ELSEVIER

Physica A 299 (2001) 127–136

PHYSICA A

www.elsevier.com/locate/physa

# A model for the growth dynamics of economic organizations

L.A.N. Amaral[a,*,1], P. Gopikrishnan[a], V. Plerou[a,b], H.E. Stanley[a]

[a] *Department of Physics, Center for Polymer Studies, Boston University, Boston, MA 02215, USA*

[b] *Department of Physics, Boston College, Boston, MA 02215, USA*

**Abstract**

We apply methods and concepts of statistical physics to the study of economic organizations. We identify robust, universal, characteristics of the time evolution of economic organizations. Specifically, we find the existence of scaling laws describing the growth of the size of these organizations. We study a model assuming a complex evolving internal structure of an organization that is able to reproduce many of the empirical findings. © 2001 Elsevier Science B.V. All rights reserved.

*Keywords:* Scaling; Universality; GDP; Countries; Business firms; Organizations; Growth; Dynamics

## 1. Introduction

At one time, it was imagined that "scale-free" phenomena are relevant to only a fairly narrow slice of physical phenomena [1,2]. However, the range of systems that apparently display power law and hence scale-invariant correlations has increased dramatically in recent years, ranging from base pair correlations in non-coding DNA [3,4], lung inflation [5], plaque aggregation in Alzeihmer's disease [6–8], and interbeat intervals of the human heart [9–15] to complex systems involving large numbers of interacting subunits that display "free will", such as ecologic food webs [16–18], city growth [19–21], network formation [22–24], stock price fluctuations [25–32] and currency exchange fluctuations [33].

We have recently shown that scale invariance holds for economic organizations [34–38]. Namely, we found that the distributions of growth rates for both business

* Corresponding author.

*E-mail address:* amaral@buphy.bu.edu (L.A.N. Amaral).

[1] WWW: http://polymer.bu.edu/∼amaral



PII: S0378-4371(01)00287-4

firms and the gross domestic product (GDP) of entire countries are described by the same functional form and that the standard deviation of the distribution depends on organization size as a power law. Our goal is to bring to bear on these problems concepts and methods of statistical physics. Specifically, we present a stochastic model that is able to reproduce the empirical findings and makes further predictions about the internal structure of economic organizations.

The remainder of this paper is organized as follows. Section describes our finding of scaling and universality in social systems. Section 3 describes our model. Section 4 presents some concluding remarks.

## 2. Scaling and universality in the growth of economic organizations

In the study of physical systems, the scaling properties of fluctuations in the output of a system often yield information regarding the underlying processes responsible for the observed macroscopic behavior [1,2,39,40]. With that in mind, we analyzed the fluctuations in the growth rates of different economic organizations.

### 2.1. Empirical results for business firms

In collaboration with an economist, Michael A. Salinger of Boston University, we investigated the growth dynamics of US business firms. A classic problem in industrial organizations is the size distribution of business firms [41–44]. For some time, it was assumed that firm size obeyed a rank-size law [45–51], that is, that the distribution of sizes decays a power law of the size. In Fig. 1(a), we show the distribution of log-sizes for US business firms, it is clear that the distribution has a fast decaying tail, inconsistent with a power law dependence.

We next consider the annual growth rate—that is to say, the fluctuation—of a firm's size,

$$g(t) \equiv \log\left(\frac{S(t+1)}{S(t)}\right), \tag{1}$$

where $S(t)$ and $S(t+1)$ are the sales in US dollars of a given firm in the years $t$ and $t+1$, respectively. We expect that the statistical properties of the growth rate $g$ depend on $S$, since it is natural that the magnitude of the fluctuations $g$ will decrease with $S$. Therefore, we partition the firms into bins according to their sales—the size of the firm. Fig. 1(b) shows a log-linear plot of the probability distribution of growth rates for three sizes. In such a plot, a Gaussian distribution has a parabolic shape. It is apparent from the graph that the distributions are not Gaussian. Furthermore, it appears from the graph that the form of the distributions for the different sizes are similar. Indeed, Fig. 1(b) suggests that the *conditional* probability density, $p(g|S)$, has the same functional form, with different widths, for all $S$.

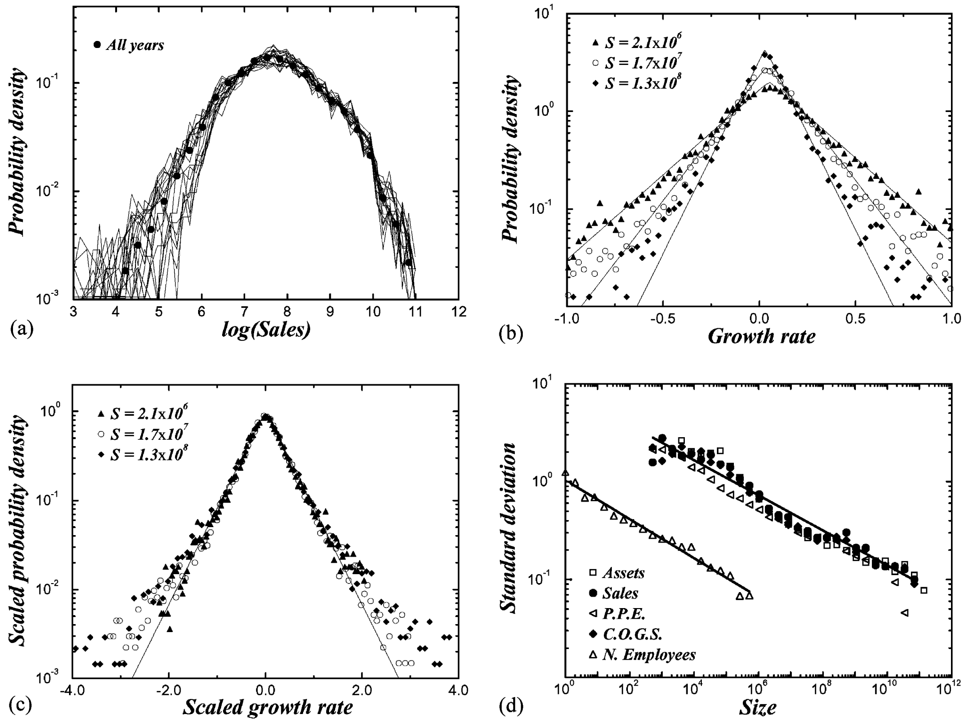


Fig. 1. (a) Histogram of the sales $S$ for publicly-traded manufacturing companies (with standard industrial classification index of 2000–3999) in the US for *each* of the years in the 1974–1993 period. All the values for sales were adjusted to 1987 dollars by the GDP price deflator. Also shown (solid circles) is the average over the 20 years. It is visually apparent that the distribution is approximately stable over the period. (b) Probability density $p(r|S)$ of the growth rate $r$ for all publicly-traded US manufacturing firms in the 1994 Compustat database with Standard Industrial Classification index of 2000–3999. The distribution represents all annual growth rates observed in the 19-yr period 1974–1993. We show the data for three different bins of initial sales. The solid lines are exponential fits to the empirical data close to the peak. We can see that the wings are somewhat "fatter" than what is predicted by an exponential dependence. (c) Scaled probability density $p_{\rm scal} \equiv \sigma p(g|S)$ as a function of the scaled growth rate $g_{\rm scal} \equiv [g - \bar{g}]/\sigma$. The values were rescaled using the measured values of $\bar{g}$ and $\sigma$. All the data collapse upon the universal curve $p_{\rm scal} = f(-|g_{\rm scal}|)$. (d) Standard deviation of the 1-year growth rates $\sigma$ for different definitions of the size of a company as a function of the initial values. We find that $\sigma \sim S^{-\beta}$. The straight lines are guides for the eye and have slopes 0.19.

To test if the conditional distribution of growth rates has a functional form independent of the size of the company, we plot the scaled quantities:

$$\sigma(S)\, p\left(\frac{g}{\sigma(S)}\middle| S\right) \quad \text{vs.} \quad \frac{g}{\sigma(S)} \,. \tag{2}$$

Fig. 1(c) shows that the scaled conditional probability distributions "collapse" onto a single curve [40], suggesting that $p(g|S)$ follows a universal scaling form

$$p(g|S) \sim \frac{1}{\sigma(S)}\, f\left(\frac{g}{\sigma(S)}\right) , \tag{3}$$

where the function $f$ is independent of $S$.

Next we calculate the standard deviation $\sigma(S)$ of the distribution of growth rates as a function of $S$. Fig. 1(d) demonstrates that $\sigma(S)$ decays as a power law

$$\sigma(S) \sim S^{-\beta}, \tag{4}$$

with $\beta = 0.19 \pm 0.05$. One may ask if these results are only valid when the size of the firm is defined to be the sales. To test this possibility, we perform similar analysis defining the size of the firms as (i) the number of employees, (ii) the assets, (iii) cost of goods sold (COGS), and (iv) plants, property and equipment (PPE). Fig. 1(d) confirms that consistent results are obtained for all the above measures.

These results are intriguing for a number of reasons. First, we find consistent results for a set of firms belonging to a wide range of industries (from services in the bin for the smallest firms to oil and car companies in the bin for the largest firms). Second, we find consistent results for quite different types of measures of a firms' size, some such as COGS, PPE, assets and number of employees are input measures, while sales is an output measure. These two points suggest that universality is present in the growth dynamics of business firms. Third, we find power law scaling in the width of the distribution of growth rates, an unexpectedly "simple" results that suggests that simple mechanisms may explain our observations.

### *2.2. Empirical results for countries*

In collaboration with another economist, David Canning from The Queen's College in Dublin and Harvard University, we extended the analysis described in the previous subsections to the economy of countries. As earlier, we first consider the distribution of sizes $S$ of a countries economy. Usually, the size of an economy is quantified by the gross domestic product (GDP) of the country [52]. Here, we detrend $S$ by the world average growth rate, calculated for all the countries and years in our database [53]. We find that $p(\log S)$ is consistent with a Gaussian distribution, implying that $P(S)$ may be a log-normal. We also find that the distribution $P(S)$ does not depend on the time period studied.

Next, we calculate the distribution of annual growth rate $g$, as defined in Eq. (1), where $S(t)$ and $S(t+1)$ are the GDP of a country in the years $t$ and $t+1$. As for business firms, we expect that the statistical properties of the growth rate $g$ depend on $S$, since it is natural that the magnitude of the fluctuations $g$ will decrease with $S$. Therefore, we partition the countries into bins according to their GDPs. We calculate the probability distribution of growth rates for three GDP sizes (small, medium and large) and find that the distributions are not Gaussian. Furthermore, as for business firms, the form of the distributions for the different sizes are consistent.

To test if the conditional distribution of growth rates has a functional form independent of the size of the company, we plot the scaled quantities (2). Fig. 2(a) shows that the scaled conditional probability distributions "collapse" onto a single curve [40], suggesting that $p(g|S)$ follows the universal functional form (3).

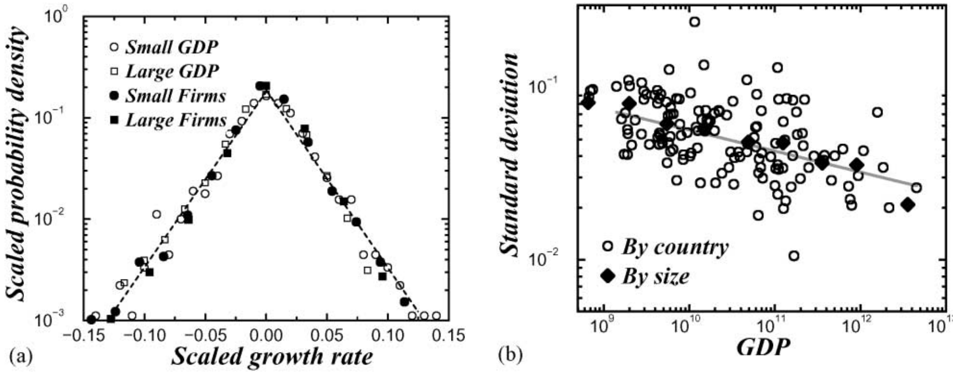


Fig. 2. (a) Probability density function of annual growth rate for two subgroups with different ranges of $G$, where $G$ denotes the GDP detrended by the average yearly growth rate. The entire database was divided into three groups: $6.9 \times 10^7 \leqslant G < 2.4 \times 10^9$, $2.4^9 \leqslant G < 2.2 \times 10^{10}$, and $2.2 \times 10^{10} \leqslant G < 7.6 \times 10^{11}$, and the figure shows the distributions for the smallest and largest groups. We consider only three subgroups in order to have enough events in each bin for the determination of the distribution. We plot the scaled probability density function, $\sigma(S)p(g/\sigma(S)|S)$, of the scaled annual growth rate, $(g - \bar{g})/\sigma(S)$ to show that all data collapse onto a single curve. (b) Standard deviation $\sigma(S)$ of the distribution of annual growth rates as a function of $S$, together with a power law fit (obtained by a least square linear fit to the logarithm of $\sigma$ vs the logarithm of $S$). The slope of the line gives the exponent $\beta$, with $\beta = 0.15$. We show the calculated standard deviation for two procedures: (i) for each individual country over the 42-yr period of the data, and (ii) for binned data according to size of GDP.

We next calculate the standard deviation $\sigma(S)$ of the distribution of growth rates as a function of $S$. Fig. 2(b) demonstrates that $\sigma(S)$ decays as a power law, $\sigma(S) \sim S^{-\beta}$, with $\beta = 0.15 \pm 0.05$. We have also confirmed these results by a maximum-likelihood analysis [54]. In particular, we find that the log-likelihood of $p(g|S)$ being described by an exponential distribution—as opposed to a Gaussian distribution—is of the order of $e^{600}$ to 1. Similarly, we test the log-likelihood of $\sigma$ obeying (4). We find that Eq. (4) is $e^{130}$ more likely than $\sigma(G) = \text{const}$, and that adding an additional nonlinear term to (4) does not increase the log-likelihood.

Surprisingly, we find that the same functional form appears to describe the probability distribution of annual growth rates for both the GDP of countries and the sales of firms; cf. Fig. 2(a). This result strongly suggests that universality, as defined in statistical physics, holds for the growth dynamics of economic organizations.

## 3. Modeling the growth dynamics of economic organizations

We next address the question of how to interpret our empirical results. We first note that an organization, such as a business firm, will comprise several subunits—the divisions of a firm. A reasonable zero-order approximation [55] is that the size of the different subunits comprising a firm will grow independently. Hence, we may view the growth of the size of each firm as the sum of the independent growth of subunits with different sizes. A model incorporating these assumptions [56] was recently

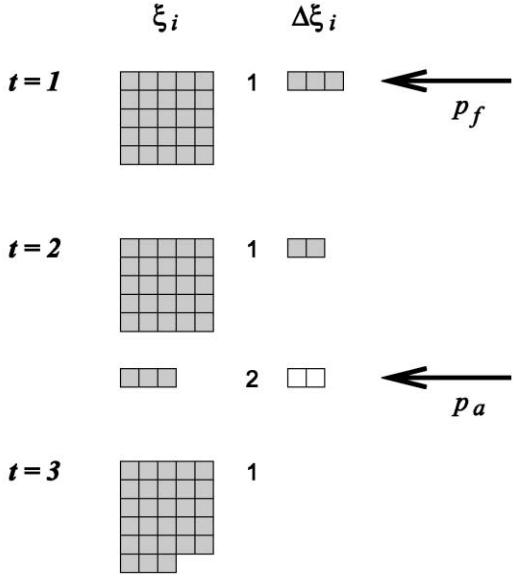


Fig. 3. Schematic representation of the time evolution of the size and structure of a firm. We choose $S_{\min} = 2$, and $p_f = p_a = 1.0$. The first column of full squares represents the size $\xi_i$ of each division, and the second column represents the corresponding change in size $\Delta\xi_i$. Empty squares represent negative growth and full squares positive growth. We assume, for this example, that the firm has initially one division of size $\xi_1 = 25$, represented by a $5 \times 5$ square. At $t = 1$, division 1 grows by $\Delta\xi_1 = 3$. A new division, numbered 2, is created because $\Delta\xi_1 > S_{\min} = 2$, and the size of division 1 remains unchanged, so for $t = 2$, the firm has 2 divisions with sizes $\xi_1 = 25$ and $\xi_2 = 3$. Next, divisions $\xi_1$ and $\xi_2$ grow by 2 and $-2$, respectively. Division 2 is absorbed by division 1, since otherwise its size would become $\xi_2 = 3 - 2 = 1$ which is smaller than $S_{\min}$. Thus, at time $t = 3$, the firm has only one division with size $\xi_1 = 25 + 2 + 1 = 28$. Note that if division 1 would be absorbed, then division 2 would absorb division 1 and would then be renumbered 1. If, division 1 is absorbed and there are no more divisions left, the firm "dies".

proposed to describe the scale-invariant growth dynamics of different types of organizations.

Our model dynamically builds a diversified, multi-divisional structure, reproducing the fact that a typical firm passes through a series of changes in organization, growing from a single-product, single-plant firm, to a multi-divisional, multi-product firm [57]. The model reproduces a number of empirical observations for a wide range of values of parameters and provides a possible explanation for the robustness of the empirical results. Indeed, our model may offer a generic approach to explain power law distributions in other complex systems.

The model, illustrated in Fig. 3, is defined as follows. A firm is created with a single division, which has a size $\xi_1(t = 0)$. The size of a firm $S \equiv \sum_i \xi_i(t)$ at time $t$ is the sum of the sizes of the divisions $\xi_i(t)$ comprising the firm. We define a minimum size $S_{\min}$ below which a firm would not be economically viable, due to the competition between firms; $S_{\min}$ is a characteristic of the industry in which the firm operates. We assume that the size of each division $i$ of the firm evolves according to a random multiplicative process [41–51]. We define

$$\Delta\xi_i(t) \equiv \xi_i(t)\,\eta_i(t)\,, \tag{5}$$

where $\eta_i(t)$ is a Gaussian-distributed random variable with zero mean and standard deviation $V$ independent of $\xi_i$. The divisions evolve as follows:

(i) If $\Delta\xi_i(t) < S_{\min}$, division $i$ evolves by changing its size, and $\xi_i(t+1) = \xi_i(t) + \Delta\xi_i(t)$. If its size becomes smaller than $S_{\min}$—i.e. if $\xi_i(t+1) < S_{\min}$—then with probability $p_a$, division $i$ is "absorbed" by division 1. Thus, the parameter $p_a$ reflects the fact that when a division becomes very small it will no longer be viable due to the competition between firms.

(ii) If $\Delta\xi_i(t) > S_{\min}$, then with probability $(1 - p_f)$, we set $\xi_i(t+1) = \xi_i(t) + \Delta\xi_i(t)$. With a probability $p_f$, division $i$ does not change its size—so that $\xi_i(t+1) = \xi_i(t)$—and an altogether new division $j$ is created with size $\xi_j(t+1) = \Delta\xi_i(t)$. Thus, the parameter $p_f$ reflects the tendency to diversify: the larger is $p_f$, the more likely it is that new divisions are created.

The present model rests on a small number of assumptions. The three key assumptions are: (i) firms tend to organize themselves into multiple divisions once they achieve a certain size. This assumption holds for many modern corporations [57], (ii) there is a broad distribution of minimum scales in the economy. This assumption has also been verified empirically [58], (iii) growth rates of different divisions are independent of one another. For an economist, the third is the stronger of the these assumptions. However, a recent study by John Sutton of the London School of Economics finds empirical support for this hypothesis [55].

## 4. Discussion

There are two features of our results that are perhaps surprising. First, although firms in our model consist of independent divisions, we do not find $\beta = 1/2$. One can derive an expression for $\beta$ in terms of the parameters of the model [56]

$$\beta = \frac{w}{2(v+w)}\,. \tag{6}$$

To gain intuition on the results predicted by this expression, consider two representative cases: (1) $v = 0$, which implies that $\beta = 1/2$, and (2) $v = w$, which implies that $\beta = w/(4w) = 1/4$. So, for a wide range of the values of the model's parameters, we find $v > w$ implying that $\beta$ is remarkably close to the empirical value $\beta \approx 0.2$.

Second, the distribution $p(g|S)$ is not Gaussian but "tent" shaped. We find this result arises from the integration of nearly-Gaussian distributions of the growth rates over the distribution of $S_{\min}$.

An additional feature of the model that is of interest is the fact that it makes predictions regarding the internal structure of the organizations. Specifically, the model predicts that the number of subunits comprising an organization and the typical size of these subunits obey scaling laws [56]. We have recently confirmed these predictions for the growth dynamic of R& D expenditures at US universities [37].

## Acknowledgements

We thank M. Barthélemy, S.V. Buldyrev, D. Canning, X. Gabaix, S. Havlin, P.Ch. Ivanov, H. Kallabis, Y. Lee, H. Leschhorn, F. Liljeros, M. Luwel, P. Maass, M. Meyer, H. Moed, B. Roehner, M.A. Salinger, M.H.R. Stanley for stimulating discussions. The CPS is supported by NSF and NIH.

## References

[1] H.E. Stanley, Introduction to Phase Transitions and Critical Phenomena, Oxford University Press, Oxford, 1971.
[2] R. Jackiw, Introducing scale symmetry, Phys. Today 25 (1) (1972) 23–27.
[3] C.-K. Peng, S.V. Buldyrev, A.L. Goldberger, S. Havlin, F. Sciortino, M. Simons, H.E. Stanley, Long-range correlations in nucleotide sequences, Nature 356 (1992) 168–171.
[4] A. Arneodo, E. Bacry, P.V. Graves, J.F. Muzy, Characterizing long-range correlations in DNA-sequences from wavelet analysis, Phys. Rev. Lett. 74 (1995) 3293–3296.
[5] B. Suki, A.-L. Barabási, Z. Hantos, F. Peták, H.E. Stanley, Avalanches and power law behaviour in lung inflation, Nature 368 (1994) 615–618.
[6] B.T. Hyman et al., Quantitative analysis of senile plaques in Alzheimer disease: observation of log-normal size distribution and of differences associated with apolipoprotein E genotype and trisomy 21 (Down syndrome), Proc. Natl. Acad. Sci. USA 92 (1995) 3586–3590.
[7] L. Cruz et al., Aggregation and disaggregation of senile plaques in Alzheimer disease, Proc. Natl. Acad. Sci. USA 94 (1997) 7612–7616.
[8] R.B. Knowles et al., Plaque-induced neural network disruption in Alzheimer's disease, Proc. Natl. Acad. Sci. USA 96 (1999) 5274–5279.
[9] C.-K. Peng, S. Havlin, H.E. Stanley, A.L. Goldberger, Quantification of scaling exponents and crossover phenomena in nonstationary heartbeat time series, Chaos 5 (1995) 82–87.
[10] L.A.N. Amaral, A.L. Goldberger, P.Ch. Ivanov, H.E. Stanley, Scale-independent measures and pathologic cardiac dynamics, Phys. Rev. Lett. 81 (1998) 2388–2391.
[11] P.Ch. Ivanov, L.A.N. Amaral, A.L. Goldberger, H.E. Stanley, Stochastic feedback and the regulation of biological rhythms, Europhys. Lett. 43 (1998) 363–369.
[12] P.Ch. Ivanov, L.A.N. Amaral, A.L. Goldberger, S. Havlin, M.G. Rosenblum, Z. Struzik, H.E. Stanley, Multifractality in human heartbeat dynamics, Nature 399 (1999) 461–465.
[13] P.Ch. Ivanov, A. Bunde, L.A.N. Amaral, J. Fritsch-Yelle, R.M. Baevsky, S. Havlin, H.E. Stanley, A.L. Goldberger, Sleep-wake differences in scaling behavior of the human heartbeat: analysis of terrestrial and long-term space flight data, Europhys. Lett. 48 (1999) 594–600.
[14] A.L. Goldberger, L.A.N. Amaral, L. Glass, S. Havlin, J.M. Hausdorff, P.Ch. Ivanov, R.G. Mark, J.E. Mietus, G.B. Moody, C.-K. Peng, H.E. Stanley, Physiobank, physiotoolkit, and physionet: components of a new research resource for complex physiologic signals, Circulation 101 (2000) e215–e220.
[15] P. Bernaola-Galván, P.Ch. Ivanov, L.A.N. Amaral, H.E. Stanley, Scale invariance in the nonstationarity of physiologic signals, arXiv:cond-mat/0005284, 2000.
[16] L.A.N. Amaral, M. Meyer, Environmental changes, coextinction, and patterns in the fossil record, Phys. Rev. Lett. 82 (1999) 652–655.
[17] J. Camacho, R. Guimerà, L.A.N. Amaral, Analytical solution of a model for complex food webs, arXiv:cond-mat/0102127, 2001.

[18] J. Camacho, R. Guimerà, L.A.N. Amaral, Robust patterns in food web structure, arXiv: cond-mat/0103114, 2001.
[19] M. Batty, P. Longley, Fractal Cities, Academic Press, San Diego, 1994.
[20] H. Makse, S. Havlin, H.E. Stanley, Modeling urban growth patterns, Nature 377 (1995) 608–612.
[21] X. Gabaix, Zipf's law for cities: an explanation, Q. J. Econ. 114 (1999) 739–767.
[22] L.A.N. Amaral, A. Scala, M. Barthélémy, H.E. Stanley, Classes of small-world networks, Proc. Nat. Acad. Sci. USA 97 (2000) 11 149–11 152.
[23] M. Barthélémy, L.A.N. Amaral, Small-world networks: evidence for a crossover picture, Phys. Rev. Lett. 82 (1999) 3180–3183.
[24] A. Scala, L.A.N. Amaral, M. Barthélémy, Small-world networks and the conformation space of a lattice polymer chain, arXiv:cond-mat/0004380, 2000.
[25] B.B. Mandelbrot, The variation of certain speculative prices, J. Business 36 (1963) 394–419.
[26] A. Pagan, The econometrics of financial markets, J. Empirical Finance 3 (1996) 15–102.
[27] R.N. Mantegna, H.E. Stanley, Scaling behavior in the dynamics of an economic index, Nature 376 (1995) 46–49.
[28] R.N. Mantegna, H.E. Stanley, Turbulence and exchange markets, Nature 383 (1996) 587–588.
[29] T. Lux, The stable paretian hypothesis and the frequency of large returns: an examination of major German stocks, Appl. Financial Econ. 6 (1996) 463–475.
[30] U.A. Muller, M.M. Dacorogna, O.V. Pictet, Heavy tails in high-frequency financial data, in: R.J. Adler, R.E. Feldman, M.S. Taqqu (Eds.), A Practical Guide to Heavy Tails, Birkhäuser Publishers, Basel, 1998, pp. 83–311.
[31] V. Plerou, P. Gopikrishnan, L.A.N. Amaral, M. Meyer, H.E. Stanley, Scaling of the distribution of price fluctuations of individual companies, Phys. Rev. E 60 (1999) 6519–6529.
[32] P. Gopikrishnan, V. Plerou, L.A.N. Amaral, M. Meyer, H.E. Stanley, Scaling of the distributions of fluctuations of financial market indices, Phys. Rev. E 60 (1999) 5305–5316.
[33] U.A. Muller, M.M. Dacorogna, R.B. Olsen, O.V. Pictet, M. Schwarz, C. Morgenegg, J. Bank. Finance 14 (1990) 1189–1195.
[34] M.H.R. Stanley, L.A.N. Amaral, S.V. Buldyrev, S. Havlin, H. Leschhorn, P. Maass, M.A. Salinger, H.E. Stanley, Scaling behaviour in the growth of companies, Nature 379 (1996) 804–806.
[35] L.A.N. Amaral, S.V. Buldyrev, S. Havlin, H. Leschhorn, P. Maass, M.A. Salinger, H.E. Stanley, M.H.R. Stanley, Scaling behavior in economics: I. Empirical results for company growth, J. Phys. I France 7 (1997) 621–633.
[36] Y. Lee, L.A.N. Amaral, D. Canning, M. Meyer, H.E. Stanley, Universal features in the growth dynamics of complex organizations, Phys. Lev. Lett. 81 (1998) 3275–3278.
[37] V. Plerou, L.A.N. Amaral, P. Gopikrishnan, M. Meyer, H.E. Stanley, Similarities between the growth dynamics of university research and of competitive economic activities, Nature 400 (1999) 433–437.
[38] H.F. Moed, M. Luwel, Science policy: the business of research, Nature 400 (1999) 411–412.
[39] H.E. Stanley, Power laws and universality, Nature 378 (1995) 554–555.
[40] H.E. Stanley, Scaling, universality, and renormalization: three pillars of modern critical phenomena, Rev. Mod. Phys. 71 (1999) S358–S366 (Special Issue for the Centennial of the American Physical Society).
[41] R. Gibrat, Les Inégalités Economiques, Sirey, Paris, 1931.
[42] Y. Ijiri, H.A. Simon, Skew Distributions and the Sizes of Business Firms, Amsterdam, North-Holland, 1977.
[43] B.H. Hall, The relationship between firm size and firm growth in the US manufacturing sector, J. Indust. Econ. 35 (1987) 583–606.
[44] J. Sutton, Gibrat's legacy, J. Econ. Lit. 35 (1997) 40–59.
[45] V. Pareto, Cours d'Economie Politique, Lausanne and Paris, 1897.
[46] P. Lévy, Théorie de l'Addition des Variables Aléatoires, Gauthier-Villars, Paris, 1937.
[47] G.K. Zipf, Human Behavior and the Principle of Least Effort, Addison-Wesley, Cambridge MA, 1949.
[48] G.R. Carroll, National city-size distribution: What do we know after 67 years of research? Prog. Hum. Geogr. VI (1982) 1–43.
[49] M. Levy, S. Solomon, Power laws are logarithmic Boltzmann laws, Int. J. Mod. Phys. C 7 (1996) 595.
[50] O. Malcai, O. Biham, S. Solomon, Power-law distributions and Lévy-stable intermittent fluctuations in stochastic systems of many autocatalytic elements, Phys. Rev. E 60 (1999) 1299.

[51] A. Blank, S. Solomon, Power laws in cities population, financial markets and internet sites (scaling in systems with a variable number of components), Physica A 287 (2000) 279–288.
[52] S.N. Durlauf, On the convergence and divergence of growth rates, Econ. J. 106 (1996) 1016–1018.
[53] R. Summers, A. Heston, The penn world tables (mark 5): an expanded set of international comparisons, 1950–1988, Q. J. Econ. 106 (1991) 327–368.
[54] W.H. Press et al., Numerical Recipes, 2nd Edition, Cambridge University Press, Cambridge, 1992.
[55] J. Sutton, The variance of firm growth rates: the 'scaling' puzzle, London School of Economics, 2000, working paper.
[56] L.A.N. Amaral, S.V. Buldyrev, S. Havlin, M.A. Salinger, H.E. Stanley, Power law scaling for a system of interacting units with complex internal structure, Phys. Rev. Lett. 80 (1998) 1385–1388.
[57] A. Chandler, Strategy and Structure, MIT Press, Cambridge, 1962.
[58] M. Gort, Diversification and Integration in American Industry, Princeton University Press, Princeton, 1962.