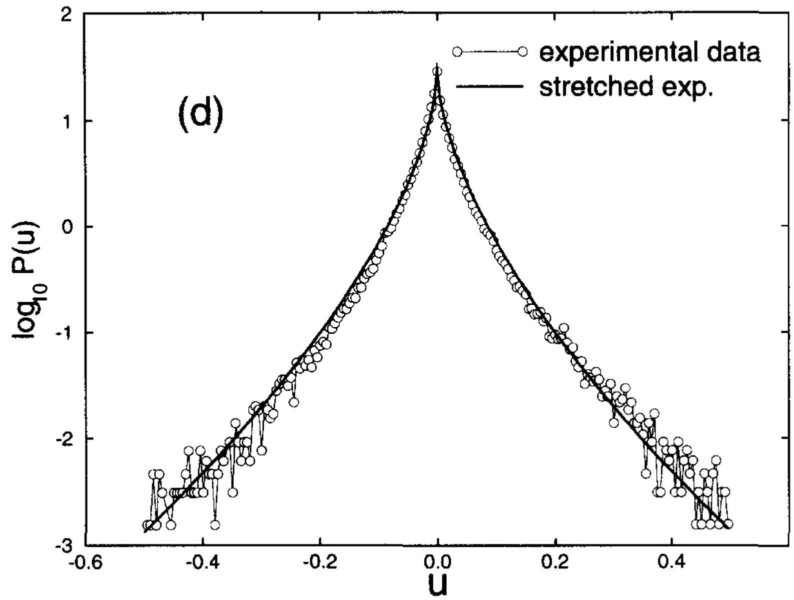

Fig. 3. contd.

together with a stretched exponential distribution characterized by the parameter $m = 0.61$.

The variance of index variations (Fig. 4(a)) and a representative component of the rate of dissipation of the kinetic energy $\varepsilon(t)$, namely $\varepsilon' \approx (dv/dt)^2$ (Fig. 4(b)) show an intermittent profile [27]. Considerable experimental evidence that the rate of dissipation of fully developed turbulence is multifractal has been obtained [28,29], but a more detailed study of stock exchange data is needed before drawing conclusions concerning the usefulness of a multifractal model in the time evolution of the index variance.

The parallel analysis of the statistical properties of an economic index and the velocity of a turbulent fluid in a three-dimensional space shows that the two processes are quantitatively different. The absence of an inertial range associated with the economic time series and the differences observed in the scaling properties of the "probability of return" to the origin clearly rule out the possibility that a Navier–Stokes type of equation might describe the dynamics of the index in a three-dimensional space. However, for d-dimensional turbulence with non-integral d [30], it is possible to select a non-integral dimension (≈ 2.05) at which the spectral density of the turbulent fluid shows the same behavior observed in uncorrelated stochastic processes. Thus, our results cannot rule out the possibility that stock indices are controlled by a Navier–Stokes type of equation in an abstract space of non-integral dimension.

The parallel analysis of the statistical properties on an important stock exchange index and the velocity of the air in a fully turbulent state shows that an interaction between economics and statistical physics may be useful – i.e., it may be fruitful to pursue analogies and differences between various stochastic models developed in economics [3,6–8] and the various approaches used in turbulence theory [18,19,28].

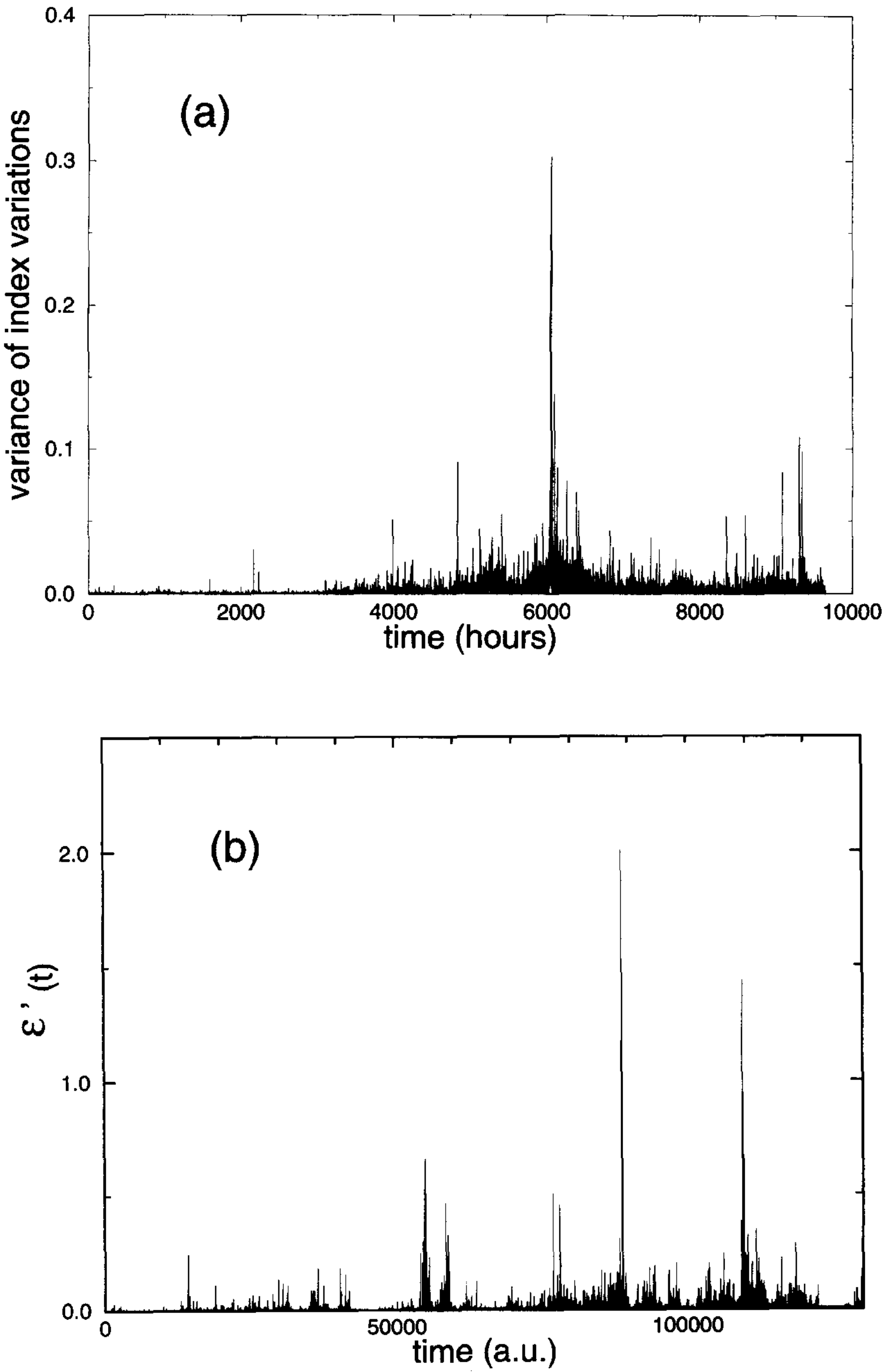

Fig. 4. (a) 1 min variance of the PDF $P(Z)$ for the variations $Z_{\Delta t}(t)$ of the S&P 500 index measured in a 1 h time interval. A strong intermittent behavior is observed on a short time scale. (b) $\varepsilon'(t)$, representative component of the rate of dissipation of the kinetic energy $\varepsilon(t)$ plotted at the highest time resolution available, and showing the typical intermittent profile of turbulent fluids.

The exchange of concepts, models and techniques of data analysis offers the opportunity to characterize qualitatively and quantitatively analogies and differences between these two stochastic processes.

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Note added in proof

Part of this work was also presented as a poster at the Sixteenth workshop on Dynamics Days held in Lyon on June 28th–July 1st 1995. Analogies and differences with turbulence have also been investigated by other groups, with a variety of different conclusions [31,32]. A discussion of some aspects of this problem is presented in [33].

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