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Scale-dependent price fluctuations for the Indian stock market

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Abstract. – Classic studies of the probability density of price fluctuations g for stocks and foreign exchanges of several highly developed economies have been interpreted using a *power law* probability density function $P(g) \sim g^{-(\alpha+1)}$ with exponent values $\alpha > 2$. To test the ubiquity of this relationship we analyze daily returns for the period November 1994–June 2002 for the 49 largest stocks of the National Stock Exchange which has the highest trade volume in India. We find the surprising result that $P(g)$ decays as an *exponential* function $P(g) \sim \exp[-\beta g]$ with a characteristic decay scale $\beta = 1.51 \pm 0.05$ for the negative tail and $\beta = 1.34 \pm 0.04$ for the positive tail. The exponential function is significantly different from the power law function observed for highly developed economies. Thus, we conclude that the stock market of the less highly developed economy of India belongs to a different class from that of highly developed countries.

Introduction. – The market index is driven by numerous players and demand-supply factors through a composite average of various stocks. These factors constitute the complex market mechanism that causes the price variation in a component stock, which in turn pulls down or pushes up the market index. Tracking many variables is tricky, making the quantification of economic fluctuations challenging.

A careful analysis of the market forces is required to provide accurate trends and indicators, which form a tool for market forecast and hence also provide solutions and key inputs for the improvement of economic policies and legislation. In this paper we investigate stock market asset price variations in a typical developing country such as India and compare the trends with those from economically developed economies.

A textbook study [1] of stock price variations suggests that stock prices —and concomitantly, stock price indices— follow a Markovian-Wiener process. This means that the stock

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price on any day is independent of the history of the stock price or its fluctuation. This results in a conventional log-normal density for stock prices [1], *i.e.*, the logarithm of the stock price follows a normal density.

However, developed markets such as those in the United States, Germany, and Japan exhibit a stock price behavior that differs from the Gaussian density frequently used in conventional theories. A key empirical finding in this regard is that the probability density of logarithmic price changes (returns) is approximately symmetric and decays with power law tails with identical exponent $\alpha \approx 3$ for both tails [2,3]. One intriguing aspect of this empirical finding is that it appears to be universal. Individual stocks appear to conform to these laws not just in US markets [3], but also in German [2] and Australian markets [4]. These same laws are obeyed by market indices such as the S&P 500, the Dow Jones, the NIKKEI, the Hang Seng, and the Milan index [5], and similar behavior is found in commodity markets [6] as well as in the most-traded currency exchange rates (*e.g.*, the US dollar *vs.* the Deutsch mark, or the US dollar *vs.* the Japanese yen [7]). The ubiquitous nature of these patterns exhibited in the statistics of daily returns is remarkable, since these markets differ greatly in their details. The observed ubiquity is consistent with a *scale-independent* behavior of the underlying dynamics.

Analysis. – We focus on the Indian stock market and find an exponential probability density function of price fluctuations, revealing an intrinsic scale. Our results are based on analyzing $\approx 10^5$ records representing daily returns for 49 largest stock of the National Stock Exchange (NSE) in India over the period November 1994–June 2002 (see table I).

TABLE I – *Indian stocks analyzed.*

Index	Symbol	Index	Symbol
1	Abb Ltd.	25	Icici Ltd.
2	Acc Ltd.	26	Indn. Hotels
3	Asian Paints.	27	Indn. Petroch
4	Bajaj Auto Ltd.	28	Infosys Tech.
5	Bhel Ltd.	29	Itc Ltd.
6	Britannia Inc.	30	Larsen Toubro
7	Bses Ltd.	31	Mah. Mah. Ltd.
8	Castrol India	32	Mtnl Ltd.
9	Cipla Ltd.	33	Nestle India.
10	Coch Refiner	34	Novartis India
11	Colgate Palmolive	35	Orient Bank
12	Dabur India	36	Procter Gamble
13	Digital Equipment	37	Ranbaxy Labs.
14	Dr. Reddys Lab.	38	Reckitt Colmann
15	Glaxo India Ltd.	39	Reliance Inds. Ltd.
16	Grasim Industries.	40	Rel. Petroleum
17	Gujrat Ambuja Cement	41	Satyam Computers
18	Hcl Info Systems.	42	Smithkl Heal.
19	Hdfc Bank Ltd.	43	St. Bk. India.
20	Hdfc Ltd.	44	Tata Chemicals
21	Hero Honda	45	Tata Power C.
22	Hindal Co. India	46	Tata Tea
23	Hindustan Lever Ltd.	47	Telco Ltd.
24	Hindustan Petrol	48	Tisco Ltd.
		49	Zee Tele Film

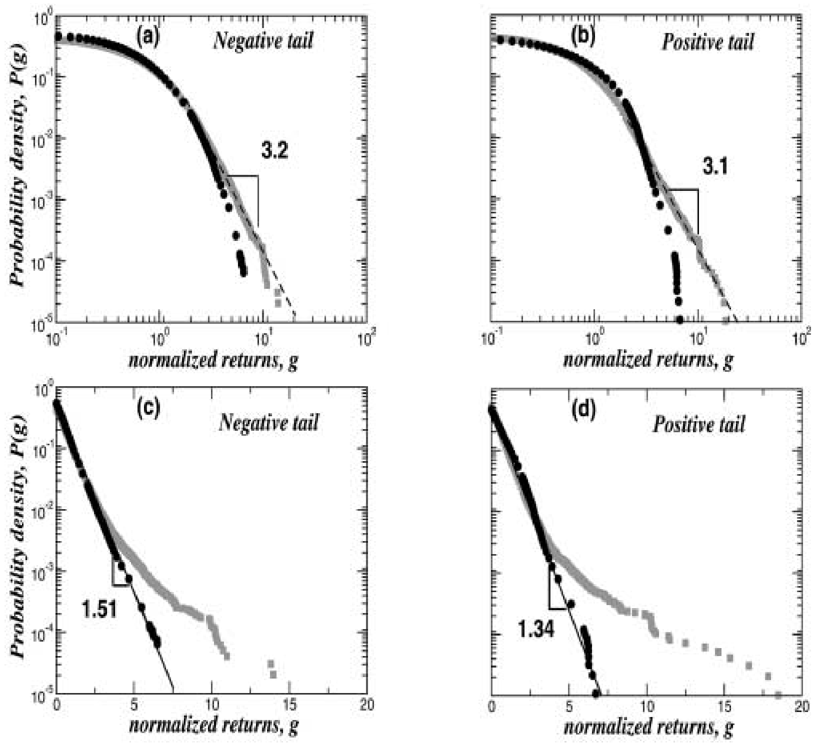

Fig. 1 – The probability density function of aggregated daily returns [9] on a log-log plot for (a) the negative tail and (b) the positive tail. Solid symbols are aggregated data from 49 Indian stocks and the open squares are aggregated data from 49 US stocks over the same period, November 1994–June 2002. The dashed lines are power law fits for $g > 2$ to the US data. The same data for aggregated daily returns on a linear-log plot for (c) the negative tail and (d) the positive tail. The solid lines have slopes $\beta = 1.51 \pm 0.05$ for the negative tail and $\beta = 1.34 \pm 0.04$ for the positive tail, where the decay parameters and the error bars are estimated by the least-square method. The least-square fitting, to the aggregated PDF or Indian stocks, with an exponential function is over the entire range of g.

We define the normalized price fluctuation (return)

$$g_i(t) \equiv \frac{\log S_i(t + \Delta t) - \log S_i(t)}{\sigma_i}\,. \tag{1}$$

Here $\Delta t = 1$ day, $i = 1, 2, \ldots, 49$ indexes the 49 stocks, $S_i(t)$ is the price of stock i at time t, and σ_i is the standard deviation of $\log S_i(t + \Delta t) - \log S_i(t)$.

To compare the probability density function of the Indian stocks with US stocks, we randomly choose 49 US stocks in the same period [8]. Next we aggregate the data [9]. Figures 1a and b display the probability density function $P(g)$ for both positive and negative tails for the daily returns in a log-log plot. The US stocks have a power law probability density function with exponent $\alpha \approx 3$ (cf. [2–5]).

Figures 1c and d display the probability density of the aggregated data for both Indian and US stocks in a linear-log plot. We observe that the probability density of the 49 Indian stocks has an exponential form of decay:

$$P(g) \sim e^{-\beta g} \tag{2}$$

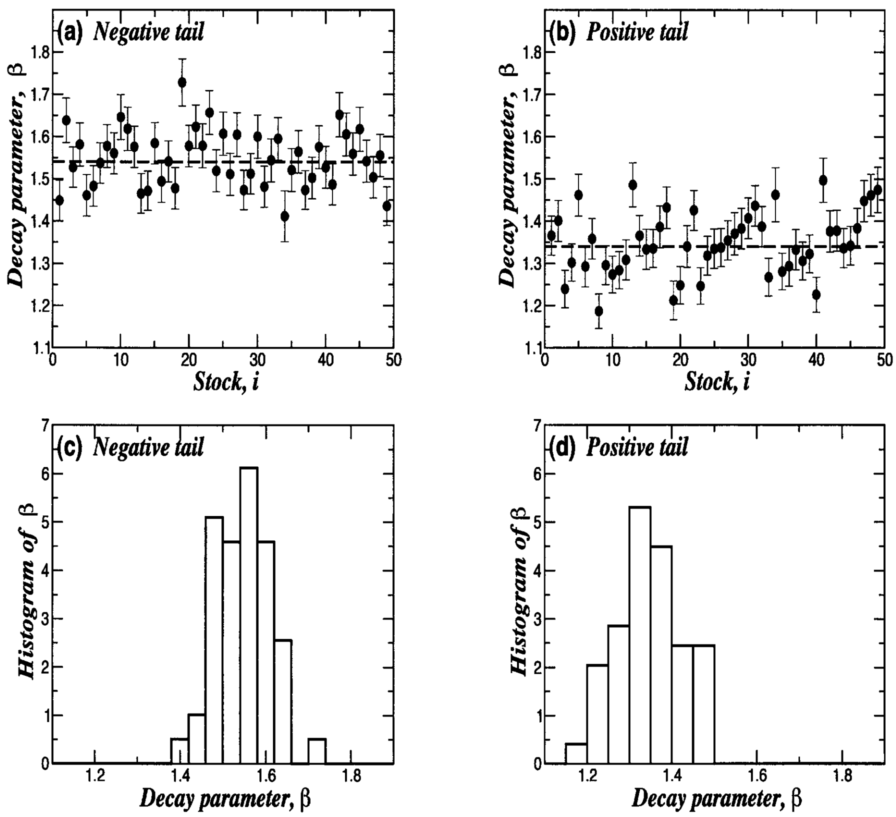

Fig. 2 – Decay parameters β_i of (a) the negative tail and (b) the positive tail, where $i = 1, 2, \ldots, 49$ indexes the 49 Indian stocks analyzed. We employ a least-square fit to estimate the parameters β_i of each stock. The dashed lines show the average values defined in eqs. (4)-(5). Histograms: (c) the negative-tail decay parameters $\beta_{\rm avg} = 1.54 \pm 0.05$, (d) the positive-tail decay parameters $\beta_{\rm avg} = 1.34 \pm 0.05$.

with

$$\beta = \begin{cases} 1.51 \pm 0.05 & \text{(negative tail)}, \\ 1.34 \pm 0.04 & \text{(positive tail)}. \end{cases} \tag{3}$$

Figure 2 displays the estimates of β_i for both positive and negative tails of the probability density function. We find the Kolmogorov-Smirnov (KS) significance probabilities for the distribution to be a power law for all 49 Indian stocks and for the aggregated data to be $\ll 5\%$, and the KS significance probabilities for the distributions to be an exponential is $> 95\%$. Further we calculate

$$\beta_{\rm avg} \equiv \frac{1}{49} \sum_{i=1}^{49} \beta_i \tag{4}$$

and find

$$\beta_{\rm avg} = \begin{cases} 1.54 \pm 0.05 & \text{(negative tail)}, \\ 1.34 \pm 0.06 & \text{(positive tail)}. \end{cases} \tag{5}$$

Discussion. – Approximately 1/6 of the world's inhabitants live in India. In 2001 India had an estimated impoverished population of 40 million, 22% of the total urban population. The National Stock Exchange averages 6×10^6 trades per day and its average daily turnover is $\approx 3 \times 10^8$ USD. The average turnover in India is $\approx 10^9$ USD and the average share volume transacted is $\approx 2 \times 10^5$. Because Indian people are traditionally extremely careful with their money, they have high individual savings and transactions in the Indian Stock Market are not distributed across all economic scales. Stock market transactions are typically carried out by those with wealth in the top 25% of the economic spectrum.

A natural question is why the Indian stock market should have statistical properties that differ from other stock markets. One possible reason can be traced to the history of trading patterns in India and to its persistent trading culture. Even after more than 127 years of stock market operations, trading in India is said to be based as much on emotional factors as on actual evaluations and quantitative analysis. Quantitative analytical skills, although available, are expensive and limited, so a majority of investors in India tend to follow archaic investment strategies, which they feel are more conservative and safe. The result is that extreme risk situations with concomitant high returns are completely avoided. There are very few large financial institutions contributing to the total volume in trade. Small investors drive panic into the market on rumors making the market susceptible to small instabilities. These factors have kept the market under a tight noose.

Furthermore, it has been reported in ref. [10] that a general 3-parameter family of distribution given as

$$P(x|b,c,d) = \begin{cases} A(b,c,d,u)x^{(b+1)}e^{-(\frac{x}{d})^c} & \text{if}\, x \geq u > 0\,, \\ 0 & \text{if}\, x < u\,, \end{cases} \tag{6}$$

where A is the normalizing constant and b, c, d are parameters, is successful in explaining transition from a power law to an exponential distribution. It has been observed that for $c = 0.3$ or less, the stretched exponential mimics a power law with exponent close to 3 and the case $c = 1$ corresponds to the Indian case reported here. Thus, the alternative interpretation to our observation is that, rather than a different universality class, there is a continuum by varying c, which becomes a measure of the maturity of the market and may describe a crossover from Gaussian thin tails to power-law–like thick tails.

The stock price fluctuations in India are intermediate to that between power law behavior and Gaussian behavior. Power law behavior is found for highly developed economies while the less highly developed economies such as India's follow a behavior which is scale dependent(¹).

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(¹)A conjecture would be whether developing economies which are less developed than India's also show Gaussian behavior [1]. To test this, we hope in the future to investigate whether stock price variations undergo a transition from Gaussian distribution to a power law distribution via an exponential distribution at intermediate time.

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[8] We download daily prices of the top, based on market capitalization, 2000 US stocks from the `http://finance.yahoo.com` database which were active in the same period as the database of Indian stocks. First, we arrange this set of 2000 stocks alphabetically from 1 to 2000. Next, we generate a list of 49 random integers between 1 and 2000 which are different from each other. Lastly, we select a set of 49 US stocks by choosing from positions indicated by the list of 49 random numbers that were generated.
[9] We aggregate the data (analyzed as a single large data set), which is justified if the prices for all 49 stocks follow the same distribution. This assumption is consistent with our experience. We also like to add that in a private discussion with Xavier Gabaix he has pointed out that generally in an emerging economy stocks are more correlated.
[10] Malevergne Y., Pisarenko V. F. and Sornette D., preprint `http://arXiv.org/abs/physics/0305089`.