Scaling of the Minimum Growth Probability for the "Typical" Diffusion-Limited Aggregation Configuration

Blumenfeld and Aharony[1] (BA) provide an interesting argument for the recently proposed[2,3] "phase transition" in the multifractal spectrum of diffusion-limited aggregation (DLA). Specifically, BA assume

$$p_{\min}(L) \sim \exp(-A_1 L^x), \tag{1}$$

where $p_{\min}(L)$ is the minimum growth probability of a typical DLA cluster of linear size L, while A_1 and x are size-independent positive constants. Then, the qth moment Z_q satisfies the inequality

$$Z_q \equiv \sum_i p_i^q \geq L^{d_{\min}} \exp(-qA_1 L^x), \tag{2}$$

where $d_{\min}$ is the fractal dimension of sites with minimum growth probabilities. Thus, for negative q, the moment Z_q should increase at least exponentially. BA thus conclude that there is a "breakdown" of power-law scaling for $q < 0$.

In this Comment, we discuss important differences between BA and Refs. 2 and 3. Based on exact enumeration data and on an analytic argument, Refs. 2 and 3 proposed a phase transition in the multifractal spectrum of *all the possible* DLA configurations. BA propose a phase transition in the multifractal spectrum of *typical* DLA configurations. Although the existence of the BA transition guarantees the existence of the transition in Refs. 2 and 3, the converse is not true.

This point can be well illustrated by considering $p'_{\min}(L)$, the minimum growth probability of *all the possible* DLA configurations with size L. In Refs. 2 and 3, the relation

$$p'_{\min}(L) \sim \exp(-A_2 L^2) \tag{3}$$

was found, where A_2 is a size-independent constant. It is also found that $p'_{\min}(L)$ arises from configurations which are not *typical* DLA configurations.

In Fig. 1, we show one of the configurations which has $p = p'_{\min}(L)$ for $L=5$. For the random walker, this configuration is equivalent to a narrow tunnel. The length of the tunnel scales as L^2, and the growth probability decreases exponentially with the length of the tunnel. Therefore, $p'_{\min}(L) \sim \exp(-A_2 L^2)$. In general, $p'_{\min}(L)$ comes from the configurations which are topologically equivalent to such "tunnel configurations."

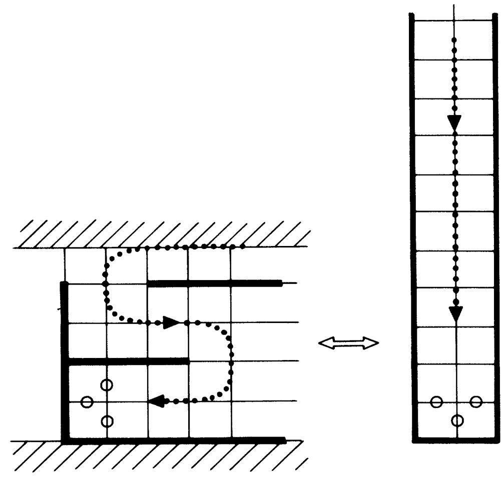

FIG. 1. One of the configurations which has $p = p'_{\min}(L=5)$. Here, we apply periodic boundary conditions in the horizontal direction. The bottom line is a line of seed particles. The light lines are unoccupied bonds, the heavy lines are occupied bonds. The broken line indicates the path of a random walker to reach the bond with $p = p'_{\min}$. The open circles indicate the bonds with minimum growth probability $p = p'_{\min}$.

However, these configurations are not typical DLA configurations (indeed, these configurations have fractal dimension 2). Therefore, the result (3) does not support the BA assumption (1), since (1) concerns a *typical* DLA cluster. In contrast to the well established size-dependence (3), the size dependence of the minimum growth probability of the *typical* configuration $p_{\min}(L)$ is not established.

Jysoo Lee, Preben Alstrøm, and H. Eugene Stanley
Center for Polymer Studies and Department of Physics
Boston University
Boston, Massachusetts 02215

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[1]R. Blumenfeld and A. Aharony, this issue, Phys. Rev. Lett. **62**, 2977 (1989); these authors discuss both the typical and average cases.

[2]J. Lee and H. E. Stanley, Phys. Rev. Lett. **61**, 2945 (1988).

[3]J. Lee, P. Alstrøm, and H. E. Stanley, Phys. Rev. A **39**, 6545 (1989); see especially the analytic argumentation in Sec. V.