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Strategy for stopping failure cascades in interdependent networks

Cristian E. La Rocca [a,*], H. Eugene Stanley [b], Lidia A. Braunstein [a,b]

[a] *Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad Nacional de Mar del Plata and Instituto de Investigaciones Físicas de Mar del Plata (IFIMAR-CONICET), Deán Funes 3350, 7600 Mar del Plata, Argentina*
[b] *Physics Department and Center for Polymer Studies, Boston University, Boston, MA 02215, USA*

HIGHLIGHTS

- A cascading failures propagates in a system composed by two interdependent networks.
- Each network fragments in clusters and the biggest one represents the functional (GC).
- In one network we reconnect with certain probability each finite cluster to the GC.
- Networks becomes more resilient to the cascading failures with the strategy.
- Strategy applied in the network with the lowest average degree is more efficient.

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ABSTRACT

Interdependencies are ubiquitous throughout the world. Every real-world system interacts with and is dependent on other systems, and this interdependency affects their performance. In particular, interdependencies among networks make them vulnerable to failure cascades, the effects of which are often catastrophic. Failure propagation fragments network components, disconnects them, and may cause complete systemic failure. We propose a strategy of avoiding or at least mitigating the complete destruction of a system of interdependent networks experiencing a failure cascade. Starting with a fraction $1-p$ of failing nodes in one network, we reconnect with a probability γ every isolated component to a functional giant component (GC), the largest connected cluster. We find that as γ increases the resilience of the system to cascading failure also increases. We also find that our strategy is more effective when it is applied in a network of low average degree. We solve the problem theoretically using percolation theory, and we find that the solution agrees with simulation results.

1. Introduction

Interdependence is a characteristic of the world in which we live. We see this in such infrastructure networks as transportation systems, electrical power grids, natural gas and water systems, telephone systems, and the Internet. Thus, transportation networks depend on petroleum supplies and electrical power, the Internet on electrical power, and the control of natural gas and water systems on telecommunications. Interdependencies among networks can produce new systemic behaviors not seen in isolated networks, and these interdependencies can both enhance network functioning or increase its

* Corresponding author.
E-mail address: larocca@mdp.edu.ar (C.E. La Rocca).

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vulnerability to catastrophic failure. For example, a transportation network can increase the propagation rate of a disease epidemic, and if the network includes airlines the propagation can become world-wide.

We see a catastrophic example of interdependence in the after-effects of Hurricane Katrina in 2005 [1–3]. Several oil rigs and refineries were destroyed and this paralyzed oil and gas extractions for a number of months. This caused the price of fuel to rise exponentially, and this affected the airlines. Forest devastation affected the Mississippi logging industry, and there was a sharp drop in activity in the ports of Southern Louisiana and New Orleans, two of the largest in the EEUU. In addition to the thousands of homes that were destroyed, the loss of thousands of jobs meant that many owners of homes who had survived the hurricane were no longer able to pay their mortgages. Some insurance companies, because of the huge indemnities they had to face, either increased their homeowner fees or stopped insuring in the area altogether. It is thus clear that we need to understand how real-world interdependencies function. We need to know how to prevent, avoid, or mitigate the catastrophic failures they can magnify because interdependencies are everywhere.

Interdependent systems have recently been treated as networks of networks (NoN), i.e., systems in which two or more networks interact, and they have been successfully used to understand epidemic spreading [4–8], failure cascades [7–11], diffusion [7,8,12,13], and synchronization [7,14–17]. We characterize single networks in terms of their internal degree distribution $P(k)$, which is the probability that a node is connected to k, with $k_{\min} \leq k \leq k_{\max}$, where $k_{\min}$ and $k_{\max}$ are the minimum and maximum connectivities in the network, respectively. The interdependence between networks involves "external" dependency links that connect nodes in one network to nodes in a second network. These dependency links can strongly affect system robustness and can facilitate such catastrophic events as failure cascades [7–11,18] in which a node failure in one network propagates through dependency links and causes nodes in the other network to also fail. This occurred in the power outage in Italy on 28 September 2003. The shutdown of some power stations caused nodes in the communication network to fail, which in turn caused breakdowns in additional power stations [19].

In a model introduced by Buldyrev et al. [18] the authors studied the cascade failures in two interdependent networks by mapping the process as random node percolation, a process that is very important due to its ubiquitous application in failure cascades and the spread of disease. In random node percolation a fraction $1-p$ of nodes fail and the network fragments into clusters. The network remains functional if there is still a connected giant component (GC). The finite clusters remaining are considered dysfunctional. In the Buldyrev model nodes in the first network depend one-to-one on nodes in the second network. At the initial stage a fraction $1-p$ of nodes and all finite clusters are removed from the first network and, as a consequence, all the interdependent nodes in the second network also fail. At each time step we remove all finite clusters and their interdependent nodes in the other network until the system reaches the state in which there are no remaining finite clusters. If both networks still have functional clusters in this final "steady" state, they are of the same size and all their nodes are supported by nodes in the opposite network. They found a threshold p_c at which all functional components disappear. This threshold is higher than in isolated networks that have the same degree distribution, which implies that a NoN is less robust than isolated networks. More important is the nature of the phase transition characterized by the order parameter $P_\infty(p)$ (the relative size of the GC), which is first order while in isolated networks it is of second order [20]. In a first order transition the GCs overcome an abrupt transition from a finite value to zero at p_c and, as a consequence, it is more difficult to forecast or control the transition than in isolated networks.

After this pioneering work, many studies focused on modeling mitigation strategies of preventing the drastic consequences of the first order phase transition, such as autonomizing a fraction q of nodes [9,21–26]. Nevertheless, it is difficult and expensive to autonomize nodes in real NoN because their infrastructure were constructed not at random, but instead by economic reasons in order to accomplish efficiently some tasks. Recently Di Muro et al. [27] proposed a node recovery strategy in a system of interdependent networks that repairs, with probability γ, a fraction of failed nodes that are neighbors of the largest connected component in each network. They found that for a given initial failure of a fraction $1-p$ of nodes, there is a threshold probability of recovery above which the cascade stops and the system is restored to its initial state and below which the system abruptly collapses. They found three distinct phases: one in which the system never collapses without being restored, a second in which the recovery strategy avoids collapse, and a third in which the repairing process cannot prevent the collapse of the system. However, it is not always possible to repair the components of a network, and sometimes the repairing process requires so much time and so many resources that the system suffers total breakdown before it is completed. Thus, in this work we propose and study another strategy for preventing total network destruction after a failure cascade is initiated. The strategy is to save finite clusters prior to their failure by connecting them to the functional network component (GC).

2. Model and simulations

Using the process proposed by Buldyrev et al. [18] we model and use a strategy for mitigating a cascade as it begins. We consider two interdependent networks A and B with the same size N and with degree distributions $P_A(k)$ and $P_B(k)$, respectively. We use the Molloy–Reed algorithm [28], disallowing self loops and multiple connections, to construct each network. The interdependency between A and B we assume to be one-to-one, i.e., each node has only one dependency link. We apply the strategy only to network B – although it could be applied to either network – because the cascading is so rapid that we must choose a network in which applying the strategy is easier, such as a communication network, which is faster and less expensive to operate than a power grid.

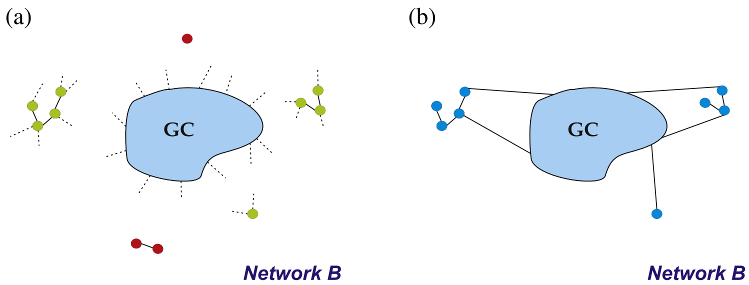

Fig. 1. Schematic of the implementation of the strategy in network B. The blue cluster represents the functional component (GC). (a) The green clusters represent the saved clusters and in red the clusters that could not be saved. The dashed lines represent the free links available to use for the reconnection of the finite clusters. (b) The new GC of network B.

At step $n = 0$ we remove a fraction $1 - p$ of nodes in network A and locate its GC. We then remove all nodes in A that belong to finite clusters, assuming that they have become dysfunctional due to lack of support, and we remove their interdependent nodes in B. We then apply the strategy of locating the GC in B and reconnecting to it every finite cluster in the same network with a probability γ (see Fig. 1). We do this by connecting two nodes in each existing finite cluster (clusters bigger than one node) with the GC. Single nodes we connect using one connection. We select two nodes rather than one when reconnecting to reduce the probability that the finite cluster will again disconnect from the GC. Note that although increasing the number of connected nodes increases network resilience, a greater number of connections requires greater economic resources, and cascading can be so rapid we are unable to achieve many reconnections. Note also that in order to preserve the initial degree distribution, the nodes we choose to reconnect must have free links, i.e., links that at earlier stages were connected to nodes that were already removed (dashed links in Fig. 1). This assumption allows us to map our model directly using node percolation theory. We assume that the clusters that are finite prior to the failure cascade will fail because they have no saveable free links.

The initial stage $n = 0$ ends when we remove with a probability $1 - \gamma$ the finite clusters in B that cannot be saved [red nodes in Fig. 1(a)]. Stage $n = 1$ begins when we propagate the failure from B back to A and remove all failed dependent nodes in B. We iterate this procedure until the system reaches the final "steady" state in which there are no remaining finite clusters. We denote by P_∞^α the relative size of the GC in network α, with $\alpha =$ A,B. Note that because nodes in one network depend one-to-one on nodes in the other network at the steady state, $P_\infty^{\rm A} = P_\infty^{\rm B} = P_\infty$. For the simulations we use an Erdös Rény (ER) random graph characterized by a Poisson degree distribution given by $P(k) = e^{-\langle k \rangle}\langle k \rangle^k/k!$, where $\langle k \rangle$ represents the average degree of the network, and a scale-free (SF) with cutoff with degree distribution $P(k) \sim k^{-\lambda} exp(-k/\beta)$, where λ is the broadness of the distribution and β is the cutoff in the connectivity. For all simulations all networks have $N = 10^6$ nodes, with a maximum connectivity $k_{\rm max} = 20$ and $\langle k \rangle = 8$ for the ER and $\lambda = 2.5$, $k_{\rm min} = 2$ and $k_{\rm max} = N^{1/2}$ [29–31] with $\beta = 20$ for the SF.

Fig. 2 plots P_∞ at the steady state as a function of p for different values of γ. Note that for $\gamma = 0$ we recover the results of Ref. [18], i.e., we obtain a first-order transition in which P_∞ jumps from a finite value to zero at p_c. As γ increases the threshold p_c decreases. This allows to the system to overcome a high level of damage before the two functional components collapse. There is still a first-order transition, but the jump in the relative size of the GC decreases as γ increases. Only when $\gamma = 1$ is there a continuous second-order transition. Note that the effect of the strategy is stronger in networks with a low average connectivity, such as for the SF networks which have an average degree $\langle k \rangle = 3.08$. Networks with a low average degree (the two SF) are more likely to fragment, which makes the system more vulnerable to failure cascades. Here and when $\gamma = 0$ the system is completely destroyed when 20 percent of its nodes fail, but using the strategy the system can sustain a higher level of damage before it collapses. The best value is $\gamma = 1$. Here the system fails only when 80 percent of its nodes fail.

When the two networks are different our strategy continues to increase systemic resilience to failure cascades, since the value of p_c decreases as γ increases. Note also that the best outcome occurs when the strategy is applied to the more fragile network – the one with the lowest average degree – in our case the SF network. Note also that when $\gamma = 0.5$ the ER–SF case with the strategy applied to the SF network has a p_c smaller than the SF–ER case with the strategy applied to the ER network. When we do not use the strategy ($\gamma = 0$), the p_c is the same for both cases and is unaffected by which network initiates the failure. However as γ increases the p_c for ER–SF and SF–ER cases increasingly differ, because it is more efficient to apply the strategy to the more fragile network, which fragments more easily and in which unsaved finite clusters ultimately fail.

3. Analytical results

Theoretically the cascading failure problem can be solved using node percolation [18,32]. In isolated networks we compute the relative size of the GC after removing a fraction $1 - \mu$ of nodes by solving the self-consistent equation for

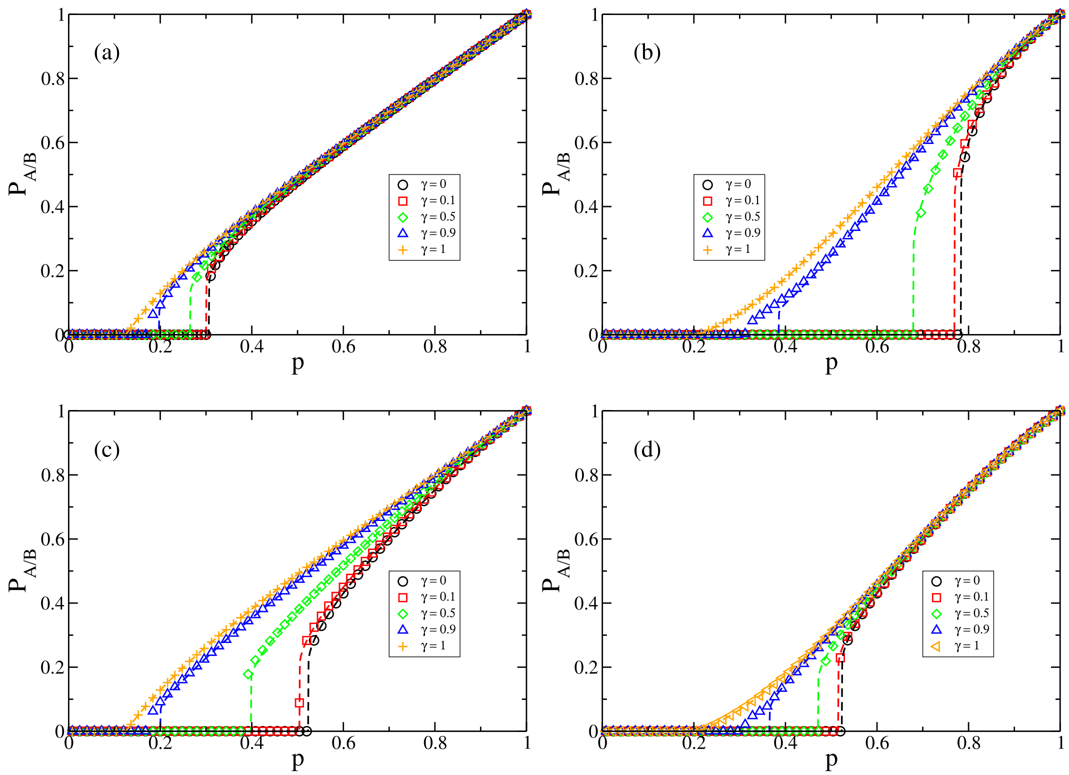

Fig. 2. P_∞ as a function of p for $\gamma = 0$ (◯), $\gamma = 0.1$ (□), $\gamma = 0.5$ (◇), $\gamma = 0.9$ (△) and $\gamma = 1$ (+) for (a) two ER, with $\langle k \rangle = 8$ (b) two SF with $\lambda = 2.5$ and $\beta = 20$, and the combination of the two kind of network ER and SF applying the strategy in (c) SF and (d) ER. The dashed curves represent the theory.

probability f. Choosing an edge of the network at random leads to a node connected to the GC, where f is

$$f = \mu(1 - G_1(1 - f)), \quad (1)$$

where $G_1(x) = \sum_{k=k_{\min}}^{k_{\max}} kP(k)/\langle k \rangle x^{k-1}$ is the generating function of the excess degree distribution [32,33], $\langle k \rangle$ is the average degree of the network, and μ is the effective fraction of remaining nodes. Thus the fraction of nodes in the GC is

$$P_\infty(\mu) = \mu(1 - G_0(1 - f)), \quad (2)$$

where $G_0(x) = \sum_{k=k_{\min}}^{k_{\max}} P(k)x^k$ is the generating function of the degree distribution [32,33]. For two interdependent networks, we start the cascading at stage $n = 0$ by removing a fraction $1 - p$ of nodes in network A ($\mu_A(0) = p$) and a corresponding fraction $S_A(0) = \mu_A(0) - P^A_\infty(0)$ of nodes in the finite clusters. The failure spreads to network B through interdependency links, thus $\mu_B(0) = P^A_\infty(0)$ and the fraction of nodes in the finite clusters in B is given by $S_B(0) = \mu_B(0) - P^B_\infty(0)$. The next stage begins when the failure $S_B(0)$ of nodes in network B returns to network A. For any step n in the cascading, the stage begins when a fraction $S_B(n - 1)$ of nodes fail in the GC of network A ($P^A_\infty(n - 1)$), which corresponds to the fraction of nodes belonging to finite clusters in B at step $n - 1$. Then the total fraction of nodes that fails in network A at step n is $\mu_A(n - 1)S_B(n - 1)/P^A_\infty(n - 1)$ and $\mu_A(n)$ is

$$\mu_A(n) = \mu_A(n - 1)\left[1 - \frac{S_B(n - 1)}{P^A_\infty(n - 1)}\right]. \quad (3)$$

Thus we obtain $P^A_\infty(n)$ using Eqs. (1) and (2), and the fraction of nodes that belongs to the finite clusters as a consequence of the fragmentation of $P^A_\infty(n - 1)$ is

$$S_A(n) = P^B_\infty(n - 1) - P^A_\infty(n). \quad (4)$$

Following the same procedure as in network A, we obtain

$$\mu_B(n) = \mu_B(n - 1)\left[1 - \frac{S_A(n)}{P^B_\infty(n - 1)}\right], \quad (5)$$

$$S_B(n) = P^A_\infty(n) - P^B_\infty(n), \quad (6)$$

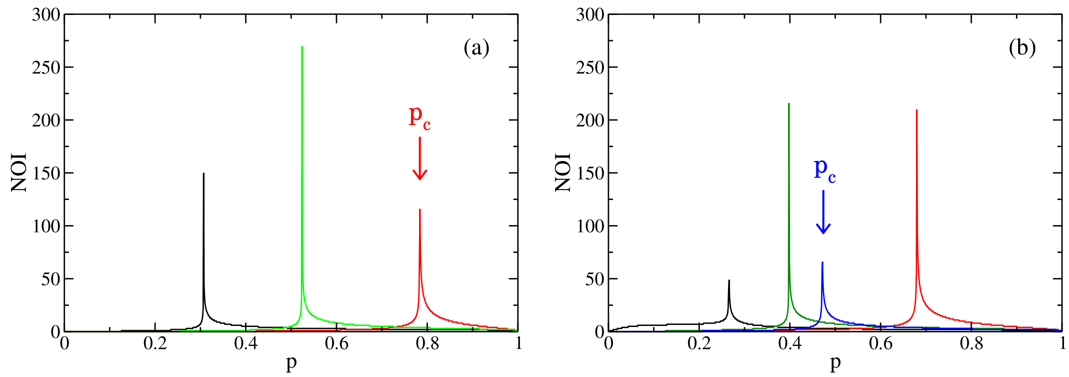

Fig. 3. NOI as a function of p from the theory for (a) $\gamma = 0$ and (b) $\gamma = 0.5$. Both cases are for two ER (black), two SF (red), ER–SF (green) and SF–ER (blue), always applying the strategy in B ($A - B$). Notice that in (a) the green curve represents the two cases ER–SF and SF–ER because without strategy the cascading failure is the same.

where $P_\infty^B(n)$ is derived using Eqs. (1) and (2). The process continues until the system reaches the steady state in which there are no finite clusters in either network $S_A = S_B = 0$ and both GCs are of the same size $P_\infty^A = P_\infty^B$.

The strategy is applied after the failure in A propagates to B and just prior to its return to A. We now save with a probability γ each finite cluster in B. Even when finite clusters have differing sizes and probabilities of existing, we assume that on average we save a fraction γS_B of nodes at each time step because the probability γ of saving each finite cluster is independent of its size (its number of nodes). We begin at stage n in network A with failure $(1-\gamma)S_B(n-1)$, which corresponds to the finite clusters in B that could not be saved during the previous step. Thus we rewrite Eq. (3) to be

$$\mu_A(n) = \mu_A(n-1)\left[1 - \frac{(1-\gamma)S_B(n-1)}{P_\infty^A(n-1)}\right] . \tag{7}$$

The cascading process evolves following Eqs. (4)–(6), with Eqs. (1) and (2). We apply our strategy and save a fraction of nodes $\gamma S_B(n)$. Then the fraction of nodes belonging to the new GC in B is

$$P_\infty^B(n) = P_\infty^B(n) + \gamma S_B(n) , \tag{8}$$

and using Eqs. (1) and (2) we obtain the new fraction of nodes $\mu_B(n)$. The process continues until the system reaches the steady state at which $P_\infty^{A/B} < \epsilon$, where $\epsilon = 1/N$ is related to finite size effects. We iterate this theoretical process numerically.

Fig. 2 compares the simulation results (symbols) with the theoretical results (dashed line). Note that both agree. Only when γ values are close to 1 and near p_c do the simulation results differ slightly from the theoretical results. To explain this we compute theoretically the number of iterations (NOI) or stages required for the system to reach the steady state (see Fig. 3). We can see that the NOI generates a sharp peak at the critical threshold. The system requires many time steps to reach the steady state when p is close to p_c. When p moves away from p_c, only a few time steps are needed for the system to reach the steady state. The internal structures of the two new GCs in B (from simulation and theory) differ after we apply the strategy. After a few steps they remain similar at the steady state for values of p far from p_c. With internal structure we are referencing to the number and form in which are made the internal connections of each GC. Note that theoretically we obtain a new GC and the internal connections change, but in the simulations we only add two new connections to each saved finite cluster. For values of p close to p_c the NOI increases exponentially and the difference between both GCs increases. Finally the deviation becomes less noticeable as we move away from $\gamma = 1$ and near p_c where the NOI is high. This is the case because the probability of saving any finite cluster decreases as γ decreases, which means that only a few nodes are saved and the restructuring of the GC is slight.

Fig. 4 plots the phase diagram in the plane γ–p for the same cases as in Fig. 2: (a) two ER with $\langle k\rangle = 8$, (b) two SF with $\langle k\rangle = 3.08$, and combinations of (c) ER–SF and (d) SF–ER where the strategy is always applied to network B (A–B). The vertical line (dashed) is the p_c for $\gamma = 0$. To the right of the dashed line in Regime I the system remains functional, even when the strategy is not applied. The middle region, Regime II, is the zone in which the strategy is needed to avoid the total destruction of the system. Note that this region is much larger when the networks have a low average degree, e.g., the two SF with $\langle k\rangle = 3.08$. Fig. 4(b) (inset) plots the phase diagram for two SF with $\langle k\rangle = 8$ ($k_{\min} = 5$). Note that the plot is similar to the one in Fig. 4(a). Networks with a low average degree have nodes with fewer connections, which means that they are more prone to fragmentation and more likely to fail completely. Fig. 4(c) and (d) show that in ER–SF and SF–ER Regime II is broader in ER–SF, and we are more able to save the system when the strategy is applied to fragile networks, i.e., those with a low average degree. This can be seen more clearly in Fig. 3(b) where for the same γ the p_c is higher (blue curve) when the strategy is applied to the less fragile network (ER). As mentioned above, the network with the lower average degree is more prone to fragmentation, and its finite clusters ultimately fail. It is thus important to apply the strategy to the more fragile network with the lower average degree. Finally the continuous curve represents the γ_c values below which the networks

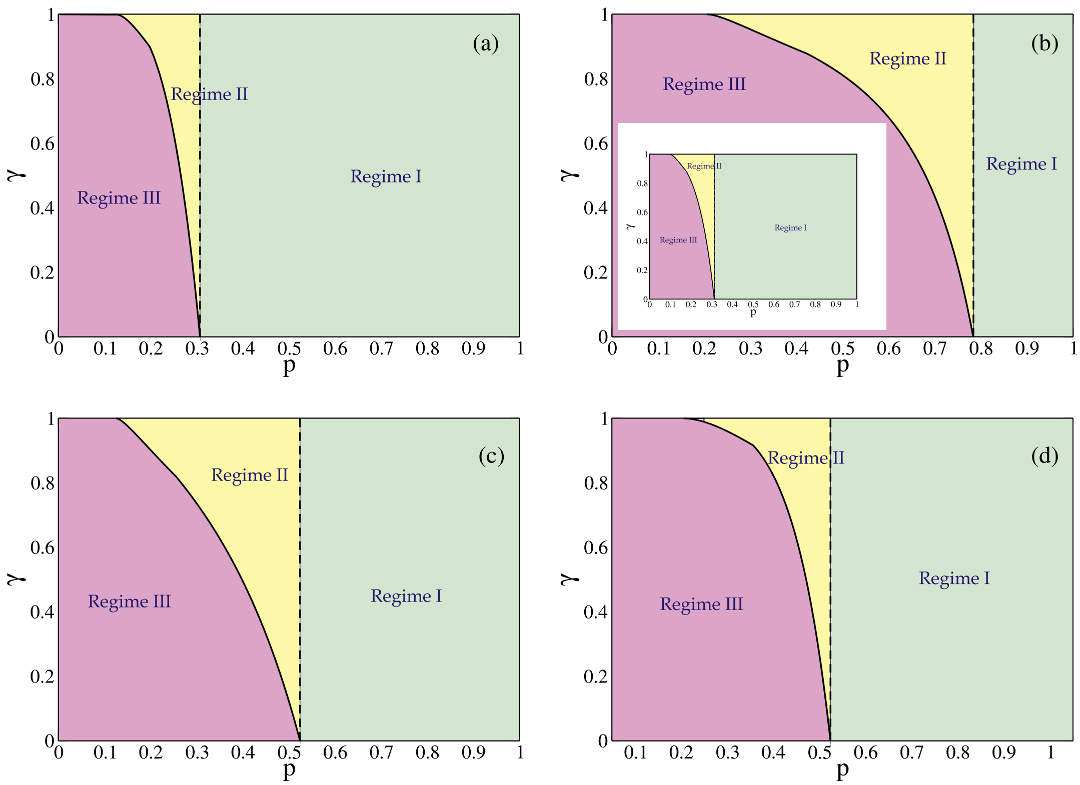

Fig. 4. Phase diagram in the plane γ–p from the theory for (a) two ER with $\langle k\rangle = 8$, (b) two SF with $\langle k\rangle = 3.08$ and the combination of and ER and SF networks applying the strategy in (c) SF and (d) ER. The continuous curve represents the value of γ_c below which the system is completely destroyed and the dashed line represents p_c for the case $\gamma = 0$. In the inset of (b) we plot the case for two SF, but with $k_{\min} = 5$ ($\langle k\rangle = 8$). Notice that the plot is very similar to the two ER case.

are completely destroyed (Regime III). We obtain this curve theoretically from the peaks in the NOI (see Fig. 3 for different γ values).

4. Conclusions

We have developed a strategy for avoiding the complete destruction of a system of two interdependent networks. We define network interconnections to be interdependent links connecting each node in the first network with its counterpart in the second. We apply our strategy to one network and prior to their failure and with probability γ save every finite cluster by connecting two of their nodes to the GC. We find that the system becomes increasingly robust to cascading failure as γ increases, and that the strategy is most effective when it is applied to the network with the lower average degree. We solve the problem theoretically using percolation theory and we find an agreement with the simulation results.

In future work we will study the implementation of the strategy in systems larger than two networks and where network interdependence is given by a degree distribution.

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