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New elements for a theory of the Barkhausen effect

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Abstract. We present a microscopical model of the domain wall motion through a disordered medium. Unlike a previous phenomenological model, which explains most of the experimental data, the disorder is supposed to be uncorrelated. The dynamical equation of the motion has a upper critical dimension of 3, so that a mean field description is suitable to describe soft magnetic materials. It is shown that the correlation of the disorder is a consequence of the nature of the magnetic interactions and not an intrinsic property of the materials. The mean field critical exponents well agree with the phenomenological model and with simulated and experimental data.

1. INTRODUCTION

The Barkhausen effect still represents an interesting method to investigate the nature of the magnetization processes and a precious tool to characterize soft magnetic materials. Up to now, a long series of papers have appeared discussing the possibility to describe the complicated features of the Barkhausen noise [1] and to obtain useful information both about the magnetic structure and the dynamics of the magnetization processes. Specifically, the main point under investigation is the explanation of the $1/f^{\beta}$ type noise and the power laws observed at low applied field rates, a feature common with many complex systems as earthquakes, superconductors, etc., displaying critical behavior. A decisive step toward a general explanation of the Barkhausen noise has been achieved in the past by a phenomenological theory [2] (know as ABBM model) based on the hypothesis of the domain wall motion over a disordered medium with spatial brownian correlations, as suggested by a previous experiment on a Fe-Si single crystal [3]. This model gives a remarkable quantitative prediction of the power law exponents, but not more than a qualitative description of power spectrum shape, as it predicts $\beta = 2$, while data on soft materials give $1.6 < \beta < 2$ [1, 2]

As a matter of fact, the true nature of the brownian pinning field remains obscure, as quenched-in disorder cannot be correlated on the spatial scale of typical Barkhausen jumps [4]. Therefore, alternative approaches, based on a microscopical description of the domain wall motion with uncorrelated disorder, has been recently proposed [5, 6]. However, the numerical values of the exponents do not agree with the experimental results. In this paper, we present a microscopical equation of the domain wall motion through an uncorrelated disordered medium at the depinning transition, and find that the corresponding critical exponents very well agree with the experimental results [7].

2. DOMAIN WALL DYNAMICS

We consider the case of a single flexible domain wall separating two regions of opposite magnetization. Assuming that the wall does not form overhangs, we describe its position by the height $h(r,t)$. As the motion is governed by eddy current damping, the dynamics equation of the domain wall is given by

$$\frac{\partial h(r,t)}{\partial t} = -\frac{dE(\{h(r,t)\})}{dh(r,t)} \qquad (1)$$

where $E(\{h(r,t)\})$ is the total energy of the system, function of the position of the wall. As the wall is flexible, we must include all the energies related to its deformation and to the appearance of free magnetic charges in the system, so that we assume the total energy to be

$$E = E_H + E_{dem} + E_{loc} + E_{dw} + E_{rnd} \qquad (2)$$

where E_H is the energy of the system due to the external field, E_{dem} and E_{loc} are the demagnetizing energies due to free magnetic charges at the ends and inside the system, respectively, E_{dw} is the elastic energy due to the domain wall deformation and the last term includes any other random energy E_{rnd} [8]. Adding up all these terms and using eq. (1), with the external field *H(t)* applied along the magnetization *M*, we obtain [7]

$$\frac{\partial h(r,t)}{\partial t} = H(t) - kM^2 \int d^2r'\, h(\vec{r}',t) + \int d^2r'\, K(\vec{r}-\vec{r}')(h(\vec{r}',t) - h(\vec{r},t)) + n\nabla^2 h(\vec{r},t) + h(\vec{r}',h) \qquad (3)$$

where the sum on the right side corresponds to the terms of eq. (2).

The demagnetizing energy E_{dem} has been assumed proportional to the total magnetization, as we do not expect very large fluctuations of wall height, with the constant *k* accounting for the sample geometry. The other demagnetizing energy E_{loc}, due the local free charge density, has been calculated [8, 9] assuming a small bending of the wall, where $K(\vec{r}-\vec{r}')$ is the non local kernel due to dipolar-dipolar interaction given by

$$K(\vec{r}-\vec{r}') = \frac{A}{|\vec{r}-\vec{r}'|^3}\left(1 + \frac{3(x-x')^2}{|\vec{r}-\vec{r}'|^2}\right) \qquad (4)$$

and *x* is the direction of the magnetization. Also the wall energy has been calculated [9] for small deformations of the wall, giving the usual term associated to elastic tension of a surface. Finally, the last term is chosen so that its derivative, i.e. the pinning field, is a random process correlated only on a small range $\langle h(\vec{r},h)\, h(\vec{r},h')\rangle = d^2(\vec{r}-\vec{r}')R(h-h')$, where $R(h-h')$ decays very rapidly along the wall.

To see how the eq. (3) describes the motion of the wall at the depinning transition, let us rewrite it in a simpler fashion

$$\frac{\partial h(r,t)}{\partial t} = H(t) - kM^2 \int d^2r'\, h(\vec{r}',t) - H_c \qquad (5)$$

where H_c is the sum of the last three fields. We first consider the case of absence of demagnetizing fields due to the sample geometry ($k = 0$). For $H < H_c$ the wall is pinned, while it takes a finite speed when $H > H_c$. The depinning field H_c has the same meaning at the coercive field in free demagnetizing field samples, like toroids and picture frames [3, 10]. At H_c, the displacements of the wall occur by avalanches (Barkhausen jumps) which show the typical power law distributions of sizes *s* and durations *T* , $P(s) \sim s^{-\tau}$ and $P(T) \sim T^{-\alpha}$, respectively, where τ and α are the critical exponents. For $k > 0$, the demagnetizing field reduces the effective applied field, thus the wall remains at the depinning transition after it has reached it [11]. This critical behavior is progressively lost with the increase of the driving field rate, as long as the field becomes strong enough to move every part of the wall.

The critical exponents can be easily calculated as eq. (3) turns out to have an interesting property: the presence of the non local kernel *K* sets the upper critical dimension to 3, instead of 5 as in conventional elastic interfaces [7, 11]. We thus expect that mean field exponents describe the experimental results on usual soft magnetic materials. Apart from logarithmic corrections, these exponents are $\tau = 3/2$ and $\alpha = 2$ [7, 11]. These are the zero driving rate limit calculated in the ABBM model using simple fractal properties of the signal, as they were found to be $\tau = 3/2 - c/2$ and $\alpha = 2 - c$, where c is proportional to the driving rate [2]. This relationship between the two models is not merely a coincidence: the starting equation of the ABBM model has the same structure of eq. (5), where the random field H_c is a space brownian process. Using a discrete infinite range version ($d \to \infty$) of eq. (3), which has the same critical behavior as the tridimensional version, we verified [7] that the effective random pinning field H_c for the average position of the wall has brownian correlations, provided that the jumps could be considered as independent, a reasonable assumption at low driving rates. This clearly states that the brownian description of the pinning field, and the experiment itself performed on the Fe-Si single crystal [3], are the consequences of nature of the magnetic interactions and do not result from any particular property of the quenched-in disorder.

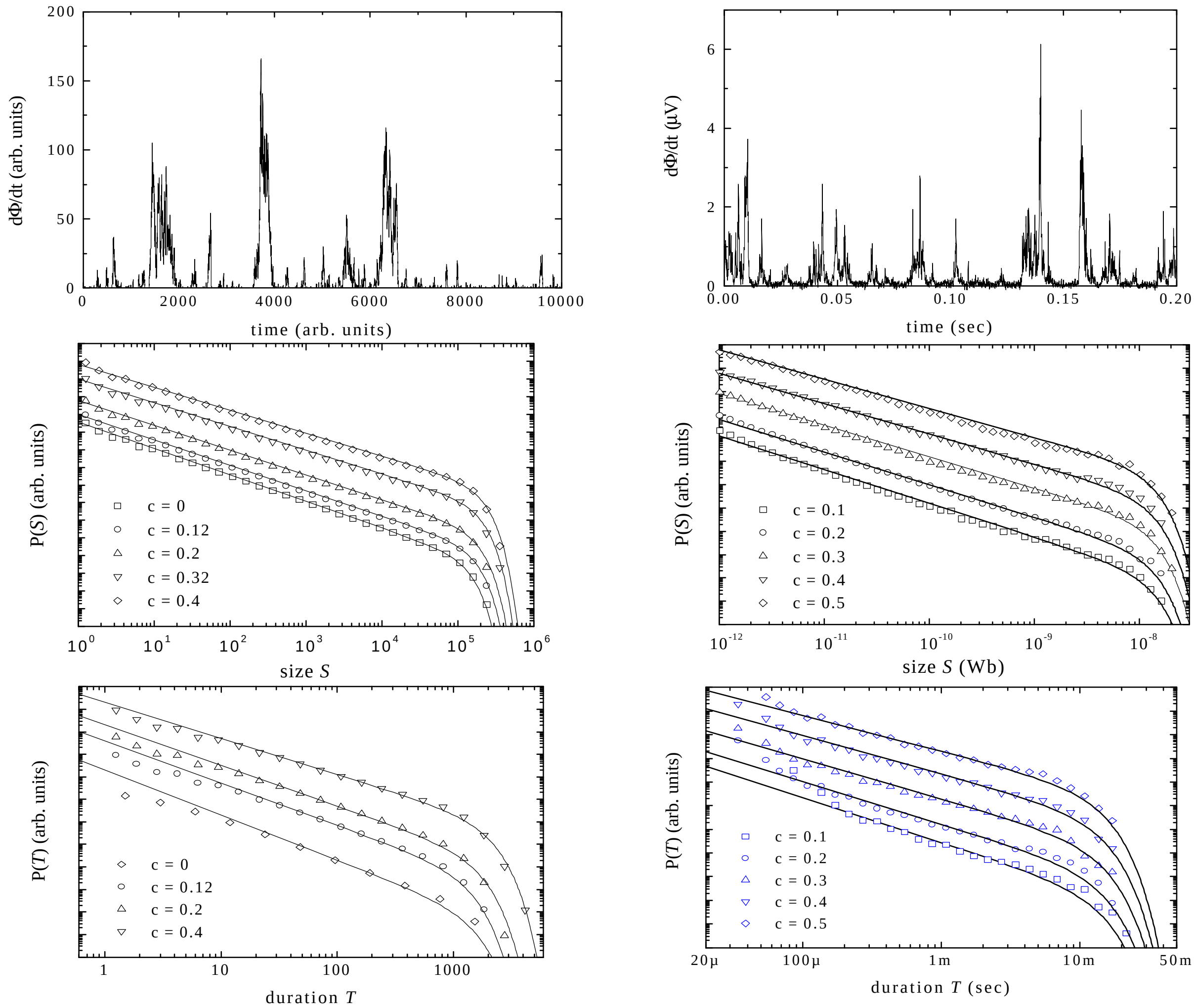

Figure 1: Simulated (left) and experimental ($Fe_{73}Co_{12}B_{15}$ stress annealed amorphous alloy, right) time signals and power law distributions. Data are compared to the theoretical predictions (see text) at different values of the parameter c, proportional to the driving rate. The same formula is used for simulations and experiments. In the figures at the bottom, short duration jumps are underestimated in simulations because of the discretization of the system and the finite number of sites, while are overestimated in the experiments because of spurious background noise.

3. SIMULATIONS AND EXPERIMENTAL RESULTS

We simulate the Barkhausen using the discrete infinite range version of eq. (2), with 32696 sites, using a linear driving rate $H(t) \propto t$, as usual in the Barkhausen experiments [2]. For comparison, the signal and the power law distribution are shown together with the experimental data of a $Fe_{73}Co_{12}B_{15}$ stress annealed amorphous alloy (fig. 1). Remarkably enough, we used the same fitting function $P(x)=x^{-y} \exp(-(x/x_o)^2)$, both for sizes ($y = 3/2 - c/2$) and durations ($y = 2-c$). The reason of this form instead of the simpler exponential cut-off is not known, but it is not surely due to the low statistics of data, as experiments were performed using about 10^5 jumps. It worth then noting that the experimental data are obtained in a very complex sample, where number of domains is of the order of 50 or more, while our theory just considers a single wall. A simple explanation of this result can be given: the domains experiment different values of the local depinning transition field but, given the low driving rate, they are kept close to it. Even if, in principle, they can interact, and thus change their local pinning fields, this does not take them too far from the depinning transition, so that we actually see the same critical behavior for each domain.

Unfortunately, the same argument does not hold when considering the shape of power spectra. Our simulations give $\beta = 2$ as a consequence of the brownian character of the effective pinning field, quite different from the experimental results. In any case, the amplitude of the simulated spectra rescales with the driving rate, which is a fundamental property of experimental data [1, 2]. Up to know, it is not yet clear how to modify the domain wall equation to explain the power spectrum exponent variability: an improved version of the model shall take into account multiple domains and time effects (flux propagation, after-effect) which surely affect the spectral properties.

To summarize, we have shown that a domain wall exhibits a critical behavior associated with the depinning transition and that the Barkhausen effect in usual soft magnetic materials (strips, bars) can be described by the corresponding mean field theory. In addition, the long range correlations of the effective pinning field are a consequence of the nature of the magnetic interactions and are not due to any intrinsic disorder property of quenched-in disorder.

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