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ABSTRACT

The static scaling law hypothesis for thermodynamic functions is formulated by the statement that the singular part of any thermodynamic potential is a generalized homogeneous function (GHF), where by definition a function $f(x, y)$ is a GHF if there exist two numbers a, b such that for all positive values of λ, $f(\lambda^a x, \lambda^b y) = \lambda f(x, y)$. We show that all Legendre transforms and all partial derivatives of a GHF are also GHFs. Since every thermodynamic function is related to a given thermodynamic potential by some combination of Legendre transforms and partial derivatives, it follows that all thermodynamic functions are GHFs. We then observe that every such function has a simple power law singularity at the critical point, and that the value of every critical-point exponent can be written down by inspection in terms of the two initial numbers a, b. Consideration of different paths of approach to the critical point lead to an important class of direct tests of the scaling hypothesis; among these are the familiar relations among certain combinations of critical-point exponents. Finally, the problems of extension to functions of more than two variables are discussed.