HOMEWORK 1

Please submit your homework to xm@bu.edu. Don't forget to attach figures and code. Feel free to ask me if you have any question. GLHF! -Sean.

Problem 1: simple statistics, PDF, and time series

For this problem you need to find the daily closing prices of the S&P 500 index from 01/03/2006 to 12/31/2009 (1007 data points).

1. Make a line-plot of the data with the x-axis being date. Can you tell from the graph when the latest global financial crisis happened? You might have become a millionaire if you had shorted the stock market at the right time!

2. Make a line-plot of the log daily returns (i.e., the differences of the logarithms of daily prices) of the S&P 500 index. Can you tell from the graph when the volatility of U.S. stock market reached a local minimum? A local maximum?

3. Make a histogram of the log daily returns. Does it look like a Gaussian distribution? Calculate the expectation and variance of the log returns and use them to fit your histogram with a Gaussian probability density function (PDF). Compare your fit with the histogram. (You can normalize your histogram first and plot it together with your Gaussian fit.) What do you find? Now plot them on a log scale to confirm your findings.

4. Create a list of 1006 random numbers sampled from the same Gaussian PDF. Make a line-plot of the random "data points" and compare it to what you have plotted in Question 2. Can you tell the difference? Which one is heteroskedastic (which one exhibits time-dependent volatility)?

5. A collection of random variables can usually be defined as a stochastic time series if indexed by time. The set of random variables $\{X_t | t = 1, 2, 3 \dots\}$ being sampled from the Gaussian PDF can be regarded as an important stochastic process called the **white noise**. Now, we define a new set of random variables, $\{Y_t = \sum_{s=1}^{t} X_s | t = 1, 2, 3 \dots\}$. Plot Y_t over time. Do you know what the name of the new time series is? What is the difference from X_t?