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Price fluctuations and market activity

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Abstract

We empirically quantify the relation between trading activity—measured by the number of transactions N—and the price change $G(t)$ for a given stock, over a time interval $[t, t+\Delta t]$. We relate the time-dependent standard deviation of price changes—volatility—to two microscopic quantities: the number of transactions $N(t)$ in Δt and the variance $W^2(t)$ of the price changes for all transactions in Δt. We find that the long-ranged volatility correlations are largely due to those of N. We then argue that the tail-exponent of the distribution of N is insufficient to account for the tail-exponent of $P\{G > x\}$. Since N and W display only weak inter-dependency, our results show that the fat tails of the distribution $P\{G > x\}$ arises from W, which has a distribution with power-law tail exponent consistent with our estimates for G. © 2001 Elsevier Science B.V. All rights reserved.

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Stock price fluctuations display distinctive statistical features that are in stark contrast to those of a simple random walk ("diffusion") model [1–4]. Consider price change $G(t) \equiv \ln S(t+\Delta t) - S(t)$—defined as the change in the logarithm of price $S(t)$ over an interval Δt. Empirical work shows that the distribution function $P_G\{G > x\}$ has tails that decay as a power law $P_G\{G > x\} \sim x^{-\alpha}$, with α larger than the upper bound ($\alpha = 2$) for Lévy stable distributions [5–7]. In particular, studies on the largest 1000 US-stocks [6] and 30 German stocks [5] show mean values of $\alpha \approx 3$ on time scales $\Delta t \leqslant 1$ day. Secondly, it is found that although the process $G(t)$ has a rapidly decaying autocorrelation function $\langle G(t)G(t+\tau)\rangle$, which at time scales $\tau < 30$ min, shows significant anti-correlations (bid–ask bounce) for individual stocks, but cease to be statistically significant for larger time scales. Higher-order two-point correlation

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functions show quite different behavior. For example, the autocorrelation function of the absolute value of price changes show long-range persistence $\langle |G(t)|\,|G(t+\tau)|\rangle \sim \tau^{-\mu}$, with $\mu \approx 0.3$ [8–10].

The problem of understanding the origin of these features is a challenging one [3,11,12]. This paper, reviews recent work which focusses on a much more modest goal of trying to understand, starting from transactions, how these statistical features—fat-tailed distributions and long-ranged volatility correlations—originate. We shall show that the price changes when conditioned on the volatility have tails that are consistent with those of a Gaussian. In addition, we shall show that the long-ranged correlations in volatility arise from those of trading activity measured by the rate of occurrence of trades N. However, the distribution characteristics of trading activity implies that the fat tails of G cannot arise solely due to N. We relate the fat-tailed behavior of G to those of "transaction-time" volatility W which, roughly speaking, measures the impact of trades.

Let us start by examining the conventionally-used "geometric"-variant of Bachelier's "classic diffusion" model. The rationale for this model arises from the central limit theorem by considering the price changes $G \equiv \Delta \ln S(t)$ in a time interval Δt as being the sum of several changes δp_i, each due to the ith transaction in that interval,

$$G \equiv \sum_{i=1}^{N} \delta p_i \,, \tag{1}$$

where N is the number of transactions (trades) in Δt. If $N \gg 1$, and δp_i have finite (constant) variance W^2, then one can apply the central limit theorem, whereby one would obtain the result that $P_G(G)$ is Gaussian with variance $\sigma^2 = W^2 N$, and therefore prices evolve with Gaussian increments. It is implicitly assumed in this description that N is almost constant, or more precisely N has only narrow (standard deviation much smaller than the mean) Gaussian fluctuations around a mean value. Let us start by asking to what extent this is true.

In a typical day, there might be as many as $N = 1000$ trades for an actively traded stock. Fig. 1a shows the time series of N for an actively traded stock sampled at 15 min intervals contrasted with a series of Gaussian random numbers. From the presence of several events of the magnitude of tens of standard deviations, it is apparent that N is distinctly non-Gaussian [13–19]. Let us first quantify the statistics of N. We first analyze the distribution of N. Fig. 1b shows that $P(N)$ decays as a power law,

$$P_N\{N > x\} \sim N^{-\beta} \,, \tag{2}$$

where $\beta \approx 3.5$ for five actively traded stocks. A more extensive analysis on 1000 stocks [19] gives values of β around the average value $\beta = 3.4$.

Since, N behaves in a non-Gaussian manner, one can ask whether the exponent α for the distribution of price changes $P_G\{G > x\} \sim x^{-\alpha}$ arises from the exponent β for P_N. To address this problem, we must first quantify the relationship between G and N. Consider the conditional distribution $P_{G|N,W}\,(G|N,W)$ for given values of N and W. If we assume that the changes δp_i due to each transaction in Δt are i.i.d., then

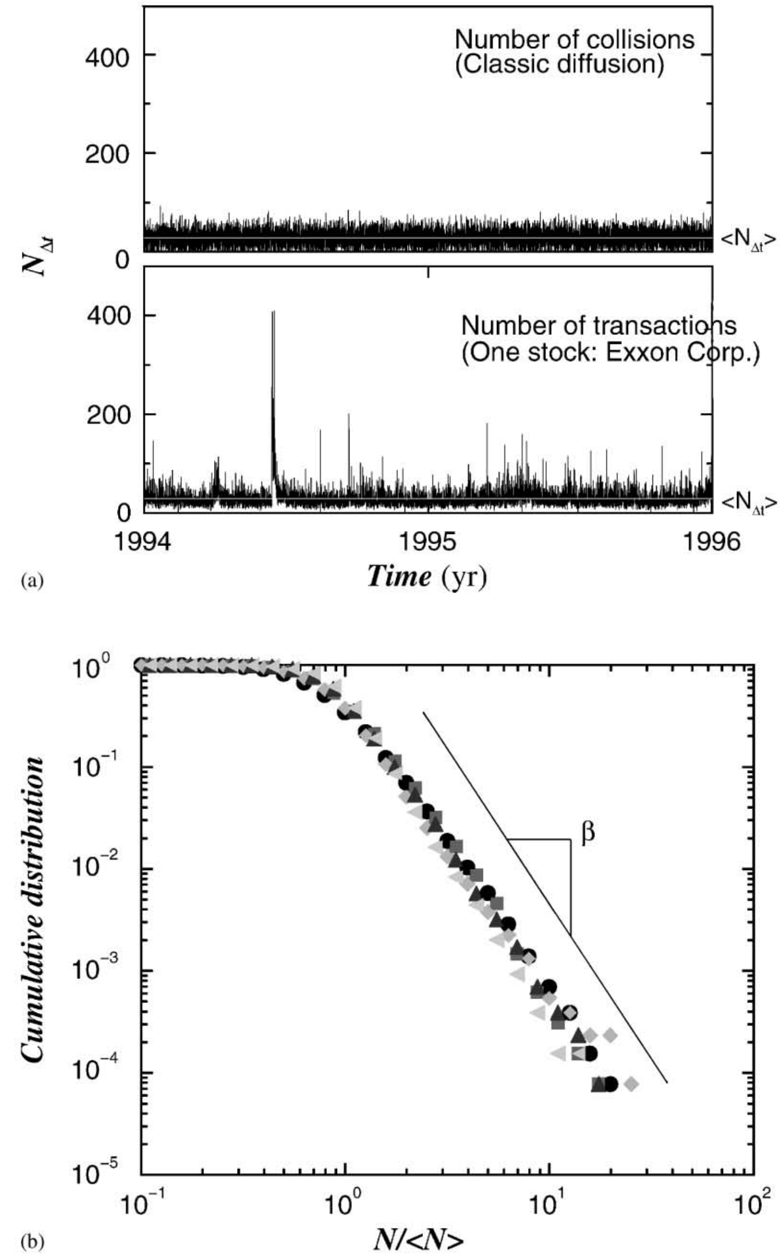

Fig. 1. Statistical properties of N. (a) The lower panel shows N for Exxon Corporation with $\Delta t = 30$ min and the average value $\langle N \rangle \approx 52$. The upper panel shows a sequence of uncorrelated Gaussian random numbers with the same mean and variance, which depicts the number of collisions in N for the classic diffusion problem. Note that in contrast to diffusion, N for Exxon shows frequent large events of the magnitude of tens of standard deviations, which would be forbidden for Gaussian statistics. (b) The cumulative distribution of N for 5 stocks: Exxon, General Electric, Coca Cola, AT& T, Merck show similar decay consistent with a power-law behavior with exponent $\beta \approx 3.4$.

the variance of $G(t)$ in that time interval will be W^2N. Thus the width of the conditional distribution $P_{G|N,W}$ $(G|N,W)$—probability density of G for given values of N and W—will be the standard deviation $W\sqrt{N}$, which measures the local volatility. If we next hypothesize that the *functional form* of $P_{G|N,W}$ $(G|N,W)$ does not depend on the values of W or N, then we can express $P_{G|N,W}$ $(G|N,W) = 1/(W\sqrt{N})f(G/(W\sqrt{N}))$, where the function f has the same form for all values of W and N.[1] In other words, during periods of large $W\sqrt{N}$ the conditional distribution $P_{G|N,W}$ $(G|N,W)$ will have large width.

We seek to quantify the functional form of the conditional distribution $P_{G|N,W}$. Under our hypothesis, determining the conditional distribution is tantamount to determining the functional form f, which is accomplished by considering a "scaled" variable

$$\varepsilon \equiv \frac{G}{W\sqrt{N}}\,, \tag{3}$$

which is free of the effects of fluctuating $W\sqrt{N}$. Our examination of the distribution $P_\varepsilon(\varepsilon)$ shows that it is consistent with Gaussian behavior [19]. Thus the conditional distribution is consistent with the functional form $P_{G|N,W}$ $(G|N,W) \simeq 1/(\sqrt{2\pi}W\sqrt{N})\exp(-G^2/2W^2N)$.[2]

We are now in a position to relate the statistical properties of G and N. One can express the distribution of price changes P_G in terms of the conditional distribution $P_{G|N,W}$ $(G|N,W)$, or equivalently in terms of f,

$$P_G(G) = \int \frac{1}{\xi} f\left(\frac{G}{W\sqrt{N}=\xi}\right) P_{W\sqrt{N}}(\xi)\,\mathrm{d}\xi\,, \tag{4}$$

where $P_{W\sqrt{N}}$ denotes the probability density function of the variable $W\sqrt{N}$. Since f is consistent with Gaussian, it is clear that the fat tails in G must arise due to the mixing of the conditional distribution, averaged over all possible widths $W\sqrt{N}$.

Next, we examine how the statistics of W and N relate to the statistics of G. First, we examine the equal-time dependence of W and N and find that the equal-time correlation coefficient is small, suggesting only weak interdependence [19]. Therefore the contribution of N to the distribution $P_{W\sqrt{N}}$ in Eq. (4) goes like the distribution of $\sqrt{N}$. We have already seen that the distribution $P_N\{N > x\} \sim x^{-\beta}$ with $\beta \approx 3.4$. Therefore, $P_{\sqrt{N}}\{y \equiv \sqrt{N} > x\} \sim x^{-2\beta}$ with $2\beta \approx 6.8$. Therefore, N alone cannot explain the value $\alpha \approx 3$. Instead, $\alpha \approx 3$ must arise from elsewhere. In fact, when we repeat the analysis through to $W_{\Delta t}$ [19], we find that the distribution $P_W\{W_{\Delta t} > x\}$ decays with an exponent $\gamma \approx 3$, which is also the contribution of W to the distribution $P_{W\sqrt{N}}$. Therefore, the averaging in Eq. (4) gives the asymptotic behavior of P_G to be a power-law with an exponent γ. Indeed, our mean estimates of γ and α are comparable

[1] The hypothesis that the conditional distribution has the same form for all W and N might strike the reader as surprising since one expects the conditional distribution to be increasingly "closer" to a Gaussian for increasing N. Strictly speaking, if W and N are independent, then the hypothesis would be exact only for a stable distribution for δp_i such as a Gaussian (consistent with our findings later in the text).

[2] The $\simeq$ sign is used because although the tails of the conditional distribution are consistent with Gaussian, the central part is affected by discreteness of price changes in units of 1/16 or 1/32 of a dollar.

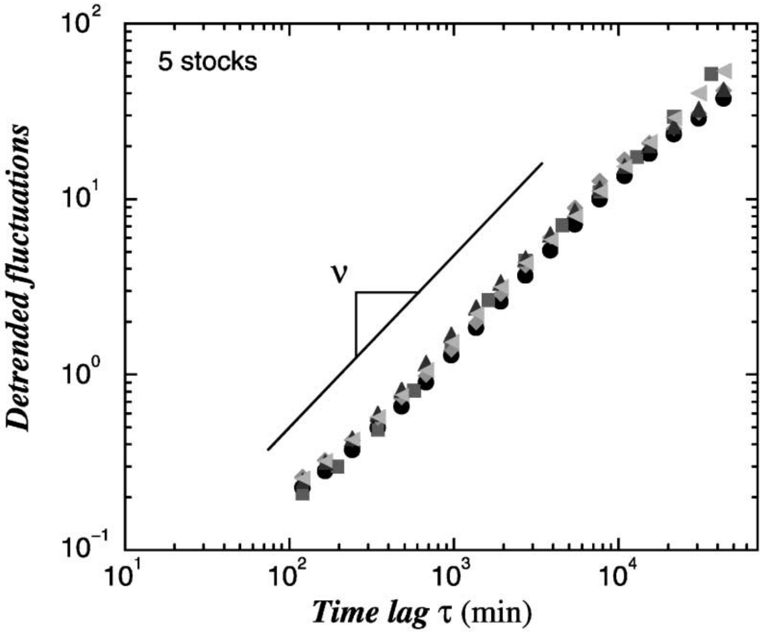

Fig. 2. Detrended fluctuation function $F(\tau)$ for the same five stocks as before. Regressions yield values of the slope $\nu \approx 0.85$, consistent with long-range correlations.

within error bounds [6,19]. Thus the power-law tails of $P_G(G)$ appear to originate from the power-law tail in $P_W(W)$.

We also analyze correlations in N. Instead of analyzing the correlation function directly, we use the method of detrended fluctuation analysis [20]. We plot the detrended fluctuation function $F(\tau)$ as a function of the time scale τ. Absence of long-range correlations would imply $F(\tau) \sim \tau^{0.5}$, whereas $F(\tau) \sim \tau^{\nu}$ with $0.5 < \nu \leqslant 1$ implies power-law decay of the correlation function,

$$\langle [N(t)][N(t+\tau)] \rangle \sim \tau^{-\nu_{cf}} \qquad [\nu_{cf} = 2 - 2\nu] \,. \tag{5}$$

We obtain the value $\nu \approx 0.85$ for the same 5 stocks as before (Fig. 2). On extending this analysis for a set of 1000 stocks we find the mean value $\nu_{cf} \approx 0.3$ [19]. It is possible to relate this to the correlations in $|G|$, which is related to the variance V^2 of G. From Eq. (1), we see that $V^2 \propto N$ under the assumption that δp_i are independent. Therefore, the long-range correlations in N is one reason for the observed long-range correlations in $|G|$. In other words, highly volatile periods in the market persist due to the persistence of trading activity.

Naturally, the mechanisms that give rise to the observed long-range correlations in N are of great interest. In Ref. [21], this problem is investigated using a continuous time asynchronous model. Recently, it was argued that these correlations could arise from the fact that agents in the market have the choice between active and inactive strategies [22].

Lastly, we discuss the role of the share volume traded in explaining the statistical properties of price fluctuations. Intuitively, one expects that the larger the trade size, the greater the price impact, and hence larger the volatility. Therefore one expects the volatility to be related to the number of shares traded (share volume). Indeed, it is a common Wall Street saying that 'it takes volume to move stock prices'. In our recent study [23], we find that the number of shares q_i traded per trade has a power-law distribution with tail-exponents ζ which are in the Lévy stable domain. Therefore one can express the number of shares Q traded in Δt as $Q = \sum_{i=1}^{N} q_i$. Due to the Lévy stable tails of the distribution of q, Q scales like $Q = \mu N + N^{1/\zeta}\xi$, where ξ is a one-sided Lévy stable distributed variable with zero mean and tail exponent ζ, and $\mu \equiv \langle q_i \rangle$.

Analyzing equal-time correlations, we find, surprisingly, that the correlation coefficients $\langle \xi N \rangle, \langle \xi W \rangle$ are small (average values of the order of ≈ 0.1). This means that even if the number of shares traded are large (large ξ), volatility $V = W\sqrt{N}$ need not be. Thus the previously found [13,15,16,24] equal-time dependence of volatility $V = W\sqrt{N}$ and share volume arises largely because of N. This is quite surprising since it means that the size of the trade, on average, does not seem to have a direct influence in generating volatility [25].

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