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Applications of statistical mechanics to finance

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Abstract

We discuss some apparently “universal” aspects observed in the empirical analysis of stock price dynamics in financial markets. Specifically we consider (i) the empirical behavior of the return probability density function and (ii) the content of economic information in financial time series. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

The analyses and modeling of financial markets started in 1900 with the pioneering work of the French mathematician Bachelier [1]. Since the 1950s, the analysis and modeling of financial markets have become an important research area of economics and financial mathematics [2]. The researches pursued have been very successful, and nowadays a robust theoretical framework characterizes these disciplines [3–6]. In parallel to these studies, starting from the 1990s a group of physicists became interested in the analysis and modeling of financial markets by using tools and paradigms of their own discipline (for an overview, consider, for example, [7–11]). The interest of physicists in such systems is directly related to the fact that predictability has assumed a meaning in physics over the years, which is quite different from the one originally associated with the predictability of, for example, a Newtonian linear system. The

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degree of predictability of physics systems is nowadays known to be essentially limited in nonlinear and complex systems. This makes the physical prediction less strong, but on the other hand the area of research covered by physical investigations and of its application may increase [12].

The research approach of physicists to financial modeling aims to be complementary to the ones of financial mathematicians and economists. The main goals are to (i) contribute to a better understanding and modeling of financial markets and (ii) promote the use of physical concepts and expertise in the multidisciplinary approach to risk management.

In this communication, we review some results of our work on (i) the statistical properties of returns in financial markets and (ii) the characterization of the simultaneous dynamics of stock prices in a financial market. Specifically, we recall statistical properties of price changes empirically observed in different markets worldwide. In particular, we discuss studies performed on the New York stock exchange [13,14], the Milan stock exchange [15] and the Budapest stock exchange [16]. The communication is organized as follows, we first discuss the results of empirical analyses performed in these different markets and then we address the problem concerning the presence of economic information in a financial time series.

2. Statistical properties of price dynamics

The knowledge of the statistical properties of price dynamics in financial markets is fundamental. It is necessary for any theoretical modeling aiming to obtain a rational price for a derivative product issued on it [17] and it is the starting point of any valuation of the risk associated with a financial position [18]. Moreover, it is needed in any effort aiming to model the system. In spite of this importance, the modeling of such a variable is not yet conclusive. Several models exist which show partial successes and unavoidable limitations. In this research, the approach of physicists maintain the specificity of their discipline, namely to develop and modify models by taking into account the results of empirical analysis.

Several models have been proposed and we will not review them here. Here, we wish to focus only on the aspects which are "*universally*" observed in various stock price and index price dynamics.

2.1. Short- and long-range correlations

In any financial market — either well established and highly active as the New York stock exchange, "*emerging*" as the Budapest stock exchange, or "*regional*" as the Milan stock exchange — the autocorrelation function of returns is a monotonic decreasing function with a very short correlation time. High-frequency data analyses have shown that correlation times can be as short as a few minutes in highly traded stocks or indices [14,19]. A fast decaying autocorrelation function is also observed in the empirical analysis of data recorded transaction by transaction. By using as time index the number of transactions emanating from a selected origin, a time memory as short as a few transactions has been detected in the dynamics of most traded stocks of the Budapest “emerging” financial market [16].

The short-range memory between returns is directly related to the necessity of absence of continuous arbitrage opportunities in efficient financial markets. In other words, if correlation were present between returns (and then between price changes) this would allow one to devise trading strategies that would provide a net gain continuously and without risk. The continuous search for and the exploitation of arbitrage opportunities from traders focused on this kind of activity drastically diminish the redundancy in the time series of price changes. Another mechanism reducing the redundancy of stock price time series is related to the presence of the so-called “noise traders”. With their action, noise traders add into the time series of stock price information, which is unrelated to the economic information decreasing the degree of redundancy of the price changes time series.

It is worth pointing out that not all the economic information present in stock price time series disappears due to these mechanisms. Indeed the redundancy that needs to be eliminated concerns only price change and not any of its nonlinear functions [20].

The absence of time correlation between returns does not mean that returns are identically distributed over time. In fact, different authors have observed that nonlinear functions of return such as the absolute value or the square are correlated over a time scale much longer than a trading day. Moreover, the functional form of this correlation seems to be power-law up to at least 20 trading days approximately [19,21–26].

A final observation concerns the degree of stationary behavior of the stock price dynamics. Empirical analysis shows that returns are not strictly sense stationary stochastic processes. Indeed the volatility (standard deviation of returns) is itself a stochastic process. Although a general proof is still lacking, empirical analyses performed on financial data of different financial markets suggest that the stochastic process is locally non-stationary but asymptotically stationary. By asymptotically stationary we mean that the probability density function (pdf) of returns measured over a wide time interval exists and it is uniquely defined. A paradigmatic example of simple stochastic processes which are locally non-stationary but asymptotically stationary is provided by ARCH [27] and GARCH [28] processes.

2.2. The distribution of returns

The pdf of returns shows some “universal” aspects. By “universal” aspects we mean that they are observed in different financial markets at different periods of time provided that a sufficiently long time period is used in the empirical analysis. The first of these “universal” or stylized facts is the leptokurtic nature of the pdf. A leptokurtic pdf characterizes a stochastic process having small changes and very large changes more frequently than in the case of Gaussian distributed changes. Leptokurtic pdfs have been observed in stocks and indices time series by analyzing both high-frequency and daily data. Thanks to the recent availability of transaction-by-transaction data, empirical analyses on a transaction time scale have also been performed. One of these studies performed by analyzing stock prices in the Budapest stock exchange shows that return pdf is leptokurtic down to a "transaction" time scale [16].

The origin of the observed leptokurtosis is still debated. There are several models trying to explain it. Just to cite (rather arbitrarily) a few of them: (i) a model of Lévy stable stochastic process [29]; (ii) a model assuming that the non-Gaussian behavior occurs as a result of the uneven activity during market hours [30]; (iii) a model where a geometric diffusive behavior is superimposed on Poissonian jumps [31]; (iv) a quasi-stable stochastic process with finite variance [32]; and (v) a stochastic process with rare events described by a power-law exponent not falling into the Lévy regime [33–35]. The above processes are characterized by finite or infinite moments. In an attempt to find the stochastic process that best describes stock price dynamics, it is important to try to preliminarily conclude about the finiteness or infiniteness of the second moment.

The above answer is not simply obtained [38] and careful empirical analyses must be performed to reach a reliable conclusion. It is our opinion that an impressive amount of empirical evidence has been recently found supporting the conclusion that the second moment of the return pdf is finite [13,33–37]. This conclusion has a deep consequence on the stability of the return pdf. The finiteness of the second moment and the independence of successive returns imply that the central limit theorem asymptotically applies. Hence, the form expected for the return pdf must be Gaussian for very long time horizons. We then have two regions — at short time horizons we observe leptokurtic distributions whereas at long time horizons we expect a Gaussian distribution. A complete characterization of the stochastic process needs an investigation performed at different time horizons. During this kind of analyses, non-Gaussian scaling and its breakdown has been detected [13,14].

3. Collective dynamics

In the previous sections we have seen that "universal" facts suggest that the stock price change dynamics in financial markets is well described by an unpredictable time series. However, this does not imply that the stochastic dynamics of stock price time series is a random walk with independent identically distributed increments. Indeed the stochastic process is much more complex than a customary random walk.

One key question in the analysis and modeling of a financial market concerns the independence of the price time series of different stocks traded simultaneously in the same market. The presence of cross-correlations between pairs of stocks has been known since a long time and it is one of the basic assumptions of the theory for the selection of the most efficient portfolio of stocks [39]. Recently, physicists have also started to investigate theoretically and empirically the presence of such cross-correlations.

It has been found that a meaningful economic taxonomy may be obtained by starting from the information stored in the time series of stock price only. This has been achieved by assuming that a metric distance can be defined between the synchronous time evolution of a set of stocks traded in a financial market and under the essential *ansatz* that the subdominant ultrametric associated with the selected metric distance is controlled by the most important economic information stored in the time evolution dynamics [40].

Another kind of study is devoted to the detection of the statistical properties of eigenvalues and eigenvectors of the covariance matrix of n stocks simultaneously traded. Also, with this approach the hypothesis that the dynamics of stock price in a portfolio of n stocks is described by independent random walks is falsified [41–43]. Moreover, information about the number of terms controlling eigenvectors can be detected.

The observation of the presence of a certain degree of statistical synchrony in the stock price dynamics suggests the following conclusion. Consideration of the time evolution of only a single stock price could be insufficient to reach a complete modeling of all essential aspects of a financial market.

4. Summary

This communication briefly discusses some of the stylized "universal" facts that are observed in financial markets and are considered robust by several researchers working in the field. Starting from these results, one can devise studies trying to enrich and expand this knowledge to provide theoreticians and computer scientists the empirical facts that need to be explained by their models progressively proposed. The ultimate goal is to contribute to the search for the best model describing a financial market, one of the most intriguing "*complex systems*".

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