Percolation transition in dynamical traffic network with evolving critical bottlenecks

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A critical phenomenon is an intrinsic feature of traffic dynamics, during which transition between isolated local flows and global flows occurs. However, very little attention has been given to the question of how the local flows in the roads are organized collectively into a global city flow. Here we characterize this organization process of traffic as "traffic percolation," where the giant cluster of local flows disintegrates when the second largest cluster reaches its maximum. We find in real-time data of city road traffic that global traffic is dynamically composed of clusters of local flows, which are connected by bottleneck links. This organization evolves during a day with different bottleneck links appearing in different hours, but similar in the same hours in different days. A small improvement of critical bottleneck roads is found to benefit significantly the global traffic, providing a method to improve city traffic with low cost. Our results may provide insights on the relation between traffic dynamics and percolation, which can be useful for efficient transportation, epidemic control, and emergency evacuation.

emergence | percolation | traffic

Traffic, as a large-scale and complex dynamical system, has attracted much attention, especially on its dynamical transition between free flow and congestion (1–3). The dynamics of traffic have been studied using many types of models (4–11), ranging from models in macroscopic scales based on the kinetic gas theory or fluid dynamics to approaches in microscopic scales with equations for each car in the network. However, there is still a gap between the microscopic behavior of individual vehicles and the emergence of macroscopic city traffic. Indeed, a fundamental question has rarely been addressed: how the local flows in roads interact and organize collectively into global flow throughout the city network. This knowledge is not only necessary to bridge the gap between local traffic and global traffic, but also essential for developing efficient traffic control strategies.

There are mainly two obstacles in studying how the collective network dynamics of real traffic emerge from local flows. The first obstacle is the lack of valid methods to quantify the dynamical organization of traffic in the road network. The second is the lack of data on traffic dynamics in a network scale. To overcome the first obstacle, we develop here a quantitative framework based on percolation theory, which combines evolving traffic dynamics with network structure. In this framework, instead of the commonly used structural topology, only roads in the network with speed larger than a variable threshold are considered functionally connected. In this way, we can characterize and understand the formation process of traffic dynamics.

To overcome the second obstacle of missing data on a network scale and understand the organization processes of real traffic in a network, we collected and analyzed velocities of more than 1,000 roads with 5-min segments records measured in a road network in a central area of Beijing (Fig. 1*A*). This area of more than 22 km^2 contains the largest train station in Beijing and is considered a typical region showing transition between free flow and congestions. The data cover a time span of 2 wk in 2013. For the road network, nodes represent the intersections and edges represent the road segments between two intersections. For each road, the velocity $v_{ij}(t)$ varies during a day according to real-time traffic. For each road e_{ij}, we set the 95th percentile of its velocity in each day as its limited maximal velocity and define $r_{ij}(t)$ as the ratio between its current velocity and its limited maximal velocity measured for that day (Fig. 1*B* and *SI Appendix*, Fig. S1). For a given threshold q, the road e_{ij} can be classified into two categories: functional when $r_{ij} > q$ and dysfunctional for $r_{ij} < q$,

$$e_{ij} = \begin{cases} 1, & r_{ij} \geq q \\ 0, & r_{ij} < q. \end{cases} \quad [1]$$

In this way, a functional traffic network can be constructed for a given q value from the traffic dynamics of the original road network, which becomes more diluted as the value of q increases.

Results

To observe the emergence of global city traffic in the network scale at a given time, we can vary the value of q and study the formation process of the dynamical traffic network. For $q = 0$, the traffic network is the same as the original road network and for $q = 1$ it becomes completely fragmented. For a certain value of q, the hierarchical organization of traffic in different scales emerges, where only clusters of roads with r_{ij} higher than q appear (clusters in Fig. 2 *A*–*C*). These clusters represent functional modules composed of connected roads with speed higher than q. For example, during a typical lunchtime instant, for $q = 0.69$, as

Significance

The transition between free flow and congestions in traffic can be observed in our daily life. Although this traffic phenomenon is well studied in highways, traffic in a network scale (representing a city) is far from being understood. A fundamental unsolved question is how the global flow in a city is being integrated from local flows. Here, we identify a fundamental mechanism of traffic organization in a network scale as a percolation process, and we show how global traffic breaks down when identified bottlenecks are congested. These bottlenecks evolve with time according to traffic dynamics and are different from structural bottleneck links found by traditional network analysis. Improvement of traffic on these bottlenecks can significantly improve the global traffic.

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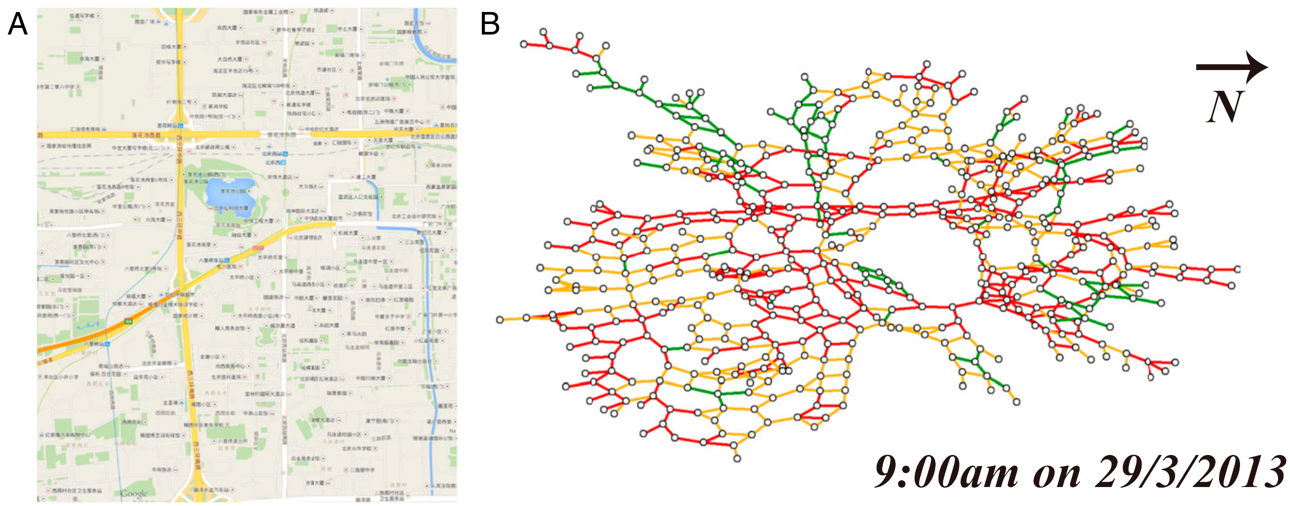

Fig. 1. Road network of the observed district. (*A*) Map of the investigated district. (*B*) Road network of the investigated district. Road network at 9:00 AM on March 29, 2013 is shown, where links are classified into three categories according to their velocity ratio r_{ij}: velocity ratio below 0.4 (red), between 0.4 and 0.7 (yellow), and above 0.7 (green). Note the clustering of each color.

shown in Fig. 2*A*, only small clusters of connected roads with high velocity emerge, which cannot maintain the global network traffic. As the value of q decreases to 0.19 in Fig. 2*C*, these small clusters merge together and a giant cluster is formed, where the functional network (with lower velocity) extends to almost the full scale of original road network. For $q = 0.38$ (Fig. 2*B*), the size of second-largest cluster becomes maximal, which signifies the phase transition point for network connectivity of a functional traffic network, according to percolation theory (12, 13). This percolation-like process can be better understood in Fig. 2*D* (more examples in *SI Appendix*, Fig. S4). As q increases, the size of the giant component decreases, and the second-largest cluster reaches a maximum at the critical threshold (q_c) separating the fragmented phase from the connected phase of the traffic network.

As an indicator of the robustness characteristics of network connectivity (14–20), the critical threshold q_c in this percolation-like process here quantifies the organization efficiency of real traffic. An individual car can travel most of the city (giant component of traffic network) only with velocity below q_c, whereas this car will be trapped in small isolated clusters when it drives with velocity above q_c. Hence, q_c measures effectively the maximal relative velocity one can travel over the main part of a network, which reflects the global efficiency of traffic in a network view.

Due to the traffic evolution, q_c is found to change dramatically during the day as seen in Fig. 2*E* (details in *SI Appendix*, Fig. S5). In a typical working day, q_c is found to be maximal from about midnight until 5:30 AM, indicating that the whole road network can function with high velocity. Close to 6:00 AM, q_c begins to drop abruptly and shows a minimum around 8:00 AM corresponding to morning rush hours in Beijing. There are usually two local minima during a typical working day, which are around 8:00 AM and 6:00 PM. Note that q_c reaches an intermediate level around noon, 12:00 PM, which might correspond to a possible third phase between free phase and congested phase. Due to the diverse commuting habit during weekends, only one local minimum appears in weekends around 2:00 PM (Fig. 2*E*).

The network at percolation criticality has a very dilute structure and behaves as the "backbone" of the original network (21). In the backbone of the traffic network, we find some links (called "red bonds" in percolation) that play a critical role in bridging different functional clusters of traffic. Therefore, these bridging links can be considered as bottlenecks because their velocities are lowest with respect to the whole backbone and q_c is determined according to their value. We identify the bottleneck links of the traffic network by comparing the functional network just below and immediately above the criticality threshold. Fig. 3 *A* and *B* demonstrates the links removed at criticality, q_c, showing that they can disintegrate the giant cluster and result in a maximal second-largest cluster. Some of these links connect different traffic clusters and are thus considered bottlenecks. Because the roads in the real data are directed, we define the connected component as the "strongly connected component" (22, 23), in which all pairs of nodes are mutually reachable from each other along a directed path. Therefore, removal of two roads in Fig. 3*A* will lead to loss of directed paths bridging different clusters and disintegration of the giant strongly connected component.

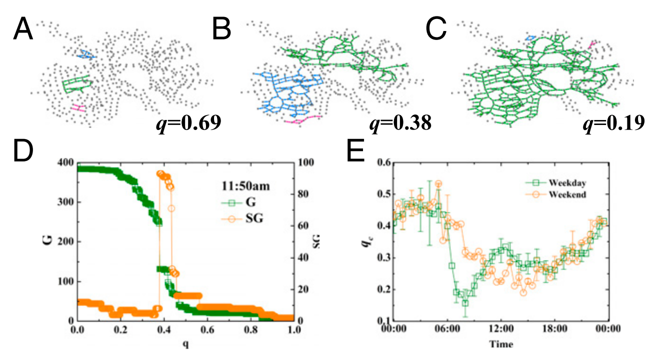

Fig. 2. Percolation of traffic networks: Traffic networks during the noon period (at 11:50 AM on March 27) for three q values corresponding to different connectivity states. *A*, *B*, and *C* exhibit the traffic networks under different q values with 0.69, 0.38, and 0.19 representing the states of high-, medium-, and low-velocity thresholds, respectively. For clarity, only the largest three clusters are plotted, which are marked in green (largest cluster), blue (second-largest cluster), and strawberry (third-largest cluster). Here the clusters are strongly connected components, considering road direction (more details in *SI Appendix*). (*D*) Size of the largest cluster (G) and the second-largest cluster (SG) of traffic networks as a function of q (more examples in *SI Appendix*). Critical value, q_c, is determined as the q value when SG becomes maximal. (*E*) q_c as a function of time, averaged separately over nine weekdays and two weekends.

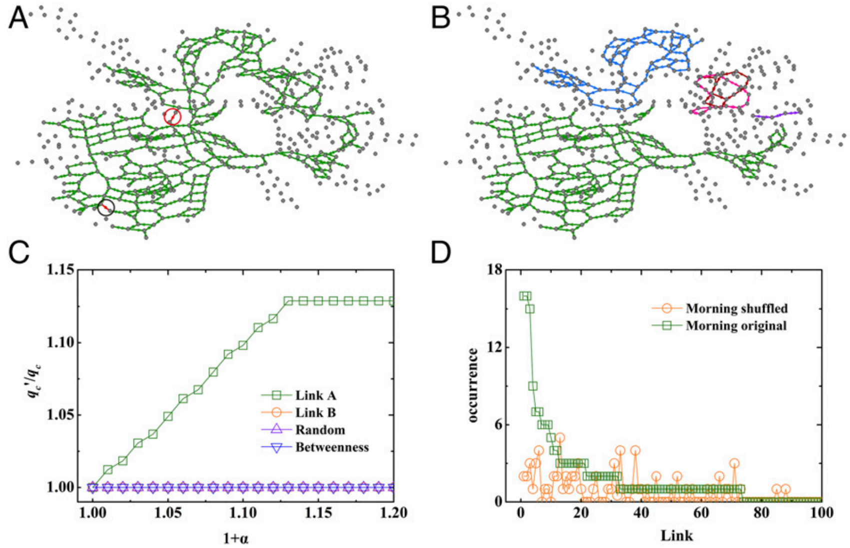

Fig. 3. Bottleneck links of a traffic network. (*A*) A typical example of a traffic network just below criticality, where two links (in red within red or black circles) are removed at criticality. Removal of them will disintegrate the giant functional network. (*B*) Same traffic network after removal of the two links, where the giant functional cluster is disintegrated into five clusters. We find all strongly connected clusters of the traffic network for each q and identify the links removed at threshold q_c when the second-largest strongly connected cluster reaches a maximum. Although some of these links are removed by chance, a few links do play a critical role of bridging different traffic clusters of higher velocities. These bridging links are identified as bottleneck links, because when increasing their velocity large clusters can join together to become the largest component (more details in *SI Appendix*, Fig. S7). (*C*) The improvement of q_c by increasing separately the ratio ($r_{ij}' = r_{ij}(1+\alpha)$) of two links marked in *A*, within which improvement of q_c can be achieved only with one (marked with red circle) of them. This link is considered a bottleneck link for global traffic. This is compared with the improvement of one link randomly chosen and the link with highest betweenness. (*D*) Zipf plot of occurrence times of links as bottlenecks during morning rush hours. It is compared with occurrence times of bottlenecks in the same network with shuffled values of r_{ij} during morning rush hours. For the shuffled case, we shuffle the r_{ij} values 100,000 times at each instant and find the bottleneck links with the same method.

To observe the impact of bottleneck links on the global traffic, we increase the road velocity (r_{ij}) of bottleneck links by a factor of $1+\alpha$ ($\alpha > 0$) and measure the new q_c of the modified traffic network. It can be seen in Fig. 3*C* that q_c of the traffic network is significantly increased only through increasing the velocity of the bottleneck link (more examples in *SI Appendix*, Fig. S7). This improvement of global traffic is dramatically higher compared with the improvement when increasing the velocity of a single link chosen randomly. Surprisingly, the improvement of q_c is negligible when the velocity of the link with the highest hop-count–based betweenness is increased, although such a link is usually considered a bottleneck link because it bridges different topological communities (24, 25). This suggests that the bottleneck links found in our dynamical network are unique and different from results of network analysis based only on structural information. In addition, the links found based on weighted betweenness (26, 27) are also compared and found to be different from bottlenecks found by our method (*SI Appendix*, Fig. S12).

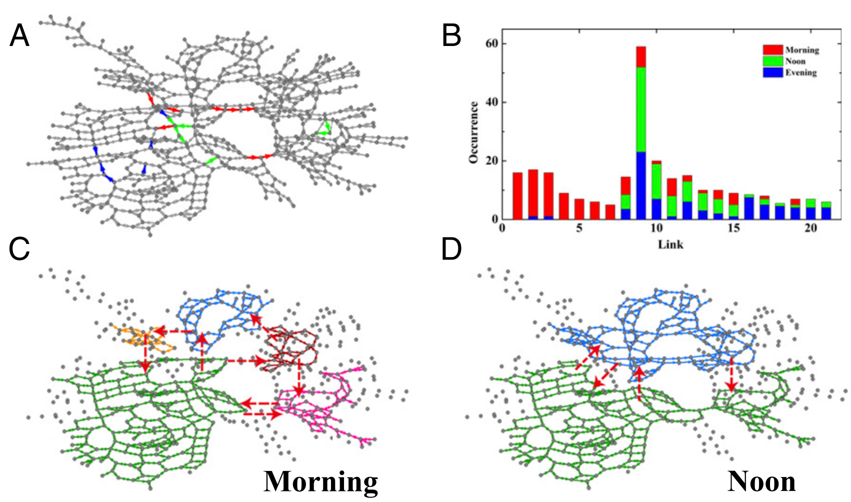

Fig. 4. Evolving bottlenecks in different periods in 1 d. (*A*) Bottleneck links with high occurrence in different periods are marked: morning (red), noon (green), and evening (blue). (*B*) The occurrence times of links (marked in *A*) as bottlenecks in different periods are plotted: morning (red), noon (green), and evening (blue). (*C* and *D*) The network breaks into several clusters after removal of bottlenecks with highest occurrence (top 10 in the morning in *C* or top 8 at noon in *D*). Red arrows in *C* and *D* are paths bridging different clusters, which are fragmented by the removal of bottleneck links.

Further discussion can be found in *SI Appendix*. The appearance of the bottleneck links is not accidental. As shown in Fig. 3*D* (*SI Appendix*, Fig. S8), some bottleneck links appear to be much more frequent than the random case. The high occurrence of bottleneck links demonstrates the dynamical percolation feature of real traffic and shows that the approach could be useful for significantly improving city traffic. Note that these bottleneck links with high occurrence are different from those found in the shuffled case, which reflect only the structural feature of the road network.

Static bottleneck links of a network are identified usually based on structural information (28–33), by considering links that are critical for network connectivity. However, traffic is a dynamical nonequilibrium system, which evolves with time as a result of collective individual competition. Therefore, we expect that bottlenecks of global traffic will also evolve accordingly, different from those found by structural methods. From the bottleneck links identified in different hours during a typical day (Fig. 4 *A* and *B* and *SI Appendix*, Fig. S10), one can conclude that the bottleneck roads are essentially different in the morning, lunchtime, and evening rush hours. This is due to the different individual travel habits and interactions, which result in different global traffic patterns during different rush hours. As seen in Fig. 4*A*, in the morning, red bonds are distributed along a central path (city highway), whose congestion disintegrates the whole network into isolated clusters; however, in the evening hours, red bonds are distributed in less central roads, whose congestion influences only local areas, and the main part of the network stays functional. Indeed, as shown in Fig. 4*B*, the occurrence of links as bottlenecks changes dramatically from morning to evening rush hours; however, they appear repeatedly in different days in the same hours (*SI Appendix*, Fig. S11).

Bottleneck links result from the interactions among local functional clusters. Different bottleneck links signify distinct organization of global traffic in different hours. As shown in Fig. 4 *C* and *D*, traffic networks become disintegrated in different ways when different bottleneck links are removed. In Fig. 4*C*, removal of bottleneck links in the morning causes the giant cluster to break into one large cluster and four smaller clusters. In Fig. 4*D*, however, removal of bottleneck links at noon breaks the giant cluster into two clusters of similar size.

Conclusion

As we reveal the percolation feature in organization of real traffic, the percolation threshold can be considered a measure for traffic efficiency, which takes into account the interaction between roads' network structure and flow. This proposed framework enables us to identify instantaneously those roads bridging different traffic clusters of higher velocity (with respect to the bottleneck). These bottleneck links identified at q_c can provide opportunities to improve significantly the global network traffic with minor cost (e.g., improving a single road). Understanding the congestion formation and dissipation mechanisms in a network view through our framework can serve to predict and control traffic, in particular in the future realization of the "smart city." Particularly, our study can be useful in mitigating congestion (34) or traffic-driven epidemics (35) through certain self-healing algorithms (36) based on real-time information on traffic dynamics in the network.

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