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Income and wealth distribution of the richest Norwegian individuals: An inequality analysis

Maciej Jagielski [a,b,c,*], Kordian Czyżewski [a], Ryszard Kutner [a], H. Eugene Stanley [c]

[a] Faculty of Physics, University of Warsaw, Pasteura 5, PL-02093 Warszawa, Poland
[b] ETHZ, Department of Management, Technology and Economics, Scheuchzerstrasse 7, CH-8092 Zürich, Switzerland
[c] Center for Polymer Studies and Department of Physics, Boston University, Boston, MA 02215, USA

HIGHLIGHTS

- We analyze the wealth and income of the richest individuals in Norway.
- Annual income and wealth level are describable using the Pareto law.
- Wealth inequality is higher than income inequality.
- Wealth inequality is more difficult to change than income inequality.
- We used high quality empirical data from the Norwegian tax office.

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ABSTRACT

Using the empirical data from the Norwegian Tax Administration, we analyze the wealth and income of the richest individuals in Norway during the period 2010–2013. We find that both annual income and wealth level of the richest individuals are describable using the Pareto law. We find that the robust mean Pareto exponent over the four-year period to be ≈2.3 for income and ≈1.5 for wealth.

1. Introduction

Income and wealth inequalities are being closely examined in current economic, sociological and econophysical literature [1–28]. The challenge is to accurately measure these inequalities.

The recent, widely-cited book on income and wealth inequalities by Piketty [1] concludes that income and wealth inequalities are different quantities and should be analyzed separately. Many authors have used the Pareto law to describe income and wealth inequalities [3–28]. Piketty uses aggregated macro-variables to describe inequality, but authors following the Pareto approach primarily use microdata, i.e., the wealth ranks of the richest individuals supplied by such periodicals as *Forbes*. In both kinds of analysis the quality of the empirical data is poor. Piketty's empirical data, although reliable, are imperfect, and those used by other researchers are also far from perfect because the only reliable source for such data is the tax office.

* Corresponding author at: ETHZ, Department of Management, Technology and Economics, Scheuchzerstrasse 7, CH-8092 Zürich, Switzerland. Fax: +48 22 5532999.
E-mail address: zagielski@interia.pl (M. Jagielski).

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Since the publication of Vilfredo Pareto's pioneering work on income distribution in the late nineteenth century many additional studies have been carried out to empirically verify the Pareto law for both individuals and households. These analyses were carried out for the United States [3–11], the European Union [6,11,12], the UK [3,5,7,8,13], Germany [3], Italy [14,15], France [7], Switzerland [16], Sweden [8], Japan [9,17–20], Australia [14,21], Canada [22], India [22,23], Sri Lanka [22], Peru [16], Egypt [22], South Korea [24], Romania [25], Portugal [22], Poland [6,26], and for the world as a whole [27,28].

Here we use the Pareto law to analyze the income and wealth rank of the 100 richest individuals in Norway.[1] In the nineteenth century, the Norwegian Tax Administration began compiling a yearly "Skattelister", a list of the yearly income and wealth levels of every citizen.[2] Although access to the Skattelister has been extremely limited, the records of the 100 richest individuals are available for the years 2010–2013. This allows a precise validation of the Pareto law for income and wealth distributions during those years [31]. Note that although current literature provides numerous analyses of this type, the Skattelister data we obtained from the Norwegian Tax Administration allows our validation to be particularly reliable.

The Norwegian Tax Administration defines income as "total wages and salaries, pensions, entrepreneurial income, and property income" [30], and the wealth of individuals as "the sum of financial and non-financial wealth, minus liabilities" [30].

2. Pareto law

At the end of nineteenth century Vilfredo Pareto formulated his law by analyzing a huge amount of empirical data that described the income and wealth distributions using the PDF universal function, i.e. [12,16,32],

$$P(m) = \begin{cases} \frac{\alpha}{m_0}\left(\frac{m_0}{m}\right)^{\alpha+1} & \text{if} \quad m \geq m_0 \\ 0 & \text{if}\, m < m_0, \end{cases} \tag{1}$$

which defines the Pareto law [32]. The m_0 value is the lowest value of variable m, and α is the Pareto exponent. By definition we choose the strongest Pareto law [32] for $\alpha = 3/2$.

Empirical studies indicate (i) that the mean value of the Pareto exponent is close to 2, and (ii) that the Pareto law is valid for large values of income and wealth [22]. For other values of income and wealth such laws as Gibrat's rule of proportionate growth [22] are valid. Thus the studies in Section 1 refer to the weak Pareto law that holds in the limit $m \gg m_0$. We will show that the mean value of the Pareto exponent for the richest Norwegians is close to 2 for income and to 3/2 for wealth.

To analyze the empirical data it is better to use the more robust global empirical complementary cumulative distribution function (CCDF) rather than the (local) Eq. (1). From Eq. (1) the CCDF can be written

$$\Pi(m) = (m/m_0)^{-\alpha}, \quad \text{for}\, m \geq m_0. \tag{2}$$

A well-known feature of Eq. (1) is the divergence of its moments of an order $\geq \alpha$. This means that we should apply quantiles (which are always finite [33]) instead of moments. For example, we use the median instead of the mean value. In any case, the moment estimates are always finite and can be calculated directly from the empirical data [12].

3. Results and concluding remarks

The annual rankings of income and wealth of Norwegian citizens for the years 2010–2013 [31] are of the 100 wealthiest people in Norway as well as in each region (fylke) of Norway. Note that income and wealth ranking lists are reported independently. Someone listed in the income ranking may not be listed in the wealth ranking and vice versa. Unlike those in, for example, *Forbes*, the data supplied by the Norwegian Tax Administration are exact, not estimates.

Fig. 1 shows log–log plots of the wealth and income rankings for the Hedmark region and for Norway as a whole for the year 2013. The slopes of these lines are equal to $-\alpha_{\text{rank}}$ and were calculated using a fitting routine.[3] The inverse of α_{rank} gives the Pareto exponent α.

Table 1 shows the Pareto exponents for each region of Norway and for all of Norway for the years 2010–2013. The error bars of the Pareto exponents fall within the range 0.01–0.13.

The mean Pareto exponent α over a four-year period for Norway is 2.3 ± 0.4 for income and 1.5 ± 0.3 for wealth. Note that the Pareto exponent for income is significantly larger than the Pareto exponent for wealth. This is the most reliable result thus far obtained for income and wealth analysis [3–28]. In addition, the Pareto exponents for income fluctuate in time more than Pareto exponents for wealth. This means that wealth inequality is more difficult to change than income

1 Note that the income of the richest individuals in Norway has been widely analyzed. See for example [29] and references therein in which the authors trace the evolution of the top of the income distribution for Norway and compare it with the evolution in other countries.

2 This body of tax assessment statistics covers all individuals, including those living abroad who owe taxes to Norway [30].

3 To obtain estimation of the parameters for theoretical formula, Eq. (2), we fitted a linear function to the log of the empirical data.

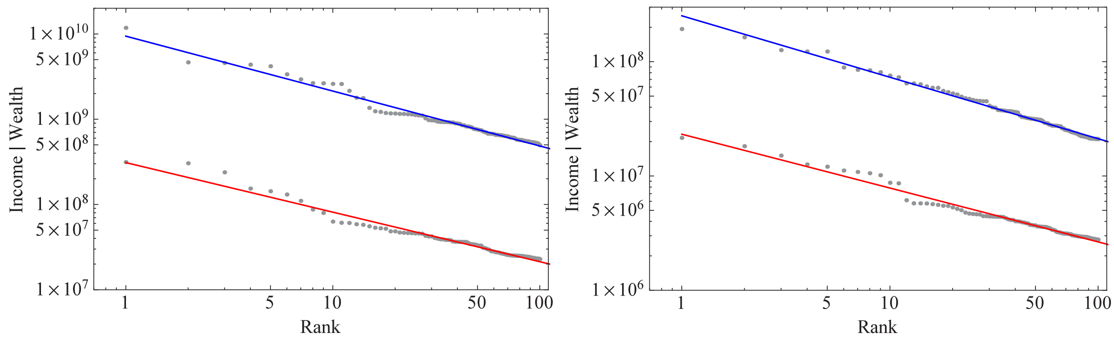

Fig. 1. Ranks of income (bottom plots) and wealth (top plots) of the richest individuals in Norway (left panel) and Hedmark region (right panel). Solid lines were obtained by fitting straight lines (in the log–log scale) to empirical data (dots) for year 2013.

Table 1
Pareto exponents for each region of Norway and all of Norway the for years 2010–2013. The errors bars of Pareto exponents fall within the range 0.01–0.13.

Region	Year							
	2010		2011		2012		2013	
	Income	Wealth	Income	Wealth	Income	Wealth	Income	Wealth
Akershus	2.45	1.14	3.00	1.21	2.34	1.23	2.45	1.43
Aust-Agder	2.22	1.20	2.19	1.13	1.70	1.14	1.58	1.16
Buskerud	2.11	1.43	1.63	1.26	1.75	1.25	2.11	1.50
Finnmark	2.90	1.63	2.69	1.63	2.94	1.61	2.90	1.81
Hedmark	2.12	1.61	2.14	1.67	1.14	1.66	2.12	1.86
Hordaland	2.13	1.25	2.13	1.18	1.78	1.23	1.69	1.13
Møre Og Romsdal	2.42	1.46	1.54	1.54	2.19	1.94	2.42	1.62
Nordland	2.15	2.08	2.59	1.84	1.88	1.75	2.47	2.25
Nord-Trøndelag	2.93	2.46	2.30	2.26	2.04	1.93	2.95	2.05
Oppland	2.26	1.88	2.53	1.85	2.12	1.83	2.47	1.92
Oslo	1.53	1.26	1.90	1.37	2.55	1.40	2.05	1.44
Østfold	1.90	1.81	2.12	2.08	2.37	2.19	1.60	1.95
Rogaland	1.82	1.65	2.32	1.57	2.28	1.54	2.58	1.54
Sogn Og Fjordane	2.55	1.44	2.19	1.36	2.50	1.36	2.05	1.36
Sør Trøndelag	1.72	1.75	1.51	1.55	1.79	1.42	2.01	1.49
Telemark	2.87	1.35	2.54	1.35	2.39	1.44	2.60	1.47
Troms	2.11	1.45	1.23	1.43	2.60	1.54	2.41	1.00
Vest-Agder	2.11	1.02	2.22	1.03	2.11	0.97	2.09	1.48
Vestfold	2.22	1.34	2.23	1.55	2.37	1.54	2.52	1.67
NORWAY	1.69	1.37	1.77	1.35	2.01	1.37	1.72	1.55
Mean (regions)	2.23	1.54	2.16	1.52	2.15	1.52	2.27	1.59
Dispersion (regions)	0.37	0.34	0.43	0.31	0.40	0.30	0.38	0.32

inequality. Note that the results obtained by authors not using data supplied by tax authorities are unsystematic and can only be approximate.

Using our comparative analysis we find that for separate regions in Norway the Pareto exponents for wealth are almost always smaller than the corresponding Pareto exponents for income. This means that wealth inequality is higher than income inequality, i.e., the lower the Pareto exponent, the higher the inequality. This is because income has no accumulation effect across the generations that acts according to the preferential choice rule, "the rich become richer". Income inequality is strongly affected by access to skills and higher education and is lowered by taxes on income, but wealth accumulation is a long-term process and is less burdened by taxes, i.e., a cadastral tax or inheritance tax does not significantly reduce wealth inequality (see [1]). Although one possible solution to this situation would be to introduce an annual tax on wealth, social and political factors make this change difficult [1].

Note also that each region of Norway tends to be more homogeneous than the country as a whole. Thus the values of the Pareto exponents are generally higher for regions (where there is lower income and wealth inequality) than for Norway itself (higher income and wealth inequality). Thus one may have high Pareto exponents at all local levels but a low one for the entire country. The discrepancy between the mean income Pareto exponent for regions and the income Pareto exponent for Norway is higher than the discrepancy between the mean wealth Pareto exponent for regions and the wealth Pareto exponent for all of Norway (see Table 1). This once again is caused by the accumulation effect for wealth and lack of this effect for income. Thus there is higher income homogeneity in regions than in the country as a whole and a similar wealth diversity between regions and the whole of Norway.

Using the highly reliable empirical data from the Norwegian Tax Administration, we have analyzed the income and the wealth of the richest Norwegian individuals. We find that income and wealth inequality must be analyzed separately because they are driven by different factors [1]. In addition, we confirm that the distribution of the top income and wealth is subject to the Pareto law.

We are aware that the analysis of income and wealth is a research area for which much has yet to be accomplished. We hope that our paper contributes some insight into the topic from a physicist's point of view.[4] This approach links economics and econophysics and demonstrates that the economic models describing the relationship between income and wealth can be supported by modeling based on methods used by physicists.

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[4] For more details on how income and wealth on microscopic and macroscopic levels are modeled by physicists see for example [10,12], where individual income or wealth is described using a Langevin equation, and a Pareto income (wealth) distribution obtained by using Fokker–Planck equation.