PHYSICAL REVIEW E **66**, 045103(R) (2002)

Different scaling behaviors of commodity spot and future prices

Kaushik Matia,[1] Luis A. Nunes Amaral,[1,*] Stephen P. Goodwin,[2] and H. Eugene Stanley[1]
[1]*Center for Polymer Studies and Department of Physics, Boston University, Boston, Massachusetts 02215*
[2]*BP Upstream, Chertsey Road, Sunbury-on-Thames, Middlesex TW16 7LN, United Kingdom*
(Received 8 March 2002; revised manuscript received 16 July 2002; published 24 October 2002)

Classic studies of spot price fluctuations for commodities like cotton and wheat have been interpreted using a power-law probability distribution with exponent α inside the Lévy-stable regime $(0<\alpha<2)$. In contrast price fluctuations for stocks have been interpreted using a power-law probability distribution with α outside the Lévy-stable regime suggesting that stock prices are in a different universality class than spot prices for commodities. To test this possibility we analyze daily returns of spot prices for 29 commodities and daily returns of future prices for 13 commodities over a period exceeding 10 years and find that the distributions of returns for futures decay as power laws with exponents $\alpha\approx 3.2$, significantly larger than $\alpha=2$ and hence outside the Lévy-stable domain, while for spot prices we find $\alpha\approx 2.3$ which appears to be marginally outside the Lévy-stable domain.

DOI: 10.1103/PhysRevE.66.045103 PACS number(s): 89.90.+n, 05.45.Tp, 05.40.Fb

Study of economic markets has recently become an area of active research for physicists. Among the reasons are (i) markets constitute complex systems for which the variables characterizing the state of the system—i.e., the price of the goods, the number of trades, the number of agents are easily quantified, and (ii) there is a large amount of data that can be accessed, since every transaction is recorded.

Much of the research interest of physicists has concentrated on stocks [1], stock averages [2], and foreign exchange rates [3]. A number of key empirical findings have been established: (i) The distribution of logarithmic price changes is approximately symmetric and decays with power law tails with an exponent $\alpha+1\approx 4$ for the probability density function (pdf) [1–3]. (ii) The price changes are uncorrelated beyond rather short time scales [4]; and (iii) the amplitude of the price changes have long-range correlations, specifically, the correlations decay as a power law with an exponent $\gamma\approx 1/3$ [1].

One of the intriguing aspects of these empirical findings is that they appear to be universal: individual US stocks appear to conform to these ''laws'' [1], as do German stocks [5], and Australian stocks [6]. Market indices such as the S&P 500, the Dow Jones, the NIKKEI, the Hang Seng or the Milan index [2] also obey these same laws. Similar results are found for the most traded currency exchange rates such as the US dollar vs the Deutsch mark, or the US dollar vs the Japanese yen [7]. The ''universal'' nature of the statistics of daily returns is remarkable since the markets described above are quite different in their details. Hence, the observed universality raises the possibility of similar underlying mechanisms.

Unlike stock and foreign exchange markets, commodity markets have received less recent attention [8–10]. Contrary to heavily traded stocks or currencies—which have a somewhat abstract character because they (i) have an almost ''elastic'' response to changes in demand, (ii) do not require storage, and (iii) are not ''consumed''—commodities are physical products that are traded because they (i) cannot be produced at will, (ii) require physical storage, and (iii) are needed for some purpose. For example, one needs gasoline to run a car, heating oil to heat a home, or electricity to light an office.

Because of the stronger constraints affecting commodity markets, one might surmise, that commodity prices show larger fluctuations than stock prices. In fact, exponents of power law tails of probability distributions of the returns of spot prices [11] of commodities such as cotton and wheat have been reported [8] to be Lévy-stable, i.e., $0<\alpha<2$, whereas the returns of future prices of commodities such as potatoes have been reported [9] to be outside the Lévy-stable domain, i.e., $\alpha>2$. Here we address the question of whether the scaling of commodity price fluctuations is statistically distinguishable from that of stocks. To this end, we study the fluctuations in the spot price for 29 commodities and in future price for 13 commodities [12] and compare our results with the statistical properties of daily returns in stock markets.

We define the normalized price fluctuation (''return'') $g_i(t)\equiv[\ln S_i(t+\Delta t)-\ln S_i(t)]/\sigma_i$, where $\Delta t=1$ day, i indexes the 29 commodity spot and 13 commodity future prices (Tables I and II), $S_i(t)$ is the price at time t and σ_i is the standard deviation of the time series $\ln S_i(t+\Delta t)-\ln S_i(t)$. The probability distributions $P(g_i>x)$ of the returns follow power law forms $P(g_i>x)\sim 1/x^{\alpha_i}$, where α_i is outside the Lévy-stable domain $0<\alpha_i<2$.

Figures 1(a)–1(d) display Hill estimates [13] of α_i for the spot price of 29 commodities and future price of 13 commodities, calculated for x above a cutoff value x_{cutoff}. Based on our analysis we choose for all commodities the same value $x_{\text{cutoff}}=2$. Note that for daily returns the number of data points beyond $x_{\text{cutoff}}=2$ is typically 50 to 150 which is about 3% of the data (the optimal range for a sample ≈ 2000 points [14]). For the spot prices the average exponents are [15]

$$\overline{\alpha}_{\text{spot}}\equiv\frac{1}{29}\sum_{i=1}^{29}\alpha_i=\begin{cases}2.3\pm 0.2 & \text{positive tail,}\\ 2.2\pm 0.1 & \text{negative tail,}\end{cases}\tag{1}$$

while for the future prices the average exponents are [15]

*Present address: Department of Chemical Engineering, Northwestern University, Evanston, IL 60208.

TABLE I. Commodities for which we analyzed spot prices

Index	Symbol	Description	Period	No. records
1	Brent	Crude oil	1/88–8/98	2770
2	BUTANE	Butane	2/93–8/98	1433
3	Gasoil	Gas oil	1/88–7/93	1433
4	HFO	Heavy fuel oil	1/88–8/98	2770
5	HSFO arg	High-sulfur fuel oil (Gulf)	1/88–8/98	2770
6	HSFO New	High-sulfur fuel oil (NYC)	1/88–8/98	2770
7	Kero New	Kerosene (NYC)	1/88–8/98	2770
8	LSFO New	Low-sulfur fuel oil (NYC)	1/88–8/98	2770
9	LSFO NYH	Low-sulfur fuel oil (NYC)	1/88–8/98	2770
10	Nap Med	Naphtha (Mediterranean)	1/88–8/98	2770
11	Nap New	Naphtha (NYC)	1/88–4/95	1897
12	Prem unl	Automobile gasoline	6/92–8/98	1619
13	USNFAMC	Aluminum	6/87–6/02	3916
14	ANTFREE	Antimony	7/88–9/01	3447
15	BISFMWS	Bismuth	6/87– 6/02	3916
16	CADFMWS	Cadmium	1/92–5/02	2714
17	COPPR3M	Copper	6/87–6/02	3916
18	GOLDBLN	Gold-bullion	6/87–6/02	3916
19	PALEUFM	Palladium	6/87–6/02	3916
20	PLTUSFM	Platinum	1/89–6/02	3504
21	SLVCASH	Silver	6/87–6/02	3916
22	ZNCHARD	Zinc	6/87–6/02	3916
23	CC.CASH	Corn	6/87–6/02	3916
24	COTNATX	Cotton	5/87–5/02	3914
25	CATLIVE	Live Cattle	11/87–6/02	3795
26	LIVEHOG	Live Hogs	6/87–6/01	3657
27	OATSMP2	Oats	6/87–6/02	3916
28	PORKBEL	Pork-bellies	6/87–6/02	3916
29	SHEEPLW	Live Sheeps	1/92–5/02	2713

$$\overline{\alpha}_{\rm future} \equiv \frac{1}{13}\sum_{i=1}^{13}\alpha_i = \begin{cases} 3.1\pm 0.2 & \text{positive tail,} \\ 3.3\pm 0.2 & \text{negative tail.} \end{cases} \quad (2)$$

As a test of our results, we generate an ensemble of 30 surrogate data sets each with 3000 points ($\approx$ same number of data points as the data analyzed) and with $\alpha = 1.7$, the value reported in [8]. We show in Fig. 1(e) the Hill estimates of α for the surrogate ensemble. We find $\overline{\alpha} = 1.80 \pm 0.08$, in agreement with the input value [16].

Next we compare our calculations of α_i for spot commodities with exponents α_i of daily returns evaluated for 7128 stocks from the CRSP database [1,17]. We select those stocks active in the same time period as the commodities analyzed, and compute tail exponents of $P(x)$ by the same procedure. Figures 2(a)–2(d) compare the probability density functions of tail exponents for spot and future prices of commodities with that for stocks. We find that the tail exponent for spot and future prices of commodities appears to be outside the Lévy-stable region. We also find quantitative similarity in the tail exponents between stocks and future prices of commodities [18] and the tail exponent for the spot prices are significantly smaller than that for future prices of commodities [18].

We next discuss time correlations of returns. The average autocorrelation function $\overline{C}(\tau) \equiv (1/N)\Sigma_i^N \langle g_i(t) g_i(t+\tau)\rangle$,

TABLE II. Commodities for which we analyzed future prices.

Index	Symbol	Description	Period	No. records
1	NHGCS	Copper	9/89–6/02	3331
2	CKICS	Gold-bullion	6/87–5/02	3892
3	NPACS	Palladium	6/87–6/02	3916
4	NPLCS	Platinum	6/87–6/02	3916
5	CAGCS	Silver	6/87–5/02	3909
6	NKCCS	Coffee	6/87–6/02	3916
7	CC.CS	Corn	6/87–6/02	3915
8	NCTCS	Cotton	6/87–6/02	3916
9	CFCCS	Feeder Cattle	6/87–6/02	3916
10	CLCCS	Live Cattle	6/87–6/02	3916
11	CO.CS	Oats	6/87–6/02	3916
12	CPBCS	Pork-bellies	6/87–6/02	3916
13	MMWCS	Wheat	6/87–6/02	3916

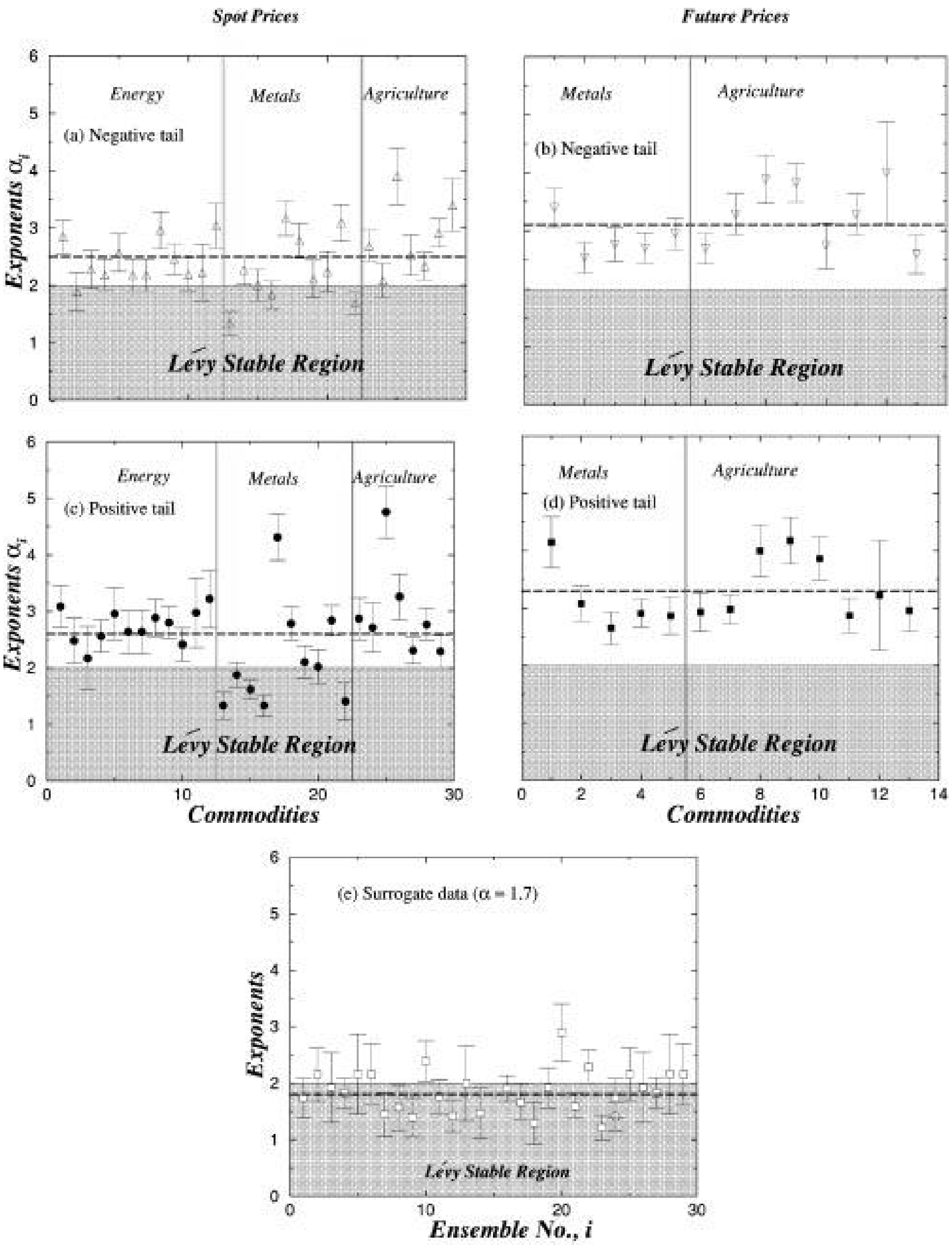

FIG. 1. Exponents α_i of the negative tail of commodity (a) spot prices and (b) future prices, where *i* indexes the 29 commodity spots and 13 commodity future prices analyzed (Tables I and II). Exponents α_i of the positive tail for commodity (c) spot prices and (d) future prices. We employ Hill's method [13] to estimate the exponent α_i of each probability distribution in the range $x \geqslant x_{\mathrm{cutoff}}$, with $x_{\mathrm{cutoff}}=2$. The dashed lines show the average values defined in Eqs. (1) and (2). Shaded regions indicate the range of Lévy-stable exponents, $0<\alpha<2$. (e) Surrogate data generated with $\alpha=1.7$ for $i=1,2,\ldots,30$ sets with $N=3000$ points. The dashed line shows $\overline{\alpha}=1.8$. Note that the mean exponent of spot prices is smaller than the mean exponent of the future prices, and that both are outside the Lévy-stable domain.

where $N=29$ for spot prices and $N=13$ for future prices, decays exponentially as $e^{-\tau/\tau_c}$. We find that $\tau_c^{\mathrm{spot}}=2.3$ days and $\tau_c^{\mathrm{future}}<1$ day. To further quantify time correlations, we use the detrended fluctuation analysis (DFA) method [19]. The DFA method calculates fluctuations $F(n)$ in a time window of size *n*, and then plots $F(n)$ versus *n*. The slope $\hat{\alpha}_{\mathrm{DFA}}$ in a log-log plot gives information about the correlations present. If $C(\tau)\sim\tau^{-\gamma}$ then $\hat{\alpha}_{\mathrm{DFA}}=(2-\gamma)/2$, while if $C(\tau)\sim e^{-\tau/\tau_c}$ then $\hat{\alpha}_{\mathrm{DFA}}=1/2$ [19]. We find that $\hat{\alpha}_{\mathrm{DFA}}=0.51\pm0.05$, $\hat{\alpha}_{\mathrm{DFA}}=0.50\pm0.05$ for spot and future prices respectively, consistent with the exponential decay of $\overline{C}(\tau)$. We also observe that $|g_i|$, the absolute value of returns (one measure of volatility), are power law correlated with

$$\hat{\alpha}_{\mathrm{DFA}}=\begin{cases}0.63\pm0.05 & \text{spot prices,}\\ 0.60\pm0.05 & \text{future prices,}\end{cases} \tag{3}$$

which implies a power law decay of the autocorrelation of the absolute value of returns with

$$\gamma=\begin{cases}=0.74\pm0.1 & \text{spot prices,}\\ =0.80\pm0.1 & \text{future prices.}\end{cases} \tag{4}$$

Note that the value of the exponent γ for commodities is larger than for stocks [1].

In summary, we analyze spot prices for 29 commodities and future prices for 13 commodities. We find quantitative similarity between stock and commodity futures markets, which strengthens the likelihood of a universal mechanism underlying both markets. We hypothesize that the the fact that nowadays a large fraction of the trading taking place at commodity markets, especially for futures, is for speculative purposes (i.e., with the intent of making a profit by buying low and selling high) is the reason why we find similar val-

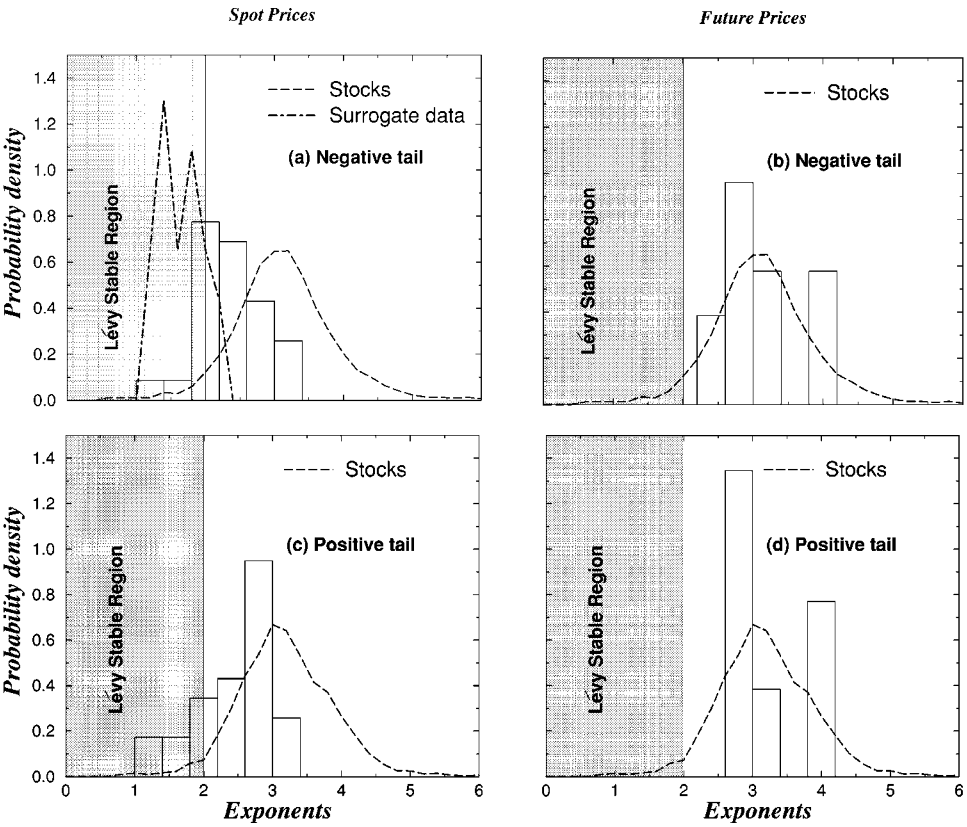

FIG. 2. Probability density function of negative tail exponents for stocks, surrogate data, and (a) spot or (b) future prices. Probability density function of positive tail exponents for stocks and (c) spot or (d) future prices. The results are based on the 7128 stocks, 29 commodity spots, and 13 commodity futures analyzed. Observe that for both stocks and commodities the mean exponent is outside the Lévy-stable region $0<\alpha<2$.

ues of α for commodity futures and stocks. Interestingly, for commodity spot prices we find *smaller* exponent values for α indicating the presence of larger fluctuations and, possibly, different underlying mechanisms.

We thank S. V. Buldyrev, X. Gabaix, P. Gopikrishnan, V. Plerou, A. Schweiger and especially P. R. King for helpful discussions and suggestions; and BP/Amoco for financial support.

[1] P. Gopikrishnan *et al.*, Eur. Phys. J. B **3**, 139 (1998); Y. Liu *et al.*, Phys. Rev. E **60**, 1390 (1999); V. Plerou *et al.*, *ibid.* **60**, 6519 (1999).

[2] R.N. Mantegna and H.E. Stanley, Nature (London) **376**, 46 (1995); P. Gopikrishnan *et al.*, Phys. Rev. E **60**, 5305 (1999).

[3] M.M. Dacorogna *et al.*, J. Int'l Money and Finance **12**, 413 (1993).

[4] For stock markets the autocorrelation function decays exponentially to the noise level with time scales of minutes [1,2].

[5] T. Lux, Applied Financial Economics **6**, 463 (1996).

[6] A. Allison and D. Abbott (unpublished).

[7] U.A. Müller, M.M. Dacorogna, R.B. Olsen, O.V. Pictet, M. Schwarz, and C. Morgenegg, J. Banking Finance **14**, 1189 (1995).

[8] B.B. Mandelbrot, J. Business **36**, 394 (1963); **40**, 393 (1967); W. Working, Wheat Studies **9**, 187 (1933); Wheat Stud. Stanford Food Inst. **2**, 75 (1934).

[9] D. Gribbin *et al.*, J. Futures Markets **12**, 475 (1992); P. Kofman, in *Advances in Futures and Options Research Vol. 6*, edited by D. and R. Trippi (JAI Press, London, 1993); P. Kofman and C. de Vries, Rev. Futures Markets **8**, 244 (1989); S. Yang and B. Brorsen, J. Futures Markets **13**, 175 (1993).

[10] R. Weron, Physica A **285**, 127 (2000); B.M. Roehner, Eur. Phys. J. B **8**, 151 (1999); **13**, 175 (2000).

[11] A commodity can have two types of prices attached to it: namely, the spot and future prices. See http://www.investorwords.com/

[12] See http://www.platts.com and Datastream.

[13] B.M. Hill, Ann. Stat. **3**, 1163 (1975).

[14] P. Kearns, and A. Pagan, Rev. Econ. Stat. **79**, 171 (1997); R. Husman *et al.*, J. Bus. Eco. Stat. **19**, 208 (2001).

[15] Assuming the spot prices for all 29 commodities follow the same distribution we aggregate the data, this gives $\alpha=2.3\pm0.06$ for the positive and $\alpha=2.4\pm0.06$ for the negative tail likewise for 13 future prices we get $\alpha=3.2\pm0.09$ for the positive and $\alpha=3.1\pm0.09$ for the negative tail.

[16] M. McCulloch, J. Bus. Eco. Stat. **15**, 74 (1997).

[17] See http://www.crsp.com

[18] We apply the Kolmogorov-Smirnov test to the distributions of tail exponents and find (i) $p<10^{-4}$ for spot versus futures prices for commodities suggesting that the tail exponent for the distributions of returns are not identical and (ii) $p<0.24$ for stocks versus commodity futures suggesting that we cannot reject the hypothesis that the tail exponents of the distributions of returns are identical.

[19] C.K. Peng *et al.*, Phys. Rev. E **49**, 1685 (1994).