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Statistical analysis of bankrupting and non-bankrupting stocks

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Abstract – The recent financial crisis has caused extensive world-wide economic damage, affecting in particular those who invested in companies that eventually filed for bankruptcy. A better understanding of stocks that become bankrupt would be helpful in reducing risk in future investments. Economists have conducted extensive research on this topic, and here we ask whether statistical physics concepts and approaches may offer insights into pre-bankruptcy stock behavior. To this end, we study all 20092 stocks listed in US stock markets for the 20-year period 1989–2008, including 4223 (21 percent) that became bankrupt during that period. We find that, surprisingly, the distributions of the daily returns of those stocks that become bankrupt differ significantly from those that do not. Moreover, these differences are consistent for the entire period studied. We further study the relation between the distribution of returns and the length of time until bankruptcy, and observe that larger differences of the distribution of returns correlate with shorter time periods preceding bankruptcy. This behavior suggests that sharper fluctuations in the stock price occur when the stock is closer to bankruptcy. We also analyze the cross-correlations between the return and the trading volume, and find that stocks approaching bankruptcy tend to have larger return-volume cross-correlations than stocks that are not. Furthermore, the difference increases as bankruptcy approaches. We conclude that before a firm becomes bankrupt its stock exhibits unusual behavior that is statistically quantifiable.

Introduction. – How to predict bankruptcy before it occurs is an open challenge. The most recent financial crisis [1] was caused by sub-prime mortgages written in 2006, and it contributed to the Lehman demise in September 2008. The bankruptcies of many other corporations at that time also resulted in substantial losses to investors. The general consensus is that if we could accurately predict bankruptcy, *i.e.*, identify a characteristic behavior exhibited by a stock before bankruptcy, it would help investors avoid such losses. Thus bankruptcy prediction is a topic of great interest, not only to investors, but also to researchers across a wide range of fields.

Beginning as far back as 1966, the attempt to predict corporate failure has been an active topic of research [2–6]. Most of this research has attempted to predict bankruptcy by using such models as neural networks, logit, quadratic interval logit, support vector machine, and AdaBoost and Bankruptcy Risk [7–12], but these models depend upon the availability of detailed financial information about the corporation being studied [3,4,7–20]. Because it is often difficult to obtain accurate internal financial information about a corporation in a timely fashion, these forecasting models are of limited utility [16].

Here we attempt to understand a corporation's risk of bankruptcy by observing the market dynamics of the price of its stock. We hypothesize that because a stock price reflects the expectation of investors, an important factor in the pool of public information, analyzing stock price movement may provide important clues for predicting bankruptcy [21]. To test this hypothesis, we begin by examining data from the U.S. stock market, comparing the statistical properties of stocks approaching bankruptcy with those of stocks that are not [21–34]. The significant differences we find may prove useful in forecasting corporate bankruptcies.

Database and variables. – Using the database from The Center for Research in Security Prices (CRSP), we collect the daily closing share prices and trading volumes

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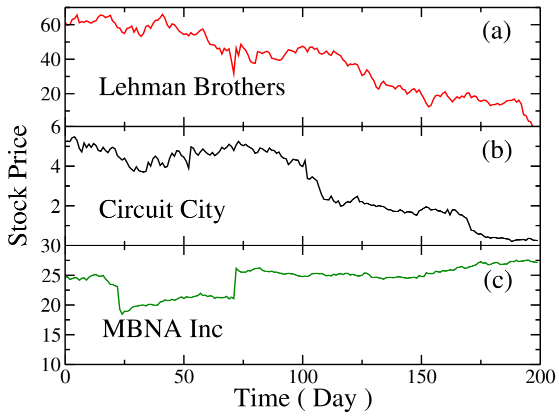

Fig. 1: (Color online) Price trends for three non-bankrupting stocks over 200 days for three different situations. (a) *Bankruptcy:* Lehman Brothers Holdings Inc. was a global financial service firm that declared bankruptcy in 2008. (b) *Delisting:* Circuit City Stores, Inc. was an American retailer that was delisted from NYSE (New York Stock Exchange) in 2008. (c) *M&A*: MBNA Corporation was the bank-holding company and the parent company of wholly-owned subsidiary MBNA America Bank, which was acquired by Bank of America in 2005.

of all 20092 securities listed in U.S. stock markets from 1 January 1989 to 31 December 2008. We choose this period because due to market rule changes, technology advances, and catastrophic events the market always evolves. Market behavior changed significantly for a period of time following the "Black Monday" market crash on 19 October 1987. In order to simplify our database, we "give it time to recover" from the crash and restart our examination approximately two years later. We also use daily data. These are more appropriate for our study than high-frequency intraday data, because the time frame for a bankruptcy procedure can extend over a period of months, and the effects of a bankruptcy do not quickly disappear.

During this 20-year period, 13249 stocks disappeared from the market due to bankruptcy, delisting, and mergers and acquisitions (M&A). Investors attempt to avoid stocks that go bankrupt or are delisted, especially when their demise appears imminent. Here we fold in the delisted stocks with those that become bankrupt (both lose the investor money) and focus our analysis on them. On the other hand, M&A are generally good news for investors, since after a M&A the stock price usually increases.

Figure 1 shows the typical trends of prices for stocks experiencing bankruptcy, delisting, and M&A during their last days in the market. We find the price trends for bankruptcy and delisting to be quite similar: they both fall until they disappear. The price trend for stocks undergoing M&A, on the other hand, increases. Thus we define a stock as bankrupting if it satisfies two requirements:

i) the stock has more than 100 days of trading records (in order to get more reliable results), and

ii) the stock price drops more than 20 percent during the previous 100 trading days.

Within this 100-day period, 4223 stocks became bankrupt, accounting for more than 20 percent of the 20092 stocks analyzed. We proceed by taking into account two basic quantities for individual companies: i) market capitalization $S(t)$, defined as the share price multiplied by the number of outstanding shares for one trading day, and ii) the market trading volume of one trading day.

We also define two basic quantities [25,26]: the daily return $R(t)$ is the logarithmic change of the successive market capitalization (which also accounts for the changes in the number of outstanding shares),

$$R(t) \equiv \log\left[\frac{S(t+1)}{S(t)}\right], \qquad (1)$$

and the volatility $V(t)$ we define as the absolute value of the return,

$$V(t) \equiv |R(t)|. \qquad (2)$$

Throughout this paper, a day means a trading day in the market, so 22 days are actually spread over a calendar month. Since we focus on the behavior of bankrupting stocks immediately before their bankruptcy, we count the time backward from the bankruptcy date. For example, the wording *22 days* means that the stock will become bankrupt in 22 days.

Yearly number of bankrupting stocks. – Stock markets are directly or indirectly influenced by large real-life events and, after a measurable time delay, will respond to them. Investors are particularly interested in reported negative events, and take them into account as they attempt to avoid big losses. In the most extreme situations, bad news will lead to bankruptcy, so detailed research about the behavior just before bankruptcy is important.

We first examine the yearly number of bankrupting stocks. This will give us a picture of how the stock market responds to bad news. In fig. 2 one sees substantial fluctuations in the fraction of yearly bankruptcies. The peak during the year 1992–1993 is due to the recession of the early 1990s, which hit much of the world in 1990–1991. Particularly for the US, the recession was largely caused by the "savings and loan crisis", which slowed the growth of the gross domestic product (GDP) until late 1992. The stock market did not respond to the recession immediately, and the peak in bankruptcies occurred almost one year later. History sometimes repeats itself, and a similar situation occurred again in the late 1990s. The period 1997–2001 (see fig. 2) was the well-known speculative "dot-com bubble" (often called the "I.T. bubble"). The NASDAQ Composite Index reached a peak of 5132 on 10 March 2000, and then fell dramatically during the remainder of 2002. The stock market responded approximately one year later. We find this kind of "delayed response" again in the most recent financial crisis of 2006, as the

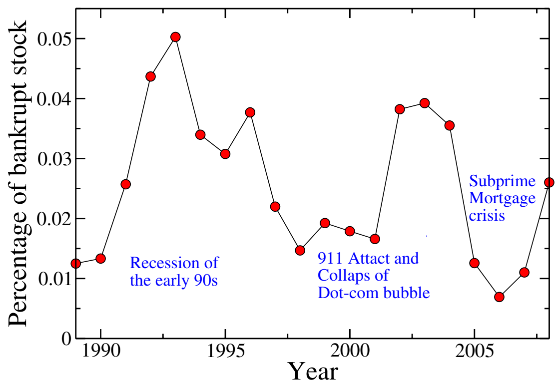

Fig. 2: (Color online) Fraction of bankrupting stocks in each year of the 20-year period analyzed, 1989–2008.

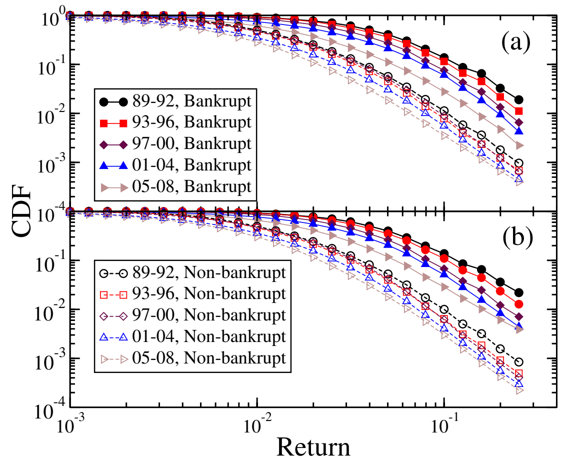

Fig. 3: (Color online) CDF of both non-bankrupting and bankrupting stock returns for each four-year period from 1989 to 2008: (a) positive returns and (b) negative returns. The solid dots represent bankrupting stocks while the empty dots represent non-bankrupting stocks and the same shape dots represent the same period of time. For both positive and negative returns, the curves have the same trend: curves for bankrupting stocks are always more pronounced than those for non-bankrupting stocks. And also as the time evolves, the curves for both bankrupting and non-bankrupting stocks tend to become less pronounced, indicating that the stock market is becoming more stable.

stock market did not begin to show significant bankruptcies until the beginning of 2007. Thus, we conclude that stock markets tend to respond to strongly negative information after a time interval of order one year —as if there were a one-year "buffering period" during which stocks struggle to survive. Some important questions arise:

i) How do stocks that go bankrupt differ statistically from those that do not?

ii) Are there quantifiable signs that emerge before bankruptcy?

Distribution of returns. – To answer these questions, we start by considering the tails of the distribution of returns. We do this because the strength of the tails of a distribution indicates the pervasiveness of large fluctuations, and large fluctuations tend to be a significant driving force as bankruptcy approaches. For a stock approaching bankruptcy, negative returns play a much stronger role than positive returns. For a stock that does not suffer bankruptcy, both negative and positive returns are significant. It is a stylized fact of econophysics research that the cumulative distribution function (CDF) of returns exhibits fat tails that are usually characterized by a power law [22].

In fig. 3, we plot the CDF of positive (daily) returns and negative (daily) returns for both bankrupting and non-bankrupting stocks taken in four-year periods beginning in 1989.

1) In the case of both bankrupting and non-bankrupting stocks, the tendency in both positive and negative returns is similar, indicating that the basic structure of the stock market is symmetric over the entire period.

2) We notice two trends:

   i) During each four-year period, for both positive and negative returns, the bankrupting stocks are more likely to have larger returns than non-bankrupting stocks. For example, the bankrupting stocks have 10 times larger probability than non-bankrupting stocks to have returns >0.1 and the price fluctuations for bankrupting stocks are more violent than those of non-bankrupting stocks. Also, when we compare the negative and positive returns, we find that negative returns exhibit a bigger difference between non-bankrupting and bankrupting stocks for large returns, which indicates that negative returns play a more important role as bankruptcy approaches.

   ii) During each four-year period, for both bankrupting and non-bankrupting stocks, the probability of having large returns decreases as the time passes. After the crash of 1987, the entire stock market became progressively more mature and stable (as the number of stocks increased every year). This confirms the result in fig. 2, where the peak of the number of bankrupting stocks decreases with time.

If stocks approaching bankruptcy have a higher probability of exhibiting large returns, we need to know exactly when these large returns begin to occur —immediately prior to bankruptcy or several months earlier? Figure 4

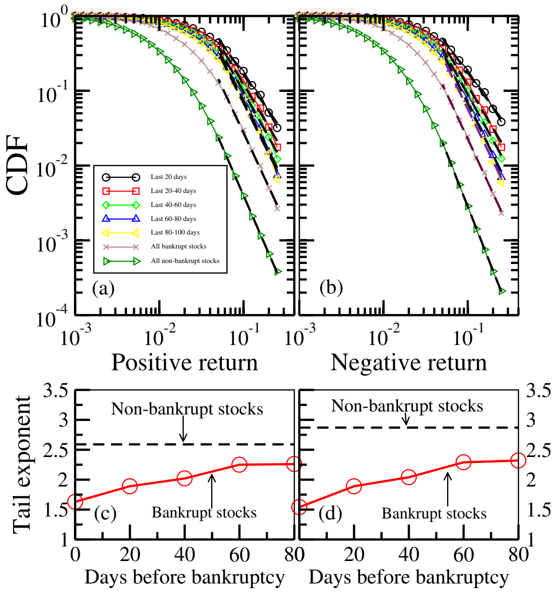

Fig. 4: (Color online) CDF of stock return for different periods: (a) positive returns and (b) negative returns. Here we set the time to start at the bankruptcy day and count backwards. The dashed lines are power-law fits. Both positive and negative returns have a similar tendency: the closer to the bankrupt day, the larger possibility to have large returns. The insets show the relation between power-law tail exponent and time: The dashed line represents the tail exponent for non-bankrupting stocks (corresponding to all non-bankrupting stocks as depicted in the legend), the curve below is approaching the dashed line as the time increases. In the abscissa of (c) and (d), 0, 20, 40, 60, and 80 respectively correspond to previous 20 days, previous 20–40 days, previous 40–60 days, previous 60–80 days, and previous 80–100 days, as indicated in the legend.

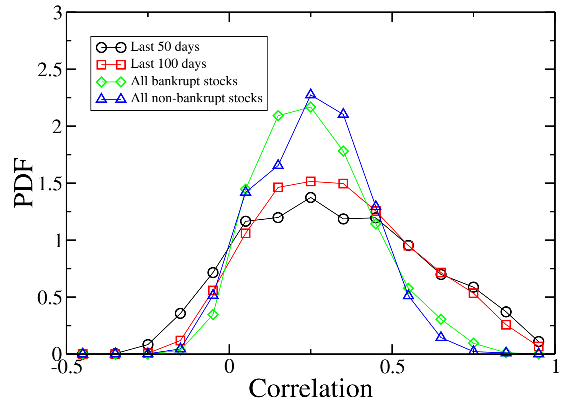

Fig. 5: (Color online) PDF of correlation between volatility and volume. For the curve of bankrupting stocks within last 50 days, the mean value of the PDF is 0.33 which is similar to that within last 100 days with mean value 0.34. For all bankrupting stocks within the full time period, their PDF is similar with all non-bankrupting stocks, and both have a mean value near 0.25. We also calculate the mean values for bankrupting stocks within last 10 and 20 days, which appear to be similar to the last 50 days (not shown).

shows the time-dependent CDF of both positive returns and negative returns for all bankrupting stocks from 1989 to 2008, using a time window of 20 days. We begin at the day of bankruptcy, shift the 20-day time window backward, and find that for both positive and negative returns the closer we approach the day of bankruptcy, the greater the possibility that the absolute values of returns will be large. When a stock approaches bankruptcy, its price changes will become increasingly dramatic. Thus we can treat this as a sign of impending bankruptcy, fit the tail of the curves, and draw a time-dependent tail exponent, see figs. 4(c) and (d). The tail exponents continue to increase, approaching the values exhibited by non-bankrupting stock as time increases. A smaller tail exponent indicates a greater possibility of having large returns, again indicating that large absolute values of returns tend to occur immediately prior to bankruptcy.

Cross-correlation between volatility and volume. – Previous research has shown that volatility and volume exhibit positive cross-correlations, which means that large changes in stock price are more commonly accompanied by large changes in trading volume [23–26]. How does this affect stocks approaching bankruptcy? Do these cross-correlations change for stocks approaching bankruptcy in ways they do not for non-bankrupting stocks?

Figure 5 shows the probability distribution function (PDF) of cross-correlations for both bankrupting stocks and non-bankrupting stocks. We find that if we consider the entire life of a stock, the PDFs for both bankrupting and non-bankrupting stocks are very similar. They all have approximately the same half-height width and the same mean value. However, comparing bankrupting stocks with non-bankrupting stocks, the results are quite interesting if we consider only the most recent months. We find that, owing to the increasing half-height width, the mean value increases, which shows that when a stock is approaching bankruptcy the volatility and volume are more strongly correlated. The explanation for this is intuitively obvious. First, as Fischer Black once suggested, volatility tends to increase as bankruptcy approaches because the threat of bankruptcy causes the stock price to drop, which reduces the equity value of the company and thus increases its financial leverage —and the larger the financial leverage, the more volatile the equity value. Second, the increased trading volume reflects the increase in speculative transactions by uninformed traders as well as the profit-taking dumping of the stock by informed insiders who are anticipating the company's demise. Thus when both volatility and volume increase as bankruptcy approaches, the two are obviously more strongly correlated. So we can say that a large return followed by a large trading volume is another statistical indication of approaching bankruptcy.

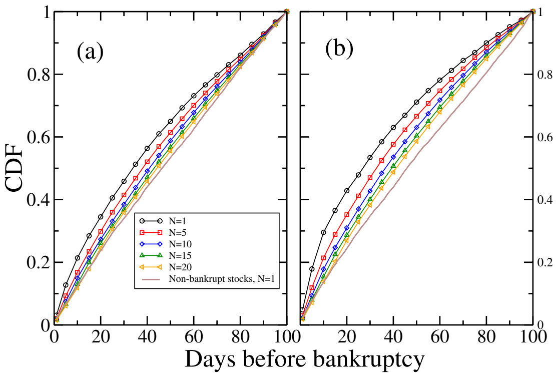

Fig. 6: (Color online) CDF of the N ($N=1$, 5, 10, 15, and 20) largest (a) positive and (b) negative returns during the last 100 days before bankruptcy date for all bankrupting stocks during 1989–2008. For comparison, we randomly choose 100 continuous days for all non-bankrupting stocks, and plot the CDF for the date when the largest return happened (non-bankrupting stocks, $N=1$).

Final 100 days before bankruptcy. – Since the above results show that bankrupting stocks have different statistical properties during their final days, we undertake more detailed research about the "final days returns" of bankrupting stocks. Figure 6 shows the CDF of the top N ($N=1$, 5, 10, 15, and 20) largest returns for bankrupting stocks in the last 100 days. For comparison, we also compute the CDF of the top N largest returns for non-bankrupting stocks for randomly chosen 100-day intervals, but here we only display the CDF of the top returns for non-bankrupting stocks since all CDF for non-bankrupting stocks is a straight line. Figure 6 shows that the largest one-day return volumes for the non-bankrupting stocks tend to be evenly spread over any given 100-day period. In contrast, as a stock approaches bankruptcy, the top N ($N=1$, 5, 10, 15, and 20) largest returns are more likely to occur close to the day of bankruptcy. Also, as the number N increases, the CDF curve of the bankrupting stocks approaches that of the non-bankrupting stocks, confirming what we show in fig. 5 —that the larger returns tend to occur during the final days of a bankrupting stock.

Discussion. – We have used statistical physics analysis to uncover several ways in which stocks approaching bankruptcy differ from non-bankrupting stocks. The tails of the distribution of returns differ significantly. In stocks approaching bankruptcy, unusually large returns are exhibited, followed by an unusually large trading volume —a behavior that is a sign of impending bankruptcy. Our analysis of stock behavior does not depend upon the availability of a firm's internal financial information, and thus can be regarded as a more reliable indicator than analyses that depend upon such information. This allows us to distinguish bankrupting stocks from non-bankrupting stocks based on the historical data alone. Further study will be required before a fully developed, reliable "early warning system" is able to precisely indicate times of bankruptcy.

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REFERENCES

[1] Jacoby M. B., *Bankruptcy Reform and the Financial Crisis* (North Carolina Banking Institute) 2009.
[2] Alfaro E., Garcia N., Gamez M. and Elizondo D., *Decis. Support Syst.*, **45** (2008) 110.
[3] Beaver W. H., *J. Account. Res.*, **4** (1966) 71.
[4] Altman E. I., *J. Finance*, **23** (1968) 589.
[5] Clark T. A. and Weinstein M. I., *J. Finance*, **38** (1983) 489.
[6] Duffee G. R., *J. Financ. Econ.*, **37** (1995) 399.
[7] Tseng F. M. and Hu Y. C., *Expert Syst. Appl.*, **3** (2010) 1846.
[8] Li H., Sun J. and Sun B.-L., *Expert Syst. Appl.*, **1** (2009) 643.
[9] Yoon J. S. and Kwon Y. S., *Expert Syst. Appl.*, **5** (2010) 3624.
[10] Tsai C. F., *Knowledge-Based Syst.*, **2** (2009) 120.
[11] Freund Y. and Schapire R. E., *Proceedings of the 13th International Conference on Machine Learning* (Morgan Kaufmann, San Francisco) 1996, pp. 148–156.
[12] Podobnik B., Horvatic D., Petersen A. M., Urosevic B. and Stanley H. E., *Proc. Natl. Acad. Sci. U.S.A.*, **107** (2010) 18325.
[13] Etemadi H., Rostamy A. A. A. and Dehkordi H. F., *Expert Syst. Appl.*, **36** (2009) 3199.
[14] Atiya A. F., *Trans. Neural Netw.*, **12** (2001) 929.
[15] Nwogugu M., *Appl .Math. Comput.*, **185** (2007)) 178.
[16] Mossman C. E., *Financ. Rev.*, **33** (1998) 35.
[17] Chava S. and Jarrow R. A., *Rev. Finance*, **8** (2004) 537.
[18] Shumway T., *J. Bus.*, **74** (2001) 101.
[19] Yu W., Gong H., Li Y. and Yue Y., *Commun. Comput. Inf. Sci.*, **35** (2009) 414.
[20] Wilson R. L. and Sharda R., *Decis. Support Syst.*, **11** (1994) 545.
[21] Beaver W. H., *J. Account. Res.*, **6** (1968) 179.
[22] Mantegna R. and Stanley H. E., *Introduction to Econophysics* (Cambridge University Press, Cambridge, UK) 2000.
[23] Yamasaki K., Muchnik L., Havlin S., Bunde A. and Stanley H. E., *Proc. Natl. Acad. Sci. U.S.A.*, **102** (2005) 9424.
[24] Wang F., Yamasaki K., Havlin S. and Stanley H. E., *Phys. Rev. E*, **73** (2006) 026117.
[25] Wang F., Shieh S.-J., Havlin S. and Stanley H. E., *Phys. Rev. E*, **79** (2009) 056109.

[26] Wang F., Yamasaki K., Havlin S. and Stanley H. E., *Phys. Rev. E*, **79** (2009) 016103.
[27] Wang F., Weber P., Yamasaki K., Havlin S. and Stanley H. E., *Eur. Phys. J. B*, **55** (2007) 123.
[28] Weber P., Wang F., Vodenska-Chitkushev I., Havlin S. and Stanley H. E., *Phys. Rev. E*, **76** (2007) 016109.
[29] Jung W.-S., Wang F., Havlin S., Kaizoji T., Moon H.-T. and Stanley H. E., *Eur. Phys. J. B*, **62** (2008) 113.
[30] Wang D., Podobnik B., Horvatić D. and Stanley H. E., *Phys. Rev. E*, **83** (2011) 046121.
[31] Podobnik B., Horvatic D., Petersen A. M. and Stanley H. E., *Proc. Natl. Acad. Sci. U.S.A.*, **106** (2009) 22079.
[32] Podobnik B. and Stanley H. E., *Phys. Rev. Lett.*, **100** (2008) 084102.
[33] Plerou V. and Stanley H. E., *Phys. Rev. E*, **76** (2007) 046109.
[34] Gabaix X., Gopikrishnan P., Plerou V. and Stanley H. E., *J. Eur. Econ. Assoc.*, **5** (2007) 564.