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# Experimental studies of stratification in a granular Hele–Shaw cell

By Hernán A. Makse†, Shlomo Havlin,† ‡, Peter R. King§ and H. Eugene Stanley†

† Center for Polymer Studies and Physics Department, Boston University, Boston, Massachusetts 02215, USA
‡ The Minerva Center for the Study of Mesocopics, Fractals, and Neural Networks, and Department of Physics, Bar-Ilan University, Ramat Gan, Israel
§ BP Exploration Operating Company Ltd, Sunbury-on-Thames, Middlesex TW16 7LN, England

Abstract

We show that, when a mixture of small and large grains is simply poured in a 'granular Hele–Shaw cell' (two transparent slabs separated by a gap of about 5 mm), the grains spontaneously stratify in alternating layers of small and large grains parallel to the surface of the pile. The spontaneous self-stratification occurs only when the large grains have a larger angle of repose than the small grains do. Conversely, we obtain only spontaneous self-segregation, that is the large grains are found preferentially near the bottom of the cell while the small grains are found near the top, when the large grains have a smaller angle of repose than the small grains. The observed stratification might be of relevance to explain similar stratified patterns observed in Aeolian rocks.

## § 1. Introduction

Granular materials (Bagnold 1941, Jaeger and Nagel 1992, Herrmann 1993, Edwards 1994, Wolf 1996) size segregate when exposed to external periodic perturbations such as vibrations. A much-studied size segregation phenomenon is known as the 'Brazil nut effect' (Williams 1976, Rosato *et al.* 1987, Gallas *et al.* 1992, Knight *et al.* 1993, Cooke *et al.* 1996) and occurs when, upon vibration, larger grains rise on a bed of smaller grains. Axial size segregation in alternating bands consisting of small and large grains also occurs when a mixture of grains is combined in a horizontal rotating drum (Oyama 1939, Weidenbaum 1958, Zik *et al.* 1994, Frette and Stavans 1997), while radial size segregation occurs in thin rotating drums (Cantelaube and Bideau 1995, Clément *et al.* 1995).

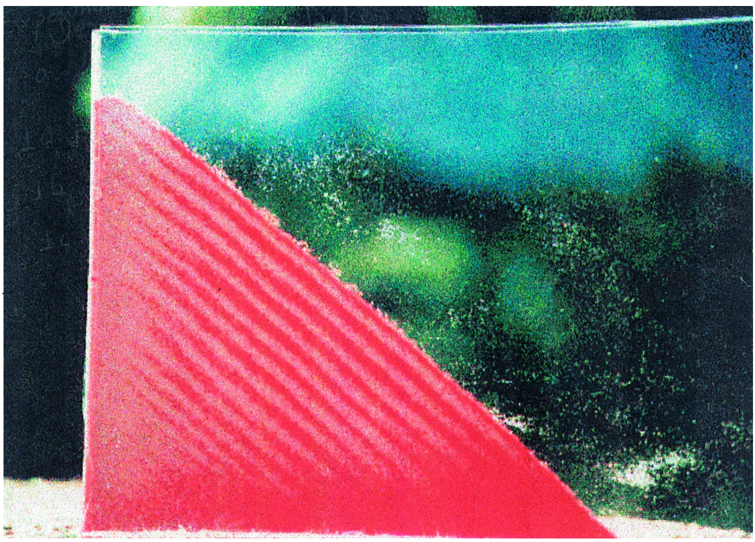
Moreover, mixtures of grains of different sizes spontaneously segregate in the absence of external perturbations; when a mixture is simply poured onto a pile, the large grains are more likely to be found near the base, while the small grains are more likely to be near the top (Brown 1939, Bagnold 1954, Drahun and Bridgwater 1983, Fayed and Otten 1984, Savage 1987, 1989, Savage and Lun 1988, Meakin 1990). Here, we report another spontaneous phenomenon (Makse *et al.* 1997b) arising when we pour a mixture between two vertical plates; the mixture spontaneously stratifies into alternating layers of small and large grains whenever the large grains have a larger angle of repose than the small grains. In contrast, we find only spontaneous segregation when the large grains have a smaller angle of repose than the small grains. The stratification is related to the occurrence of avalanches; during each avalanche the grains comprising the avalanche spontaneously stratify into a pair of layers, through a 'kink' mechanism, with the small grains forming a sublayer underneath the layer of large grains. Many sedimentary rocks are formed by an avalanche of sand (e.g. sand dunes) (Bagnold 1941). Layers of coarse and fine sand are present in these rocks (cross beds) and so this phenomenon might be of relevance to geological systems. In this paper we review mainly the experimental aspects of stratification and the mechanism for stratification. Details can be found in the studies of Makse *et al.* (1996a,b, 1997a,b). In another paper (Cizeau *et al.* 1997) we shall discuss a discrete and a continuum model for stratification and segregation.

## § 2. Experiments

Our experimental system consists of a vertical 'quasi-two-dimensional' cell with a gap of 5 mm separating two transparent plates (made of Plexiglass or of glass) measuring 300 mm × 200 mm (see figure 1 (a)). To avoid the effects of electrostatic interaction with the wall, the wall is cleaned with antistatic cleaner.

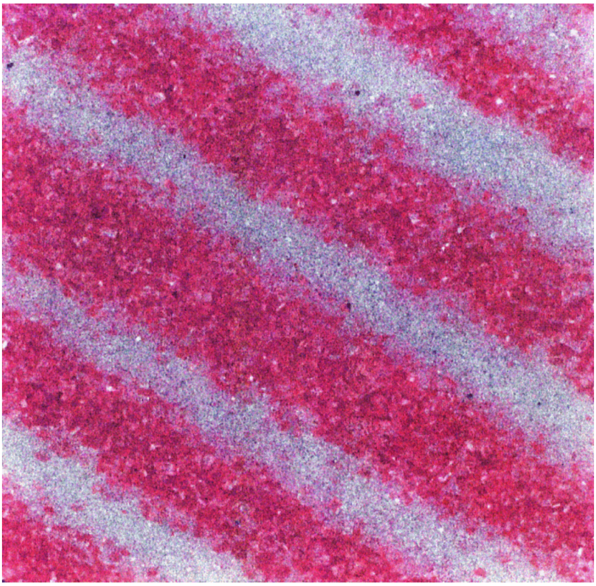
In a first series of experiments, we close the left edge of the cell, leaving the right edge free, and we pour, near the left edge, an equal-volume mixture of white sand grains (typical size, 0.3 mm; irregular rounded shape; repose angle, 35°), and red sugar crystals (typical size, 0.8 mm; cubic shape; repose angle, 39°). Figure 1 (a) shows the result of the first series of experiments. We note two features.

(i) *Spontaneous stratification.* We see the formation of alternating layers consisting of small and large grains, with a 'wavelength' of about 1 cm (Makse *et al.* 1997b).

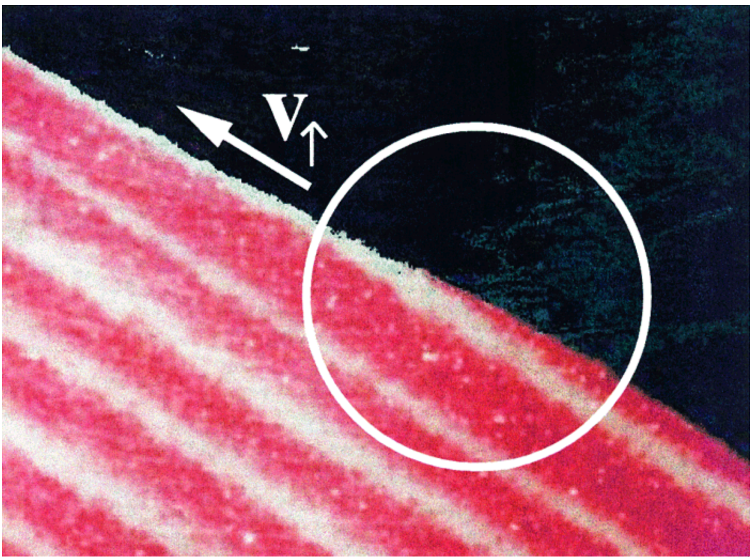


(*a*)

Figure 1. Stratification. (*a*) Experiment showing the formation of successive layers of fine and coarse grain (here the white grains are sand grains of typical size 0.3 mm, while the larger grains are red sugar crystals of typical size 0.8 mm). (*b*) Close-up photograph of the stratification pattern obtained using a mixture of small spherical glass beads and red sugar crystals (field of view, 40 mm × 40 mm). (*c*) Close-up photograph of the kink where the grains stop during an avalanche. The small white grains stop first, and then the large red grains; hence the small grains form a sublayer underneath the large grains.



1 (*b*)



1 (*c*)

(ii) *Spontaneous segregation*. We find that the smaller grains segregate near the left edge and the larger grains segregate farthest from it and near the base (Brown 1939, Bagnold 1954, Drahun and Bridgwater 1983, Fayed and Otten 1984, Savage 1987, 1989, Savage and Lun 1988, Meakin 1990).

In a second series of experiments, we confirmed the results of these initial experiments by testing for stratification and segregation using a mixture of grains of same density, consisting of fine sand (typical size, 0.4 mm) and coarse sand (typical size, 1 mm), suggesting that the density of the grains may not play an important role in stratification.

In all the above experiments we used mixtures composed of two types of grain with different shapes and therefore with different angles of repose. In particular we obtain stratification (plus segregation) when we use larger cubic grains and smaller spherical grains: the angle of repose of the large species is then larger than the angle of repose of the small species. Otherwise we obtain only segregation and not stratification when the large grains are less facetted than small grains, that is the large grains have a smaller angle of repose than the small grains.

To confirm this, we performed a series of experiments using mixtures of irregularly shaped sand grains (repose angle, 35°; mean size, 0.3 mm), and spherical glass beads (repose angle, 26° (*smaller* than the repose angle of the sand grains)).

We find that stratification (plus segregation) occurs for two different experiments using spherical beads of size 0.07 and 0.11 mm (so that the larger grains have a larger repose angle). In contrast, we obtain only segregation *but not stratification* for two experiments using spherical beads of size 0.55 and 0.77 mm (so that the larger grains have a smaller repose angle). In all cases the segregation of grains occurs with the smaller grains being found near the left edge of the cell and the larger grains near the base of the cell. These results suggest that the phenomenon of segregation is always expected when pouring a granular mixture of grains of different sizes, irrespective of the values of the angles of repose of the species. However, the phenomenon of stratification is only expected when the large species have a larger angle of repose than the small species.

Additionally, we performed a series of experiments in which we find similar stratification by using different mixtures of various size ratios of large to small grains (1.66, 2.1, 2.25, 3.25 and 6.66), suggesting that the phenomenon occurs for a broad regime of grain size ratios. We find a similar stratification when we double the gap between the vertical plates of the cell and simultaneously double the flow rate of grains.

## § 3. Kink mechanism

We propose a physical mechanism responsible for the observed stratification that is related to the fact that not one but rather a pair of layers is formed in the course of each avalanche through a 'kink' mechanism (Makse *et al.* 1997a).

Starting from a surface layer of large grains (at their angle of repose) the flow of small and large grains rides on top (figure 2). The small grains tend to be trapped on this layer (forming a thin layer of small grains) while the large grains 'ride' down to the base because large grains do not tend to get trapped (in local minima of the surface profile) as easily as small grains, that is large grains roll down more easily on top of small grains than small grains on top of large grains (for rolling *large* grains on top of a surface of small grains the surface appears smoother than for rolling *small* grains on top of a surface of large grains). When the flow of grains reaches the base of the pile, we find that the grains develop a profile characterized by a well defined 'kink', at which the grains are stopped (see figures 1 (c) and 2), but we find that the small grains stop first; so a pair of layers forms with the small grains *underneath* the large grains. As more grains are added, the kink appears to move upwards in the direction opposite to the flow of grains. Once the kink reaches the top, the pair of layers is complete and the cycle is then repeated: a new avalanche occurs, the kink develops, and a new pair of layers forms.

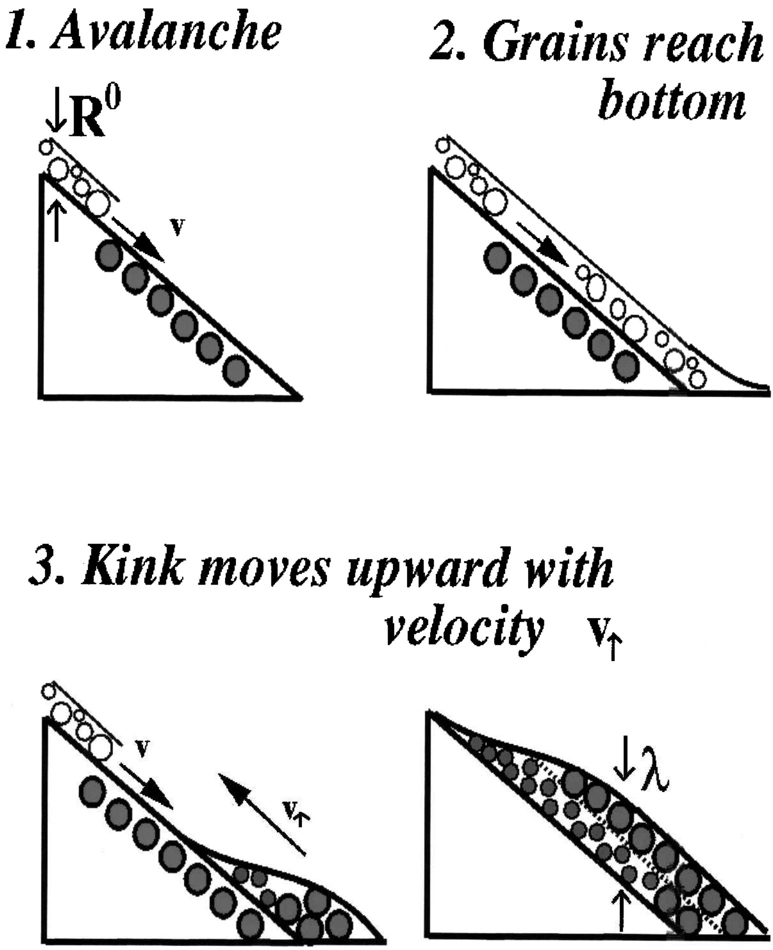



Figure 2. Description of the formation of the layers and ‘kink’ mechanism.

The ‘wavelength’ $\lambda$ of a pair of layers can be determined by the mean value of the downward velocity v of the rolling grains during an avalanche, the upward velocity $v_\uparrow$ of the kink, and the thickness $R^0$ of the layer of rolling grains during the avalanche (see figure 2). If the volume of grains in an avalanche scales as the volume of grains in a well formed kink, we predict that

$$\lambda = \frac{R^0(v + v_\uparrow)}{v_\uparrow}, \tag{1}$$

and we confirmed this relation experimentally (Makse *et al.* 1996b).

Next we test the above principles by generalizing from two grain sizes to three. The experiment results in stratification with three layers, with the finest grains on the bottommost of each triplet of layers and the coarsest grains on the topmost layer (see figure 3 where the same experimental set-up as in figure 1(a) is used to obtain an alternation of *three* layers of grains of *three* different sizes, namely 0.15 mm, 0.4 mm and 0.8 mm, and three different repose angles, namely 26°, 35° and 39° respectively).

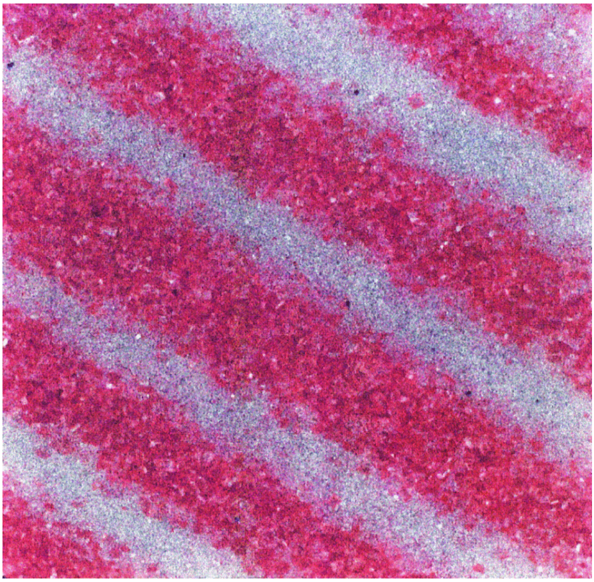


Figure 3. Close-up photograph of the stratification pattern obtained using a mixture of three different types of grain: nearly spherical glass beads (0.15 mm; angle of repose, 26°), blue sand (0.4 mm; angle of repose, 35°) and red sugar crystals (0.8 mm; angle of repose, 39°). The field of view is 40 mm × 40 mm. Note the grading (from bottom to top) in a triplet of layers: small (white), medium (blue) and large (red) grains.

We note that Boutreux and deGennes (1996) have recently made considerable progress (Boutreux and deGennes 1996) in developing a general theoretical framework (Bouchaud *et al.* 1995, deGennes 1995) to treat the case of granular flows of two different grains. Their conclusions (Boutreux and deGennes 1996, Bouchaud *et al.* 1995, deGennes 1995) are consistent with the experiments presented here.

Based on this theoretical approach, we have develop a model for stratification and segregation (Cizeau *et al.* 1997, Makse *et al.* 1997a) in which we treat the individual grain motion in accord with microscopic rules that depend on the local angles formed between each grain and its neighbours. The dynamics of the small and large rolling grains are governed by the critical repose angles $\theta_{\alpha\beta}$ ($\alpha, \beta = 1, 2$ for the small and large grains respectively), corresponding to the interaction between a rolling grain $\alpha$ and a static grain $\beta$ of the sandpile surface. We find stratification and also find that the profile of the sandpile displays a kink at which rolling grains are stopped (Makse *et al.* 1997a) just as in the experiment.

## § 4. Stratification instability in granular flows

According to our experimental findings, a segregation profile is always observed in the initial regime of the experiments. This segregation profile is 'stable' (showing the segregation of the mixture) so long as the control parameter

$$\delta \equiv \theta_{22} - \theta_{11} < 0, \tag{2}$$

and 'unstable' (evolving to stratification for large enough systems) when $\delta > 0$.

Why is $\delta$ the control parameter for stratification?

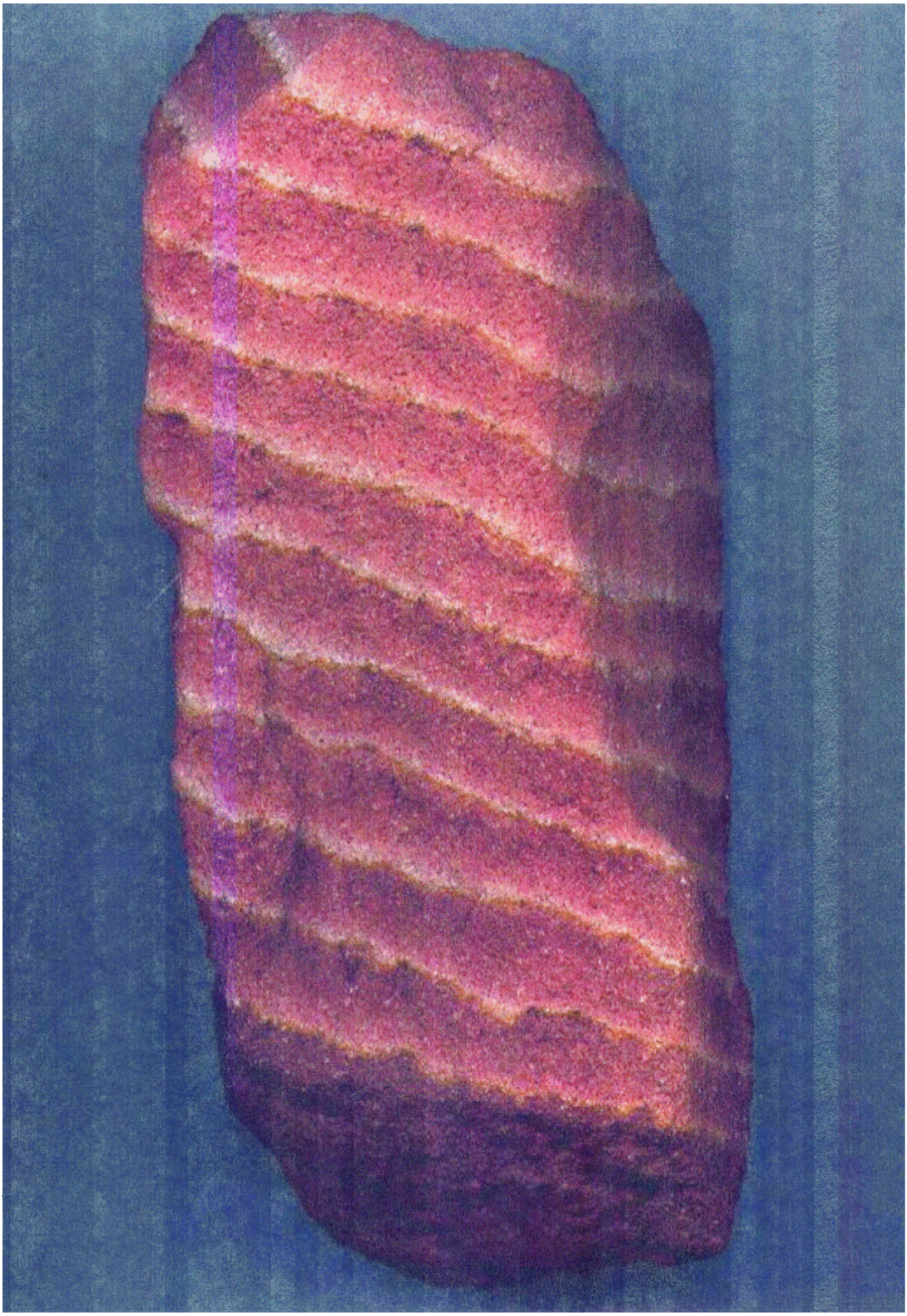
The fundamental stable state of a granular mixture of grains differing in size and shape poured in a granular Hele–Shaw cell is achieved when the mixture is composed of type A grains (larger and rounded shape) and type B grains (smaller and irregular shape). In this case, the segregation of the mixture is found with grains A located near the bottom of the cell, and grains B located near the top of the cell; grains A roll down to the bottom of the cell because the larger grains roll down more easily on top of smaller grains, and the rounded grains roll down more easily on top of the rougher grains, too. This situation corresponds to $\delta < 0$, the stable steady state of the system. On the other hand, when pouring a mixture of type A grains (larger and irregular shape) and type B grains (smaller and rounded shape), an instability develops in the system. Upon pouring the mixture, grains A would tend to go to the bottom, and grains B would tend to be trapped near the top of the cell, because grains A are larger than grains B. However, at the same time, grains A would tend to be trapped near the top and grains B would go to the bottom of the cell since type A grains are more irregularly shaped than grains B. This unstable situation corresponds to $\delta > 0$ and leads to the appearance of the oscillation characteristic of stratification.

The onset of stratification occurs as follows. As seen in all the stratification experiments, before the layers appear there is an initial regime where only segregation is found. At the onset of stratification, a few large grains are captured on top of the region of small grains near the centre of the pile where the angle of the pile (denoted $\theta$) is $\theta \approx \theta_{11}$ ($\theta_{11}$ denotes the angle of repose of the small grains). However, the repose angle $\theta_{22}$ for large grains is different from $\theta_{11}$. When $\theta_{22} > \theta_{11} \approx \theta$, more large grains can be trapped (since the angle $\theta$ of the surface is smaller than the repose angle $\theta_{22}$), leading to an incipient sublayer of large grains and then to stratification. On the other hand, when $\theta_{22} < \theta_{11} \approx \theta$, no more large grains can be captured and therefore the segregation profile remains stable.

According to this picture, we note that experiments where $\theta_{11} = \theta_{22}$ (i.e. mixtures of perfect spherical beads differing only in size) should not show stratification. We confirmed this prediction experimentally. However, some oscillations might still be present around the stable segregation profile, as seen in previous experiments using mixtures of spherical beads (Williams 1963, 1968, Allen 1982), especially when the beads are not perfectly spherical.

## § 5. How the wind may work

The formation of alternating laminae of fine and coarse grains in large-scale sedimentary structures is a widespread phenomenon familiar to specialist and non-specialist alike (Bagnold 1941, McKee *et al.* 1967, Jopling and Walker 1968, Borges 1945, Bridge 1978, Fryberger and Schenk 1981, Allen 1985, Cheel and Middleton 1986). However, the question of how such periodic patterns are generated remains unanswered. Previous attempts to explain the occurrence of stratified structures in rocks have been related to the existence of periodic fluctuations in sedimentary condition, such as oscillations in wind velocity (McKee *et al.* 1967, Jopling and Walker 1968, Borges 1945, Bridge 1978, Fryberger and Schenk 1981, Allen 1985, Cheel and Middleton 1986). Here we argue that stratification in rocks might occur in the absence of any periodic external perturbation (Makse *et al.* 1996a). Specifically, we comment on the connection between the stratification experiments shown above and the stratified patterns observed in sedimentary structures.



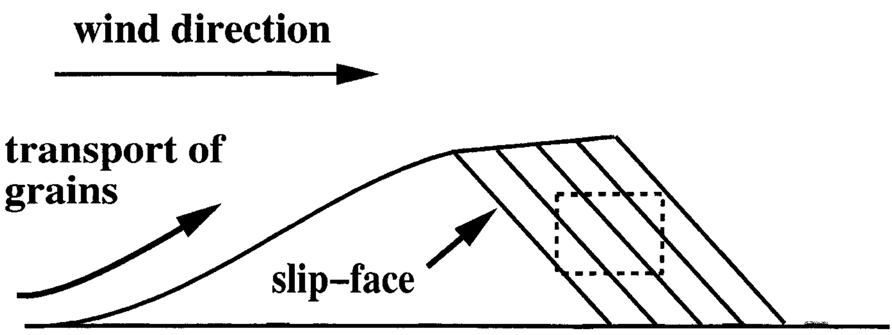
(*a*)

Figure 4. (*a*) Photograph of sandstone taken from Petra, Jordan, on 31 March 1997 by H. A. Makse and H. Hlalat. The size of the sample is 6 cm by 5 cm by 12 cm tall. The wavelength of the layers is 0.9 cm.

A typical example of a stratification pattern in rocks is shown in figure 4 (a). Sedimentary rocks are usually formed by grain flow (avalanches) of wind-blown sand. Other processes, such as grainfall ('raining'), also contribute to the formation of real sand dunes. However, the effects of grainfall are known to be insignificant at the length scales of interest, that is from tens of centimetres to tens of metres (Bagnold 1941). Indeed, grainfall gives rise to the opposite size segregation (small-scale ripple formations which are typically 1 cm in amplitude) (Forrest and Haff 1992, Anderson and Bunas 1993), where large grains are observed at the crests and small grains at the bottom (Anderson and Bunas 1993), as opposed to the size segregation that we study which involves large grains on the bottom of the dune.

As a unidirectional wind moves sand along a bed, a small sand accumulation (incipient dune) is formed (figure 4 (b)). As the wind continues, sand moves from the upstream side of the dune to the crest of the dune. When the slope of the dune becomes steeper, a downstream slip face is developed where avalanches of sand begin (Bagnold 1941). As new material is brought to the top of the dune, another avalanche occurs.

Since the actual geological system is translationally invariant along the transverse direction (owing to the unidirectional flow of sand), our experiment performed in a quasi-two-dimensional geometry might be relevant for the avalanche dynamics in the slip face of the dune. The three-grain experiment shown in figure 3 shows a grading in a triplet of three consecutives sublayers of the form: from bottom to top, small–medium–large, small–medium–large, etc. The same grading can be observed in the rock sample shown in figure 4 (a), indicating that similar grading mechanisms might be acting in the slip face of the dune as in the experiments presented here.




(*b*)

Figure 4. (*b*) Schematic representation of the formation of a slip face of a dune. After an incipient dune is formed, the wind moves sand from the upstream side to the top, and dune avalanches (*grain flow*) begin. This kind of large-scale sedimentary structure occurs on length scales ranging from tens of centimetres to tens of metres and is characterized by avalanching of grains (Bagnold 1941) (as opposed to small-scale ripple formations which are typically 1 cm in amplitude (Forrest and Haff 1992) and are mainly dominated by *grainfall*). Grain flow (avalanches) and grain fall (small ripples) lead to size segregation of opposite sign; in grain flow, the large grains tend to be on the bottom of the dune while, in grainfall, the large grains tend to be on the top (see, for example, figure 3 of Anderson and Bunas (1993)). Grain flow also occurs in other sedimentary systems, such as fluvial, but in that case the geological process is modified by the turbulent water flow.

According to Fineberg (1997), the self-stratification phenomenon might be relevant to another puzzle: long-run-out rock slides. Such rock slides are known to destroy entire towns as was the case of Frank, a town in Alberta, Canada, which was wiped out by 74 million tonnes of rock crashing down from Turtle Mountain on 29 April 1903. The rock slide began when stones fell 1 km down the mountain into the valley but, when the rocks reached the bottom of the mountain, they kept moving for 4 km across the valley, sweeping out Frank and stopping at the foot of another slope (McConnel and Brock 1904). During avalanches, the rocks spontaneously form layers, with the small rocks at the bottom of the layers, a phenomenon called kinematic sieving (Bridgwater 1976, Drahun and Bridgwater 1983, Savage 1984). These small rocks act as the best 'ball-bearing' reducing friction and providing an overall lubrication effect, so that the mass of material can continue to slide over for large distances after reaching the ground.

## § 6. Conclusion

In summary, we show that, when a mixture of grains of different sizes and shapes is simply poured in a vertical Hele–Shaw cell, the grains spontaneously stratify whenever the large grains have a larger angle of repose than the small grains have. Conversely, we find only spontaneous segregation when the large grains have a smaller angle of repose than the small grains have. We argue that the experiments presented here might be able to help to explain the origin of similar stratified structures found in Aeolian rocks.

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