Tricritical Behavior of an Ising Antiferromagnet with Next-Nearest Neighbor Ferromagnetic Interaction

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(Received December 13, 1976)

Using high temperature series expansions, we have studied an Ising model with nearest-neighbor antiferromagnetic interaction ($J_1<0$) and next nearest neighbor ferromagnetic interaction ($J_2>0$), which is thought to exhibit a tricritical point (TCP).

Assuming that tricritical exponents are mean-field like, we locate TCPs for several values of the exchange coupling parameter J_2/J_1 by the method of log-Padé analysis.

Recently, several authors[1,2] have studied an Ising model with nearest-neighbor antiferromagnetic interaction ($J_1=-1$) and next-nearest neighbor ferromagnetic interaction ($J_2=+1/2$) (NNN model), which has been assumed to exhibit tricritical behavior.

By analyzing high-temperature series expansions, Harbus and Stanley suggested that the direct susceptibility diverges at the tricritical point with an exponent $\bar{\gamma}=1/4$.

This contradicts the value $\bar{\gamma}=1/2$, obtained by renormalization group techniques.[3] In a further analysis of the same model, Wortis *et al.*[4] pointed out that if the true tricritical temperature were actually somewhat lower than the Harbus-Stanley value, then standard ratio analysis would give tricritical exponents consistent with the Gaussian-tricritical-fixed-point values of Riedel-Wegner.[3]

An analysis using renormalization group techniques (Fisher and Nelson[5]) has demonstrated that the NNN model displays the same values for tricritical exponents mean field, with logarithmic corrections as does the 'meta-model', in which 2-dimensional planes of ferromganetically-coupled spins are linked by antiferromagnetic interactions.

Furthermore, Landau's[6] recent Monte Carlo data provide experimental verification that the simple cubic Ising antiferromagnet with next-nearest neighbor ferromagnetic interaction does have mean-field tricritical exponents.

Kincaid and Cohen[7] have shown that different types of critical behavior can be expected for such next-nearest neighbor (NNN model). In particular, for $|J_2/J_1|$ sufficiently small, no tricritical point is anticipated. Instead, the second order line is thought to terminate at a critical end point, where 2 phases become critical in the presence of 2 other coexisting phases.

Therefore, it is of interest to study the problem for other values of the next-nearest neighbor exchange coupling parameter J_2, first to test whether the predictions of Kincaid and Cohen can be confirmed and second to attempt to resolve the actual values of $\bar{\gamma}$.

In this note we focus our attention on the simple cubic, spin$=1/2$ Ising antiferromagnet with fixed values of the ratio of coupling parameters $|J_2/J_1|$.

The Hamiltonian for our system can be written as

$$\mathscr{H}=-J_1\sum_{i,j} S_iS_j-J_2\sum_{i,j}^{nnn} S_iS_j-\mu H\sum_i S_i \quad (1)$$

where $S_i=\pm 1$, on each lattice site i; $\sum^{nn}$ and $\sum^{nnn}$ denote sums over nearest-neighbor and next-nearest neighbors, respectively.

Our analysis is based on the high-temperature series for the direct (χ) and the staggered (χ_{st}) susceptibility.[2)] The coefficients in these series are obtained from the renormalized linked cluster expansion up to eighth order in the inverse temperature, and for arbitrary field values. Full original data of the coefficients are available on request.

First we point out that the results from standard ratio analysis and log-Padé analysis[8)] for $|J_2/J_1|=1/2$ do agree very well, as can be seen in Table I.

We therefore analyzed the χ and χ_{st} for other values of the ratio of the exchange coupling parameter $|J_2/J_1|$, using the same log-Padé method and standard ratio analysis. The pole of the P.A. to the logarithmic derivative of the series which give a TCP exponent that agrees best with the renormalization group results is taken to be an estimate for the TCP. The results of this analysis are shown in Table II.

Next, we examine the Padé approximants to the direct susceptibility series raised to the power 2 for some values of the parameter $|J_2/J_1|$. We may expect the tricritical susceptibility exponent to be $\bar{\gamma}=1/2$ with error bars less than or about 10%, as is given in Table III.

Table I. Estimates of exponents γ_{st} and critical temperatures T_c for various $h=\mu H/k_BT_c$ values by ratio method (R) and log-Padé approximant (LP).

$J_2/J_1=-1/2$

h	$k_BT_c^R$	$k_BT_c^{LP}$	γ_{st}^R	γ_{st}^{LP}
0.00	10.17	10.16	1.23	1.23
0.10	10.09	10.09	1.23	1.23
0.20	9.86	9.85	1.22	1.23
0.30	9.50	9.50	1.21	1.22
0.40	9.03	9.02	1.20	1.21
0.50	8.49	8.48	1.18	1.19
0.60	7.90	7.89	1.15	1.16
0.70	7.29	7.29	1.12	1.12
0.80	6.69	6.69	1.07	1.07
0.84	6.45	6.46	1.05	1.05
0.90	6.10	6.11	1.02	1.02
0.94	5.88	5.89	1.00	0.99

Table II. Estimates of critical fields ($h_c\equiv\mu H/k_BT_c$) and critical temperatures T_c for various $|J_2/J_1|$ values with an uncertainty of at most 0.02.

$\|J_2/J_1\|$	0.3	0.4	0.45	0.5	0.55	0.60	0.8	1.0
h_t	1.18	1.00	0.96	0.94	0.88	0.74	0.56	0.36
k_BT_t	4.78	5.55	5.80	5.88	6.22	7.27	9.22	12.65

Table III. Padé tables for the leading singularity of χ^p.

χ^2 at $h=1.00$ for $|J_2/J_1|=0.4$

D/N	3	4	5
3	5.17	5.26	4.81
4	5.24	5.17	
5	5.39		

χ^2 at $h=0.74$ for $|J_2/J_1|=0.6$

D/N	3	4	5
3	6.46	6.65	6.72
4	7.04	6.78	
5	6.77		

χ^2 at $h=0.56$ for $|J_2/J_1|=0.8$

D/N	3	4	5
3	7.84	8.27	8.53
4	6.38	8.79	
5	8.76		

The authors would like to thank Dr. F. Harbus for invaluable correspondence concerning various aspects in the initial stage of this work.

They are also very grateful for numerous discussions with Professor T. S. Chang, Dr. J. Nicoll and Dr. F. Family. One of authors (C.K.) is indebted to Professor M. Wortis, Dr. G. Baker, Professor D. D. Betts and Professor M. Blume for useful advices.

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