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# Quantifying and understanding the economics of large financial movements

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**Abstract**

Financial market data offer the exciting possibility of quantifying and understanding the physics of a complex dynamical system, and the hope that this line of thinking may give some insights into understanding collective human behavior. Various measures of stock market activity have been found to exhibit puzzling features that have recently attracted much research attention. These features include the power law distributions of return, volume, number of trades, assets under management of trading institutions, and other power-law relations linking them. Here, we review these empirical results and show that some of these findings can be usefully interpreted within the framework of a reduced-form model [Gabaix, X., Gopikrishnan, P., Plerou, V., Stanley, H.E., 2003. A theory of power-law distributions in financial market fluctuations. Nature 423, 267–270] and an economic model [Gabaix, X., Gopikrishnan, P., Plerou, V., Stanley, H.E., 2006b. Institutional investors and stock market volatility. Quarterly Journal of Economics 121, 461–504]. The features not only present a challenge to models of market fluctuations, but their specific power-law nature also suggests

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new modeling directions, which include ideas from statistical physics which proved useful in understanding similar relationships that occur in the physics of critical phenomena.


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## 1. Introduction

Statistical physics deals with systems comprising a very large number of interacting subunits, for which predicting the exact behavior of the individual subunit would be impossible. Hence, one is limited to making statistical predictions regarding the collective behavior of the subunits. In the last century, statistical physics has begun to address systems that are out of equilibrium, that is, are driven by external 'forces' for which the exact interactions between the subunits comprising the system are not known. Recently, it has come to be appreciated that many such systems which consist of a large number of interacting particles obey universal laws that are independent of the microscopic details. The finding, in physical systems, of universal properties that do not depend on the specific form of the interactions gives rise to the intriguing hypothesis that universal laws or results may also be present in economic systems.

The interest of physicists in economic systems has roots that date back at least as far as 1936, when the Italian physicist Majorana wrote a paper on the analogy between statistical laws in physics and in the social sciences (Majorana, 1936). Although Majorana's work was initially considered of marginal interest, physics research activity in this field has become less episodic and a research community has begun to emerge. The hope of some is that their efforts could in time provide a complementary approach to the approaches in economics, and indeed there is some cause for optimism along these lines.[1]

One of the key conceptual elements in modern statistical physics is the concept of scale invariance, codified in the scaling hypothesis that functions obey certain functional equations whose solutions are power laws.[2] The scaling hypothesis has two categories of predictions, both of which have been remarkably well verified by a wealth of experimental data on diverse systems. The first category is a set of relations, called *scaling laws*, that serve to relate the various critical-point exponents characterizing the singular behavior of functions such as thermodynamic functions. The second category is a sort of *data collapse*, where under appropriate axis normalization, diverse data 'collapse' onto a single curve called a scaling function.

[1] See, e.g., Anderson et al. (1988), Arthur et al. (1997), Blume and Durlauf (2005), Roehner (1995), Lux (1997, 1998), Brock and Hommes (1998), Durlauf (1993, 1996, 1997, 1999), Blume and Durlauf (1998a,b), Blume (1993), and Lux and Marchesi (1999).

[2] See, e.g., Bouchaud and Potters (2003), Mantegna and Stanley (2000) and Sornette (2000).

Our research has been addressing a key question of interest: quantifying and understanding large stock market fluctuations. Our work focussed on the challenge of quantifying the behavior of the probability distributions of large fluctuations of relevant variables such as the return, the volume, and the number of trades. Sampling the far tails of such distributions require a large amount of data, and indeed, analyzing the wealth of stock data has proven to be a challenge in itself. Literally every transaction of every stock is recorded. As a result, for each stock there are in the order of a few million data entries per year, and we analyze over one trillion observations. Our empirical focus has been to quantify and test the robustness of power-law distributions that characterize large movements in stock market activity. Using estimators that are designed for serially and cross-sectionally independent data, our findings thus far support the hypothesis that the power-law exponents that characterize fluctuations in stock price, trading volume, and the number of trades[3] are seemingly 'universal' in the sense that they do not change their values significantly for different markets, different time periods, or different market conditions.

Our second focus was on developing a theoretical understanding for the empirical facts. We have proposed a model that appears to explain the statistical regularities, first in a reduced-form (Gabaix et al., 2003), then in a fuller economic model (Gabaix et al., 2006b). In our model, large movements in stock market activity arise from the transactions of large participants. A key ingredient in our model is the empirical size distribution of large market participants (mutual funds), which we systematically studied. We demonstrated that when mutual funds trade in an optimal way, one finds the empirically observed power-law tails of the pdf's of returns, volumes and number of trades. Our model also explains the empirically observed relationships between large fluctuations in prices, trading volume, and the number of transactions. Such a theory, in which large participants 'move the market,' is consistent with independent evidence that stock market movements are difficult to explain only with changes in fundamental values (Cutler et al., 1989).

The second theme of modern statistical physics goes by the name 'universality.' [4] It was found empirically that one could form an analog of the Mendeleev table if one partitions all critical systems into 'universality classes.' Two systems belonging to the same universality class are described by the same power-law exponents, though the converse is not true. Remarkably, details of the interactions among the subunits constituting a complex system appear to matter less than one might have initially expected.

The 'inverse cubic law' of Eq. (1) holds over as many as 80 standard deviations for some stock markets, with $\Delta t$ ranging from 1 min to one month, across different sizes of stocks, different time periods, and also for different stock market indices (Jansen and de Vries, 1991; Lux, 1996; Gopikrishnan et al., 1999). These findings raise the possibility that the power-law distribution describing the return distribution is

[3]See, e.g., Mandelbrot (1963), Fama (1963), Lux (1996), Guillaume et al. (1997), Gopikrishnan et al. (1998, 1999, 2000) and Plerou et al. (1999, 2000, 2001).

[4]See, e.g., Stanley (1971, 1999), Stanley et al. (2001), and Mantegna and Stanley (2000).

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'universal'. Our analysis shows that the power laws (2) and (3) obtained for U.S. stocks also hold for a distinctly different market, consistent with the possibility that (2) and (3) hold across other markets as well. Moreover, data associated with extreme events – including the 1929 and 1987 market crashes – still conform to Eq. (1) (Gabaix et al., 2005).

## 2. Empirical results

We next describe the power laws and their associated exponents that we have found to characterize stock market variables. We primarily analyzed two databases (i) the NYSE's Trades and Quotes database, from which we analyzed every transaction for the 1,000 largest U.S. stocks for the 2-year period 1994–1995 – the stocks we selected are the largest by market value on January 1, 1994; and, (ii) the CRSP daily data from which we analyzed the 6,000 stocks for the 35-year period 1962–1996. In addition, we have analyzed returns of other major international equity indices.

Our analysis proceeded as follows. Define $R_t = \log P(t) - \log P(t - \Delta t)$ to be the return at time $t$ over a given time interval $\Delta t$, and $P(t)$ denotes the stock price at time $t$. We found that the probability that $R_t$ is in absolute value larger than $x$ was consistent with the power-law form (cf. Fig. 1) (Lux, 1996; Gopikrishnan et al., 1999).

$$P(|R_t| > x) \sim x^{-\zeta_R} \quad \text{with } \zeta_R \approx 3. \tag{1}$$

The specific values of $\zeta_R$ were calculated using the quasi-maximum likelihood estimator of Hill (1975). We calculate the inverse local slope $\gamma = \zeta_R^{-1}$ of the cumulative distribution function $P(R)$, $\gamma \equiv -(d \log P(R)/d \log R)^{-1}$ for the negative and the positive tail. We obtain an estimator for $\gamma$, by sorting the normalized increments by their size, $R^{(1)} > R^{(2)} > \cdots > R^{(N)}$. The cumulative distribution can then be written as $P(R^{(k)}) = k/N$, and we obtain for the local slope

$$\gamma = \left[ (N-1) \sum_{i=1}^{N-1} \log R^{(i)} \right] - \log R^{(N)},$$

where $N$ is the number of tail events used. We use the criterion that $N$ does not exceed 10% of the sample size, simultaneously ensuring that the sample is restricted to the tail events (Pagan, 1996). Using this estimator individually for each of the 1,000 stocks, we obtain the mean value over $\Delta t = 15$ min[5]

$$\zeta_R = \begin{cases} 3.10 \pm 0.03 & \text{(positive tail),} \\ 2.84 \pm 0.12 & \text{(negative tail).} \end{cases} \tag{2}$$

[5]Our preliminary analysis using generated time series with similar statistical properties both distributional and heteroskedastic show that a large proportion of the dispersion in measured exponents $\widehat{\zeta}_R^i$ for each stock $i$ is due to measurement noise. The asymmetry of the tail exponent, however, seems to be genuine.

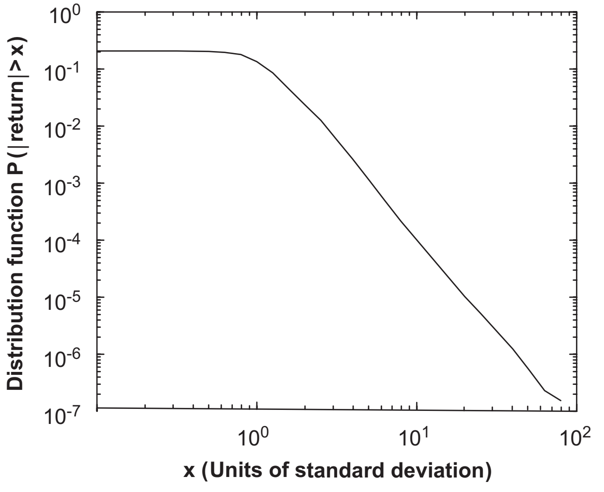


Fig. 1. Empirical cumulative distribution of the absolute values of the normalized 15 min returns of the 1,000 largest companies in the Trades And Quotes database for the 2-year period 1994–1995 (12 million observations). We normalize the returns of each stock so that the normalized returns have a mean of 0 and a standard deviation of 1. For instance, for a stock $i$, we consider the returns $r'_{it} = (r_{it} - r_i)/\sigma_{r,i}$, where $r_i$ is the mean of the $r_{it}$'s and $\sigma_{r,i}$ is their standard deviation. In the region $2 \leqslant x \leqslant 80$ we find an ordinary least squares fit $\ln P(|r| > x) = -\zeta_r \ln x + b$, with $\zeta_r = 3.1 \pm 0.1$. This means that returns are distributed with a power-law $P(|r| > x) \sim x^{-\zeta_r}$ for large $x$ between 2 and 80 standard deviations of returns. *Source*: Gabaix et al. (2003).

To explore the origins of the cubic distribution of returns, we also analyzed the same databases to calculate the distribution of aggregate trading volume $Q$,[6] which we found to have a distribution (Gopikrishnan et al., 2000; Maslov and Mills, 2001)

$$P(Q > x) \sim x^{-\zeta_Q} \quad \text{with } \zeta_Q \approx 1.5. \tag{3}$$

Further, we found a power-law distribution (Gopikrishnan et al., 2000) to hold for individual transaction size $q$ (Fig. 2)

$$P(q > x) \sim x^{-\zeta_q} \quad \text{with } \zeta_q \approx 1.5. \tag{4}$$

Finally, we found that the number of transactions $N$ in $\Delta t$ follows (Plerou et al., 2000):

$$P(N > x) \sim x^{-\zeta_N} \quad \text{with } \zeta_N = 3.4. \tag{5}$$

The mean values of exponents that we obtain for the 1,000 stocks in our database for $\Delta t = 15$ min are $\zeta_q = 1.53 \pm 0.07$, $\zeta_Q = 1.7 \pm 0.1$, and $\zeta_N = 3.40 \pm 0.05$. Here the error-bars on the mean are calculated under the assumption that the exponent estimates for each stock are independent.

[6]The volumes are here in normalized number of shares. Because the volumes $V_{it}$ for each share $i$ are normalized by their average value $V_i$, defining volume as normalized fraction of shares outstanding or normalized dollar value makes no material difference to the preliminary results.

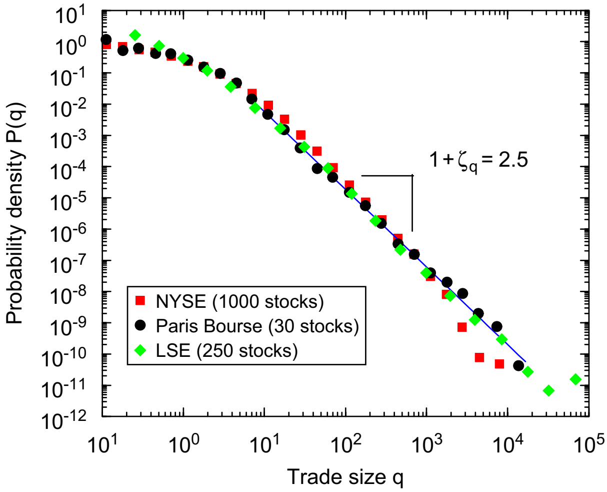


Fig. 2. Probability density of normalized individual transaction sizes $q$ for three stock markets: (i) NYSE for 1994–1995; (ii) the London Stock Exchange for 2001; and (iii) the Paris Bourse for 1995–1999. OLS fit yields $\ln p(x) = -(1+\zeta q)\ln x + \text{constant}$ for $\zeta q = 1.5 \pm 0.1$. This means a probability density function $p(x) \sim x^{-(1+\zeta_q)}$, and a countercumulative distribution function $P(q > x) \sim x^{-\zeta_q}$. The three stock markets appear to have a common distribution of volume, with a power law exponent of $1.5 \pm 0.1$. The horizontal axis shows invididual volumes that are up to $10^4$ times larger than the absolute deviation, $\overline{|q-\bar{q}|}$. *Source*: Gabaix et al. (2006b).

In the following, we refer to Eqs. (3) and (4) as, respectively, the '*half-cubic laws of trading volume*.'

### 2.1. *Theoretical framework*

Motivated by the empirical regularities we uncovered, we have begun to develop a theory in which we can interpret these results. Specifically, in Gabaix et al. (2003) we proposed a reduced-form model that provides an explanation for these empirical power-laws.

To test the assumptions of the theory, we first performed an empirical analysis of the distribution of the largest market participants – mutual funds. We found that, for the largest 10% of mutual funds, the market value of the managed assets $S$ for each year of the period 1961–1999 obeys the power law

$$P(S > x) \sim x^{-\zeta_S} \quad \text{with } \zeta_S = 1.05 \pm 0.08. \tag{6}$$

Exponents of $\approx 1$ have also been found for the cumulative distributions of city sizes (Zipf, 1949) and firm sizes (Axtell, 2001; Stanley et al., 1995), and the origins of this 'Zipf' distribution are becoming better understood.[7] Below, we show that if managers of large funds trade on their intuitions about the future direction of the

[7]See, e.g., Simon (1955), Gabaix (1999), Gabaix and Ioannides (2004), and Fu et al. (2005).

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market, and if they adjust their trading speed to avoid moving the market too much, then their trading activity leads to $\zeta_r = 3$ and $\zeta_Q = 1.5$.

We presented empirical evidence for the curvature of the price impact function, proposed an explanation for this curvature, and showed how the resulting trading behavior generates power laws (1)–(5).

The price impact $\Delta p$ function of a transaction of size $V$ is increasing and concave in $V$ (e.g., Hasbrouck, 1991; Plerou et al., 2002).[8] We hypothesized that for large volumes its functional form is

$$r = \Delta p \sim k V^{1/2} \tag{7}$$

for some constant $k$. When we aggregate over several transactions of size $V$, Eq. (6) predicts that the square return, $r^2$, varies linearly with the aggregate trading volume, $Q$ – a result consistent with our preliminary results (Plerou et al., 2004) that for large volumes $Q$

$$\mathrm{E}[r^2|Q] \sim Q. \tag{8}$$

Since Eq. (7) implies $P(r > x) \sim P(kQ^{1/2} > x) = P(Q > x^2/k^2) \sim x^{-2\zeta_Q}$, it follows that

$$\zeta_r = 2\zeta_Q. \tag{9}$$

Thus, the power law of returns, Eq. (1), follows from the power law of volumes, Eq. (3), and the square root form of price impact, Eq. (7). We are developing a framework for explaining Eqs. (3) and (7).

In our reduced-form model (Gabaix et al., 2003), large volumes and large returns are created by the transactions of large investors. Qualitatively, our theory can be motivated by the following argument. Consider a large investor who decides to trade in a stock because he perceives mispricing. For a sufficiently large investor, the desired quantity of the particular stock will be comparable to its daily turnover. Therefore, if the investor executes the transaction quickly, an undesirably large price impact will result. However, the alternative to performing the transaction quite slowly over time is not attractive either, since the mispricing cannot be expected to remain indefinitely. So, the desire to profit quickly from the transaction is balanced by the loss that will occur from the large price impact. Under a set of plausible conditions, we show that the investor's optimal trading behavior generates power-law distributions for returns, volumes and the number of transactions with the specific values of exponents consistent with our empirical results, i.e., the exponents $\zeta_Q = \zeta_q = \frac{3}{2}$ for the volume, and $\zeta_r = \zeta_N = 3$ for the returns and the number of transactions. The model makes 'out of sample' predictions about the relation between trading imbalance and returns which are qualitatively borne out by the data (Gabaix et al., 2003, Fig. 2).

[8] Here $V$ denotes the size of the block trade of a given trader and $Q$ denotes the aggregate volume traded by all market participants over a given time interval. The aggregate volume $Q$ is directly observable by the econometrician, but $V$ is not.

## 3. Large returns – are they driven by number of trades, volume, or something else?

The existence of scale-free power-law distributions, and the seemingly universal nature of exponents for stock markets are important for the applicability of the statistical physics paradigm. A further challenge is to understand the origin of these power-law distributions.

The question of understanding the power-law behavior of large returns is especially interesting since extreme stock market movements present challenging problems for existing theories of efficient markets. For example, there is enough evidence that crashes have occurred without significant causative news (see Cutler et al., 1989; Fair, 2002), and there seems to be significant excess volatility (Campbell and Shiller, 1989; French and Roll, 1986; Roll, 1988; Shiller, 1989; Shleifer, 2000), perhaps linked to limited arbitrage (Gabaix et al., 2007) or bounded rationality (Gabaix and Laibson, 2002; Gabaix et al., 2006a). Many theories have been proposed to accommodate crashes (Romer, 1993; Gennotte and Leland, 1990; Bernardo and Welch, 2004) but, as yet, there is no consensus. A typical view is to regard market movements in terms of 'normal events' and a few outliers for which a special model must be introduced. But if there is only one law describing both everyday events and rare events, then there is likely to be only one economic mechanism at work.

### 3.1. Returns and number of trades

We propose to analyze first the relationship between $R$ and $N$. Previous work has proposed that market activity measured by volume or number of trades may give rise to large returns, and in particular Clark (1973), Tauchen and Pitts (1983), Stock (1988), Ane and Geman (2000) have suggested that the fat-tailed behavior of returns may arise from the underlying process occurring in a subordinated intrinsic time. In our context, this question translates to whether $\zeta_R$ arises from $\zeta_N$. Our work detailed in the next sections shows that the value of $\zeta_N$ is not sufficient to explain $\zeta_R = 3$.

Our work is in the spirit of time deformation proposed by Clark (1973), Tauchen and Pitts (1983), Stock (1988), Lamoureux and Lastrapes (1990), and Engle and Russell (1998). Returns $R$ over a time interval $\Delta t$ can be expressed as the sum of several changes $\delta p_i$ due to the $i = 1, \ldots, N$ trades in the interval $[t, t + \Delta t]$,

$$R = \sum_{i=1}^{N} \delta p_i. \tag{10}$$

If $\Delta t$ is such that $N \gg 1$, and $\delta p_i$ have finite variance, then one can apply the classic version of the central limit theorem, whereby one would obtain the result that the unconditional distribution $P(R)$ is Gaussian (Clark, 1973). It is implicitly assumed in this description that $N$ has only *narrow* Gaussian fluctuations, i.e., has a standard deviation much smaller than the mean $\langle N \rangle$.

A investigation of $N$ suggests stark contrast with a Gaussian time series with the same mean and variance – there are several events of the magnitude of tens of standard deviations which are inconsistent with Gaussian statistics.[9] For each stock analyzed, we chose sampling time intervals $\Delta t$ such that it contains sufficient $N$; for actively traded stocks $\Delta t = 15$ min, and for stocks with the least frequency of trading, $\Delta t = 390$ min (1 day) (Plerou et al., 2000). We find that the distribution of $N$ appears to display an asymptotic power-law decay

$$P\{N > x\} \sim x^{-\zeta_N} (x \gg 1). \tag{11}$$

For the 1,000 stocks that we analyze, we estimate $\zeta_N$ using Hill's method (Hill, 1975) and obtain a mean value $\zeta_N = 3.40 \pm 0.05$. Note that $\zeta_N > 2$ is outside the Lévy stable domain $0 < \zeta_N < 2$ and is inconsistent with a stable distribution for $N$, and with the lognormal hypothesis of Clark (1973).

Since we have found that $P\{R > x\} \sim x^{-\alpha}$, we can ask whether the value of $\zeta_N$ we find for $P\{N > x\}$ is sufficient to account for the fat tails of returns. To test this possibility, we shall implement, for each stock, the ordinary least squares regression

$$\ln |R(t)| = a + b \ln N(t) + \psi(t), \tag{12}$$

where $\psi(t)$ has mean zero and the equal time covariance $\langle N\psi(t) \rangle = 0$. Preliminary results on 30 actively traded stocks yield the average value of $b = 0.57 \pm 0.09$.

We note that values of $b \approx 0.5$ are consistent with what we would expect from Eq. (10), if $\delta p_i$ are *i.i.d.* with finite variance. In other words, suppose $\delta p_i$ are chosen *only from the interval* $[t, t + \Delta t]$, and let us hypothesize that *these* $\delta p_i$ are mutually independent, with a common distribution $P(\delta p_i | t \in [t, t + \Delta t])$ having a finite variance $W^2$. Under this hypothesis, the central limit theorem, applied to the sum of $\delta p_i$ in Eq. (10), implies that

$$|R(t)| \sim W(t) \sqrt{N(t)}. \tag{13}$$

Eq. (13) implies that the fat tails of $P\{|R(t)| > x\} \sim x^{-\alpha}$ cannot be caused solely due to $P\{N > x\} \sim x^{-\zeta_N}$, because by conservation of probabilities $P\{\sqrt{N} > x\} \sim x^{-2\zeta_N}$ with $2\zeta_N \approx 6.8$. Eq. (13) then implies that $N$ alone cannot explain the value $\alpha \approx 3$.

### 3.2. Large returns and volume

The second possibility that we examine is the connection between large returns and volume. Indeed, as the old Wall Street adage goes, 'it takes volume to move prices.' The present subsection presents evidence for that view, which is generally supportive.

To examine this relationship in detail, we analyze the price impact function for the 1,000 largest stocks in our NYSE TAQ database. Our results support the hypothesis that large returns arise from large volumes. Indeed this is also predicted by our theory (Gabaix et al., 2006b) that we outlined above. Under some circumstances, this theory predicts that the price impact function takes a square-root form.

[9] See, for example, Clark (1973), Mandelbrot and Taylor (1962), Epps and Epps (1976), Tauchen and Pitts (1983), Stock (1988), Engle and Russell (1998), Guillaume et al. (1995), Ane and Geman (2000), Ghysels et al. (1996), Jones et al. (1994), and Plerou et al. (2000).

This particular functional relationship gives a natural connection between return and volume, i.e., $\zeta_R = 2\zeta_Q$ – a relationship bourne out by the empirical data.

Here we present results of our analysis of the price impact function, i.e., the relation between returns and volume. In order to quantify variations in demand and supply, we distinguish two types of trades – *buyer and seller initiated* – based on which of the two participants in the trade is more eager to execute the trade. When such distinction is not possible, we label the trade as *indeterminate*. We quantify demand-supply fluctuations through the *order imbalance*, defined as the difference between the number of shares traded in buyer-initiated and seller-initiated trades in a time interval $\Delta t$,

$$\Phi \equiv \sum_{i=1}^{N} q_i a_i, \tag{14}$$

where $a_i \in \{-1, 0, 1\}$ has value $-1$ if the trade is seller initiated, 1 if the trade is buyer initiated, and 0 if the trade is indeterminate, $q_i$ is the number of shares traded in each trade, and $N$ is the number of trades in $\Delta t$. We use the procedure of Lee and Ready (1991) to determine $a_i$.

We quantify the equal-time dependence of order-imbalance and price change using the conditional expectation function

$$F(\Phi) \equiv \mathbf{E}(\Delta p|\Phi), \tag{15}$$

which gives the equal-time expectation value of price change $\Delta p$ for a given order imbalance $\Phi(t)$. Analysis indicates $F(\Phi) \sim \Phi^{\beta}$ for small $\Phi$ while for large $\Phi$ the curve seems to plateau (Plerou et al., 2002; Lillo et al., 2003; Farmer and Lillo, 2004).

The above estimation, however, suffers from the fact that in practice, large orders are executed by splitting into orders of smaller size (Chan and Lakonishok, 1995; Keim and Madhavan, 1997) which are observed in the trade time series as the trade size $q_i$. The true impact function $\mathbf{E}(\Delta p|V)$ is indeed notoriously difficult to measure since the information about the unsplit order size is usually proprietary and not available. The quantity of interest pertains to $\Delta p$, the total impact in price of a large order of size $V$.

Consider an example. Suppose that a large fund wants to buy a large number $V$ of shares of a stock whose price is \$100. The fund's dealer may offer this large volume for a price of \$101. Before this transaction, however, the dealer must buy the shares. The dealer will often do that progressively in many steps, say 10 in this example. In the first step, the dealer will buy $V/10$ shares, and the price will go say, from \$100 to \$100.1, and in the second the price will go from \$100.1 to \$100.2. After some time elapses, the price will have gone to \$101 in increments of \$0.1. At this stage, the dealer has his required number of shares, and hands them over to the fund manager at a price of \$101. The true price impact here is 1%, since the price has gone from \$100 to \$101. But in any given transaction, the price has moved by no more than \$0.1. So an analysis of $\mathbf{E}(\Delta p|V)$ would find an 'apparent' price impact of no more than \$0.1, i.e., 0.1% of the price. Since as the transaction is executed the price of the stocks goes from \$100 to \$101, the true price impact is 1%. As a result the procedure above will measure a value 10 times smaller than the true value. This downward bias perhaps explains the plateau behavior of $\mathbf{E}(\Delta p|V)$ for large $V$.

We can quantify the bias in the above example. Suppose that a trade of size $V$ is split into $K = V^{\alpha}$ (10 in our example) trades of equal size $q = V/K = V^{1-\alpha}$, with $0<\alpha<1$. Then the apparent impact $\delta p$ incurred by each trade (0.1% in our example) will be $1/K$ ($\frac{1}{10}$ in our example) of the total price impact $V^{\beta}$ (1% in our example), i.e., $\delta p = V^{\beta}/K = V^{\beta-\alpha}$. So a power law fit of $\delta p$ vs $q$ will give $\delta p \sim q^{\beta'}$ with

$$\beta' = (\beta - \alpha)/(1-\alpha) < \beta.$$

So the measurement of price impact directly from Eq. (15) leads to a biased measurement $\beta'$ of the exponent $\beta$ of the true price impact.

To address this bias we examine $\mathbf{E}(r^2|Q)$ in Gabaix et al. (2003). As is well established empirically, the sign of returns is unpredictable in the short term, so the reasoning in Gabaix et al. (2006b) shows that $\mathbf{E}(r^2|Q)$ will not be biased.

Our analysis (Gabaix et al., 2003) was presented with data for the 116 most actively traded stocks. To check if the result of $\beta = 0.2$ for large volumes presented in Lillo et al. (2003) could arise from increasing the size of the database, we now extend our analysis to the 1,000 largest stocks in our database for the 2-year period 1994–1995. Fig. 3 shows that $\mathbf{E}(r^2|Q) \sim Q$ is consistent with our theory. This regression is, however, not definitive evidence for Eq. (7). This regression is performed in fixed $\Delta t$ so is exposed to the effect of fluctuations in the number of trades – i.e., if $N$ denotes the number of trades in $\Delta t$, $r^2 \sim N$ and $V \sim N$ so Eq. (8) could be a consequence of this effect. Fortunately, when we perform the analysis

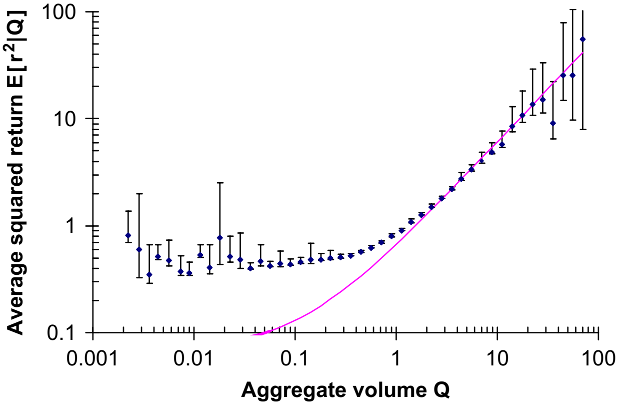


Fig. 3. Conditional expectation $E[r^2|Q]$ of the squared return $r^2$ in $\Delta t = 15$ min, given the aggregate volume $Q$ in $\Delta t$. $r$ is in units of standard deviation, and $Q$ in units of absolute deviation, $\overline{|Q - \bar{Q}|}$. The results are averaged over the largest 100 stocks in the New York Stock Exchange market capitalization on January 1, 1994. The data spans the 2-year period 1994–1995 and is obtained from the Trades and Quotes database, which records all transactions for all listed securities in the NYSE, AMEX and NASDAQ. One cannot reject $E[r^2|Q] = \alpha + \beta Q$ large enough ($Q \geqslant 3$). This is consistent with a square root price impact of large trades. *Source*: Gabaix et al. (2006b).

with a constant number of trades (Plerou et al., 2004), we find again support for Eq. (8).

In sum, we find empirical support for the view that large trades move prices, and can explain quantitatively the cubic power law of returns. This result is potentially important because, as articulated in Gabaix et al. (2006b), it provides support for a unified explanation of the power laws of volumes and returns.

Encouraging though it is, this result does not close the debate, as there are surely other sources of large returns. For instance, work by Farmer and Lillo (2004) suggests that large fluctuations are not due to large volumes, but rather due to liquidity fluctuations (see also Plerou et al., 2005). This is indeed a valid possibility, and further work is needed to carefully discern the role of volume and liquidity in price formation. The main other candidate explanation is that prices would be purely driven by news, that would at the same time move number of trades, volume, and liquidity. Further research should investigate this possibility systematically.

## 4. Quantifying and understanding the origin of long-ranged volatility persistence

A third focus of our proposed research is to quantify and understand long-range dependencies in financial data. Although we have focused above on distributional aspects of returns, volume, and market activity, these variables also show remarkable time dependence. Indeed, it is well known that the volatility displays correlations that persist for significant periods of time. Several previous studies have reported that the volatility of price fluctuations has correlations that decay slowly.[10] Our analysis not only confirms the long-persistence of volatility correlations, but more importantly shows that these correlations decay as a power-law function. This is especially interesting in the view of the market as a strongly interacting system, since power-law decay of correlation functions is one of the signatures of strongly interacting physical systems.

The origin of these correlations are especially puzzling, since most models predict short-ranged correlations. We extend the analysis of time correlations to related variables such as $N$ and $Q$. Our results show the existence of power-law correlations in both $N$ and $Q$ with almost identical values of power-law exponents describing the decay of correlation functions. Our work suggests that the long-ranged correlations in volatility and in volume arise from those of $N$.

The dependencies in the data can be seen even in the behavior of the distribution of returns on varying time scales. Since the values of $\zeta_R$ we find are inconsistent with a statistically stable law, we expect the distribution of returns $P(r > x)$ on larger time scales to converge to Gaussian. In contrast, our analysis of daily returns from the CRSP database suggests that the distributions of returns retain the same functional form for a wide range of time scales $\Delta t$, varying over three orders of magnitude, $5\ \text{min} \leqslant \Delta t \leqslant 6240\ \text{min} = 16\,\text{days}$. The *onset* of convergence to a Gaussian starts to

[10]See, e.g., Campbell et al. (1997), Ding et al. (1993), Granger and Ding (1996), and Andersen et al. (2001).

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occur only for $\Delta t > 16$ days (Plerou et al., 1999). In contrast, $n$-partial sums of computer-simulated time series of the same length and probability distribution display Gaussian behavior for $n \geqslant 256$ (Gopikrishnan et al., 1999). Thus, the rate of convergence of $P(R)$ to a Gaussian is remarkably slow, indicative of time dependencies (Campbell et al., 1997; Lo and MacKinlay, 1988) which violate the conditions necessary for the central limit theorem to apply.

To test for time dependencies, we performed a preliminary analysis of the autocorrelation function of returns, which we denote $\langle G(t)G(t+\tau)\rangle$, using 5 min returns of 1,000 stocks. Our preliminary results show pronounced short-time ($<30$ min) anti-correlations, consistent with the bid-ask bounce (Campbell et al., 1997). For larger time scales, the correlation function is at the level of noise, consistent with the efficient market hypothesis.[11] Lack of linear correlation does not imply independent returns, since there may exist higher-order correlations. Our recent studies (Liu et al., 1997, 1999) show that the amplitude of the returns measured by the absolute value or the square has long-range correlations with persistence[12] up to several months,

$$\langle |G(t)||G(t+\tau)|\rangle \sim \tau^{-a}, \tag{16}$$

where $a$ has the average value $a = 0.34 \pm 0.09$ for the 1,000 stocks studied. In order to detect genuine long-range correlations, the effects of the U-shaped intra-day pattern (Wood et al., 1985; Admati and Pfleiderer, 1988) for $|G|$ has been removed (Liu et al., 1997). This result is consistent with earlier studies[13] which also noted long-range correlations. In addition to analyzing the correlation function directly, we are also applying power spectrum analysis and the recently developed detrended fluctuation analysis (Liu et al., 1997; Peng et al., 1994). Both of these methods yield consistent estimates of the exponent $a$. We shall also apply estimators such as those developed in Robinson (1994, 1995) to obtain accurate estimates of the exponent $a$.

To better understand the origin of correlations in $|R|$, we shall analyze time correlations in $N$, and attempt to relate it to the time correlations of $|R|$. If $N$ is a long-range correlated variable, it is indeed possible that the volatility correlations arise from $N$ since $R_t^2 \sim N$, from Eq. (13), so correlation structure of $N$ translates to the same in $R_t^2$.

Preliminary studies on the same 30 actively traded stocks indicate that the autocorrelation function $\langle N(t)N(t+\tau)\rangle \sim \tau^{-\nu}$, with a mean value of the estimates of $\nu = 0.32 \pm 0.09$ using the detrended fluctuation analysis (Peng et al., 1994). To detect genuine long-range correlations, the marked U-shaped intra-day pattern (Wood et al., 1985; Admati and Pfleiderer, 1988) in $N_{\Delta t}$ is removed (Liu et al., 1997). It would be interesting to substantiate the analysis of long memory using semi-parametric estimators such as those due to Robinson[14], and test the dependence of the exponent $\nu$ on the type of industry sector, and market capitalization.

[11] See, e.g., Campbell et al. (1997) and Fama (1965, 1970, 1991).

[12] See, for example, Granger (1966, 1980), Granger and Joyeux (1980), and Beran (1994).

[13] See, e.g., Campbell et al. (1997), Ding et al. (1993), Granger and Ding (1996), and Andersen et al. (2001).

[14] See, e.g., Robinson (1994, 1995).

Finally, investigations on the 30 stocks seem to indicate the absence of long-range correlations in $W$, the above investigation of correlations could yield the interesting statement that the long-range correlations in volatility are due to those of $N$. Together with the above discussion on distribution functions, these preliminary results are indicative of an interesting dichotomy – that the fat tails of returns $R$ arise from $W$ and the long-range volatility correlations arise from trading activity $N$.

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