Angle restriction enhances synchronization of self-propelled objects

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Understanding the synchronization process of self-propelled objects is of great interest in science and technology. We propose a synchronization model for a self-propelled objects system in which we restrict the maximal angle change of each object to θ_R. At each time step, each object moves and changes its direction according to the average direction of all of its neighbors (including itself). If the angle change is greater than a cutoff angle θ_R, the change is replaced by θ_R. We find that (i) counterintuitively, the synchronization improves significantly when θ_R decreases, (ii) there exists a critical restricted angle θ_{Rc} at which the synchronization order parameter changes from a large value to a small value, and (iii) for each noise amplitude η, the synchronization as a function of θ_R shows a maximum value, indicating the existence of an optimal θ_R that yields the best synchronization for every η.

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I. INTRODUCTION

Over the past 10 years, there has been widespread scientific interest in the collective motion exhibited by such groups as a school of fish, a flock of birds, or a swarm of robots [1–11]. Collective motion due to synchronization processes (called also polarization or orientational ordering of velocity) plays an important role in many different fields, including biology, ecology, climatology, sociology, and technology, and is even a factor in the arts [12–18]. A surprisingly simple but useful model, proposed by Vicsek *et al*. [19], shows the existence of phase transition in a self-propelled objects (SPO) system. More recently, various specialized models have been proposed to describe, e.g., accelerating convergence [20], avoiding collision [21,22], and enhancing the efficiency of convergence [23]. Also, other properties of SPO such as coherence [24] and collective intelligence [25] were studied. These in turn have promoted the development of such applications as distributed sensor networks [26], unmanned aerial vehicles [27], underwater vehicles [28], and altitude alignment of satellite clusters [29]. However, one of the most important issues, in particular for applications, is the question as to how to improve synchronization of SPO based on local information. Even more important is to identify optimal synchronization conditions in the presence of noise, which exists in a group of SPO machines such as robots or vehicles.

In this paper, we develop a restricted angle SPO model (RASPO), which generalizes the Vicsek model (VM) [19] and counterintuitively improves the synchronization of SPO systems. Specifically, we find that (i) the synchronization is significantly improved when the restricted angle decreases; (ii) there exists a critical restricted angle θ_{Rc} above which the synchronization order parameter changes from a large value to a small value; and (iii) for a given noise amplitude η, the synchronization shows a peak as a function of θ_R, which yields the best synchronization conditions.

II. THE ANGLE RESTRICTION MODEL

In the VM, a group of n objects move in a $L \times L$ square with the same constant speed in different directions. Initially, the objects are randomly distributed, and their directions are also uniformly randomly distributed in the interval $(0,2\pi)$. At each time step, the direction of each object is determined by the average directions of all the objects within a circle centered at the given object, the influencing radius of which is R. At time t, the position of a specific object is updated according to

$$x_i(t+1) = x_i(t) + v_0 e^{\mathrm{i}\theta_i(t)}. \tag{1}$$

Its direction is updated as

$$e^{\theta_i(t+1)} = e^{\mathrm{i}\Delta\theta_i(t)} \frac{\sum_{j\in\Gamma_i(t+1)} e^{\mathrm{i}\theta_j(t)}}{\left\| \sum_{j\in\Gamma_i(t+1)} e^{\mathrm{i}\theta_j(t)} \right\|}, \tag{2}$$

where $\|\cdots\|$ is the standard norm [30] defined as $\|(z_1,z_2,\ldots,z_n)\| = (|z_1|^2 + |z_2|^2 + \cdots + |z_n|^2)^{1/2}$, $\Delta\theta_i \in [-\eta,\eta]$ denotes the white noise, $e^{i\theta_i(t)}$ is a unit directional vector, and $\Gamma_i(t+1)$ is the set of neighbors of object i at time step $t+1$. In order to measure the synchronization of the system, an order parameter is introduced as [19,31]

$$V_\alpha = \frac{1}{n} \left\| \sum_{i=1}^{n} e^{\mathrm{i}\theta_i(t)} \right\|, \quad 0 \leqslant V_\alpha \leqslant 1. \tag{3}$$

A larger value of V_α indicates a better consensus, and when $V_\alpha = 1$, all the objects are moving in the same direction. Numerical simulations show that when the density is high and the noise low, all the objects will have reached consensus, i.e., will be moving in the same direction after a finite number of time steps (the convergence time) [32,33].

In the VM and all related models of SPO, the rotation and rectilinear motions of each object can be treated separately. Each object changes direction to the average direction of its neighbors. It then moves a distance v_0 rectilinearly.

However, because the directions and positions of all the objects are initially randomly distributed, most of the objects make sharp changes in direction that bear little similarity to behavior found in nature, and are thus impractical when developing applications in engineering. The movie supplied in Ref. [34] clearly demonstrates that a flying bird can not execute sharp changes in direction within a single step. From

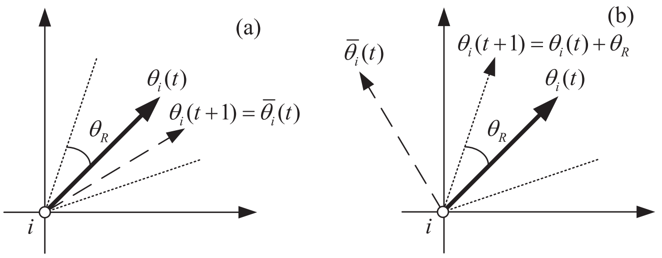

FIG. 1. Description of two cases of direction updated in the RASPO model: (a) the change from $\theta_i(t)$ to $\bar{\theta}_i(t)$ is smaller than θ_R, so $\theta_i(t+1)=\bar{\theta}_i(t)$; (b) the change from $\theta_i(t)$ to $\bar{\theta}_i(t)$ is greater than θ_R and $\bar{\theta}_i(t)$ is closer to $\theta_i(t)+\theta_R$, so $\theta_i(t+1)=\theta_i(t)+\theta_R$. Note that θ_R is defined as half of the restricted angle and, therefore, $\theta_R=\pi$ represents no angle restriction as in Vicsek's model.

the point of view of an engineer, any robot or vehicle powered by an engine can not make an acute-angle turn in a very short time period. In order to more closely resemble behavior found in nature and to be useful in developing real-world applications, we introduce a restricted angle model for an SPO system and find that this restriction dramatically improves the synchronization properties.

Figure 1 describes the RASPO model in which each object with direction $\theta_i(t)$ updates its direction and position within a radius R. The model (i) calculates an average for the directions of all its neighbors $\bar{\theta}_i(t)$, (ii) calculates the changes from $\theta_i(t)$ to $\bar{\theta}_i(t)$, and determines whether it is smaller [Fig. 1(a)] or greater [Fig. 1(b)] than θ_R [if the change is smaller, the result is $\theta_i(t+1)=\bar{\theta}_i(t)$ and if it is greater, the result is $\theta_i(t+1)=\theta_i(t)+\theta_R$], and (iii) each object then updates its position according to Eq. (1). (For a rigorous mathematical description of the RASPO model, see Appendix.) The RASPO model, which generalizes the VM, has six main parameters n, L, R, v_0, η, and θ_R, each of which affects the synchronization differently. In order to study the different effects, we perform computer simulations of RASPO in which the density is fixed [3] at $\rho=n/L^2=1$ with periodic boundary conditions [19].

III. SIMULATION AND DISCUSSION

To investigate the main effect of introducing a restricted angle, we perform computer simulations of synchronization as a function of restricted angle θ_R for different n, and for several values of R and v_0 (see Fig. 2). Our simulation results show that synchronization increases significantly as θ_R decreases. This is counterintuitive since one would assume that giving more freedom to the objects would facilitate greater synchronization. It is clear that when θ_R is small, synchronization is high, while for $\theta_R=\pi$, where the model corresponds to the original VM, the synchronization is relatively low [35]. A plausible explanation for this finding is as follows. It seems that, even without any input of external noise, there exists some internal noise in the system, which is larger when θ_R is larger. Therefore, reducing θ_R is effectively like reducing the internal noise, which improves synchronization. Support for this conjecture can be found from Fig. 3. From Fig. 3(a), we can see that inherent noise is increasing with noise η, and from Fig. 3(b), we can also see that inherent noise is increasing with noise θ_R. So, decreasing the restricted angle θ_R may be regarded as

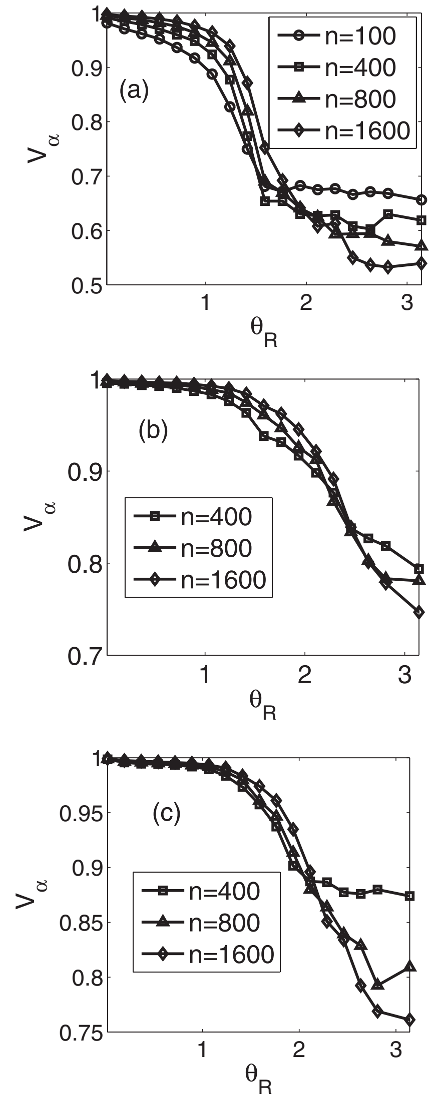

FIG. 2. The synchronization V_α as a function of restricted angle θ_R for different system size n. In the simulation, the density $\rho=1$, (a) $R=0.3$ and $v_0=0.1$, (b) $R=0.3$ and $v_0=0.4$, (c) $R=0.6$ and $v_0=0.4$. All the data points above are obtained by averaging over 300 different realizations. The synchronization is much larger when θ_R is small. Note that while the synchronization is significantly improved in the RASPO model, the convergence time increases when θ_R is smaller.

decreasing the inherent noise. We also find that when θ_R is large, a system of large n exhibits low synchronization, but when θ_R is small, a system of large n exhibits high synchronization. These results suggest that when n approaches infinity, the synchronization shows a phase transition as a function of θ_R. Thus, there seems to exist a critical restricted angle θ_{Rc}, below which the synchronization is high, and above which the synchronization is low. The simulation results also indicate that θ_{Rc} increases with v_0 and decreases with R. Thus, it seems that our realistic angle restriction assumption significantly improves the synchronization of the SPO and thus also makes our model more practical for technological applications.

To study the synchronization V_α as a function of the absolute velocity v_0, we perform numerical simulations of the RASPO

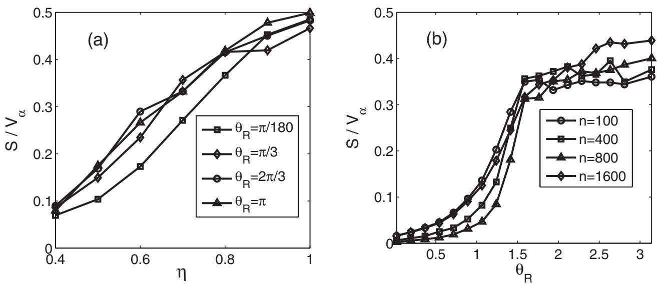

FIG. 3. (a) The relative standard deviation S/V_α as a function of η for different θ_R, where S is defined as $S = [\frac{1}{N-1}\sum_{i=1}^{N}(V_\alpha - \bar{V}_\alpha)^2]^{\frac{1}{2}}$. In the simulation, the density $\rho = 1, n = 200, R = 0.6$, and $v_0 = 0.1$. (b) The relative standard deviation of S/V_α as a function of θ_R for different n. All data points above are obtained by averaging over 400 different realizations.

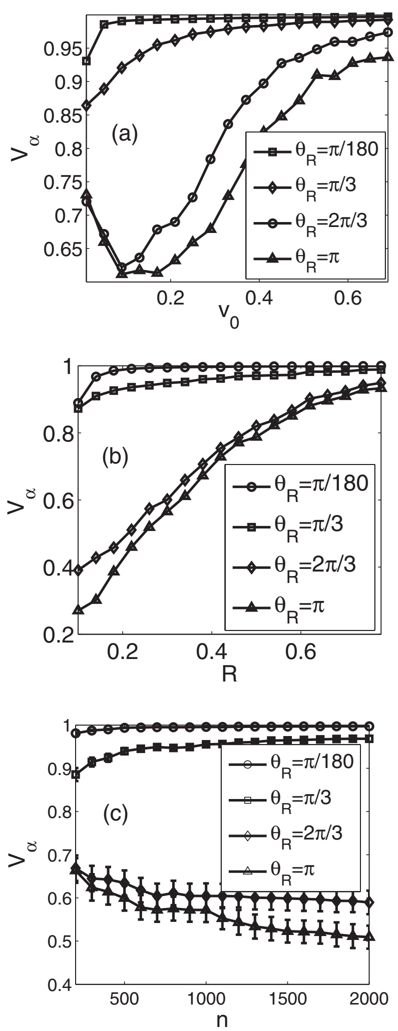

FIG. 4. (a) The synchronization V_α as a function of the absolute velocity v_0 for different restricted angle θ_R. In the simulation, $R = 0.3$, $\rho = 1$, and $n = 800$. (b) The synchronization V_α as a function of radius R for different restricted angle θ_R. In the simulation, $v_0 = 0.1$, $\rho = 1$, and $n = 800$. (c) The synchronization V_α as a function of system size n for different angle restricted angle θ_R. In the simulation, $R = 0.3$, $\rho = 1$, and $v_0 = 0.1$. All quantities are averaged over 300 realizations.

model for $n = 400$, $L = 20$, and $R = 0.3$ for various restricted angles θ_R [Fig. 4(a)]. The simulation results demonstrate that the synchronization V_α increases monotonically as the absolute velocity v_0 increases when the restricted angle θ_R is small, but V_α decreases and increases (has a minimum) as the absolute velocity v_0 increases when the restricted angle θ_R is large. Thus, it is implicit that the synchronization is significantly improved by a small restricted angle, in contrast to the original VM for a wide range of absolute velocity v_0.

We also study the synchronization V_α as a function of radius R. We perform the numerical simulations of RASPO for $n = 400$, $v_0 = 0.1$, and various restricted angles θ_R [see Fig. 4(b)]. Our simulation results indicate that, for a fixed θ_R, V_α is an increasing function of R, which implies that the synchronization is significantly improved when the restricted angle is small. In particular, the synchronization is surprisingly improved more for worse conditions, i.e., when R is small.

The synchronization V_α as a function of system size n for different restricted angles θ_R is shown in Fig. 4(c). It is particularly surprising that the synchronization V_α increases with n when the restricted angle θ_R is small, which is in sharp contrast with being a decreasing function of n when the restricted angle θ_R is large. Our simulation results also suggest that the synchronization converges to constant finite values when n is large.

We next study the effect of noise on restricted angle synchronization. In Fig. 5(a), we show the synchronization V_α as a function of noise amplitude η for different restricted angles θ_R. We see that V_α is a decreasing function of η and, as θ_R increases, V_α decreases slowly and then quickly, indicating

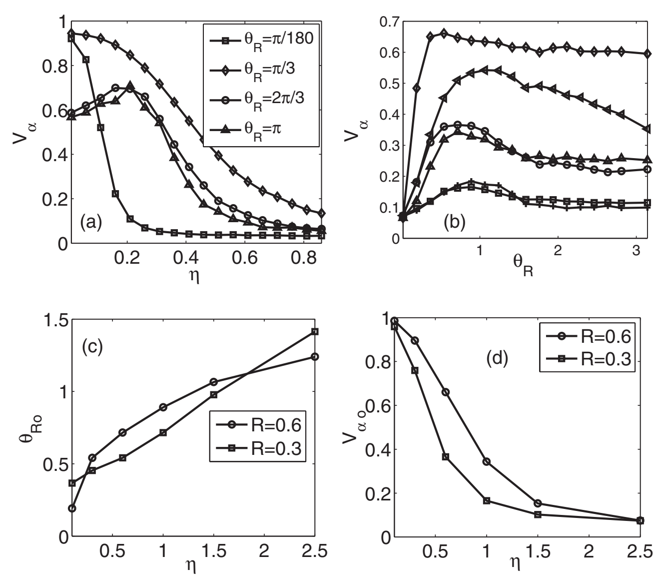

FIG. 5. (a) Synchronization V_α as a function of noise amplitude η for different restricted angle θ_R, $R = 0.6$, $v_0 = 0.1$. (b) Synchronization V_α as a function of restricted angle θ_R for $\bigcirc$: $\eta = 0.6, R = 0.3, v_0 = 0.1$; $\square$: $\eta = 1, R = 0.3, v_0 = 0.1$; $\diamond$: $\eta = 0.6, R = 0.6, v_0 = 0.1$; $\triangle$: $\eta = 1, R = 0.6, v_0 = 0.1$; $\triangleleft$: $\eta = 0.6, R = 0.3, v_0 = 0.3$, $+$ $\eta = 1, R = 0.3, v_0 = 0.3$. (c) Optimal restricted angle θ_{Ro} as a function of noise amplitude η, for different R values. $v_0 = 0.1$ (d) Optimal synchronization $V_{\alpha o}$ as a function of η for different R. In the simulation, $n = 200$ and all quantities are averaged over 300 realizations.

the existence of a peak when we plot the synchronization V_α as a function of the restricted angle θ_R [see Fig. 5(b)]. This means that, for each noise amplitude η, we can obtain an optimal value θ_{Ro} for θ_R [see Fig. 5(c)] and a corresponding optimal synchronization $V_{\alpha o}$ [see Fig. 5(d)].

IV. CONCLUSION

In summary, we have proposed a restricted angle model that significantly (i) improves the synchronization of SPO systems when the restricted angle decreases because reducing θ_R is effectively like reducing the internal noise which leads to improving synchronization, (ii) demonstrates the existence of a critical restricted angle θ_{Rc} above which the synchronization order parameter changes sharply form a large value to a small value, and (iii) reveals that for each noise amplitude η the synchronization shows a peak as a function of θ_R, so there exists an optimal θ_{Ro} for which one will obtain the best synchronization $V_{\alpha o}$. Note that a model of SPO with restricted *vision* was studied, where an optimal view angle was found to obtain the fastest convergence speed [36]. However, this model is very different from the RASPO model since the average direction of the objects within view angle can be in any direction and the angle change will be of all sizes. In contrast, in the RASPO model, the change of the angle is restricted.

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APPENDIX

In order to describe the RASPP model mathematically, we define the angle change function $C(\alpha,\beta)$,

Definition 1. The angle change $C(\alpha,\beta)$ denotes the change from angle α to angle β, where $\alpha,\beta \in [0,2\pi)$. So it is easy to obtain $C(\alpha,\beta) = C(\beta,\alpha)$.

The direction of the RASPP is updated as:

$$\theta_i(t+1) = \begin{cases} |\bar{\theta}_i(t) + \theta_R|_{2\pi} & \text{if} \quad C(\theta_i(t),\bar{\theta}_i(t)) > \theta_R, \quad \text{and} \quad C_l < C_r \\ \bar{\theta}_i(t) & \text{if} \quad C(\theta_i(t),\bar{\theta}_i(t)) \leqslant \theta_R \\ |\bar{\theta}_i(t) - \theta_R|_{2\pi} & \text{if} \quad C(\theta_i(t),\bar{\theta}_i(t)) > \theta_R, \quad \text{and} \quad C_l > C_r \end{cases}$$

where $\bar{\theta}_i(t)$ is obtained from Eq. (2), $C_l = C([\bar{\theta}_i(t) + \theta_R]_{2\pi},\bar{\theta}_i(t))$, $C_r = C([\bar{\theta}_i(t) - \theta_R]_{2\pi},\bar{\theta}_i(t))$, and $|\bullet|_{2\pi} = \bullet + 2k\pi \in [0,2\pi)$, $k \in Z$.

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