Phase synchronization in coupled nonlinear oscillators

Limei Xu

Department of Physics
Boston University

Collaborators: Plamen. Ch. Ivanov, Zhi Chen, Kun Hu

H. Eugene Stanley

Z. Chen et al., Phys. Rev. E 73, 031915 (2006)

L. Xu et al., Phys. Rev. E (R) 73, 065201 (2006)

Why study synchronization?

- Synchronization is often present in physiological systems

Example:

• In human heart coupled cells fire synchronously, producing a periodic rhythm governing the contractions of the heart

- Alteration of synchronization may provide important physiological information:

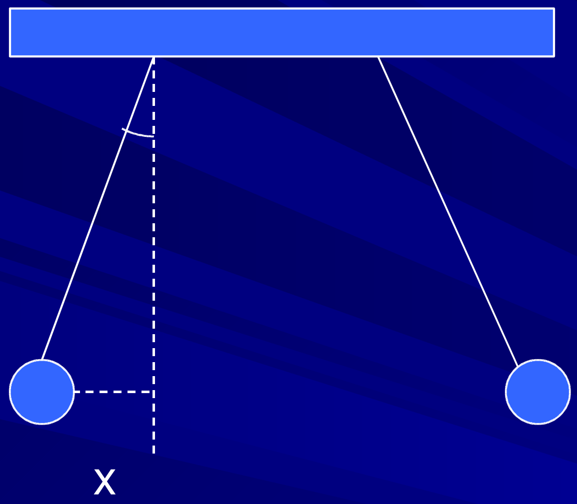
Example:

•Parkinson’s disease, synchronously firing of many neurons results in tremor

Concept of synchronization

Huygens clock (1657)

two identical pendulums + common bar



Phenomenon:

anti-phase synchronized

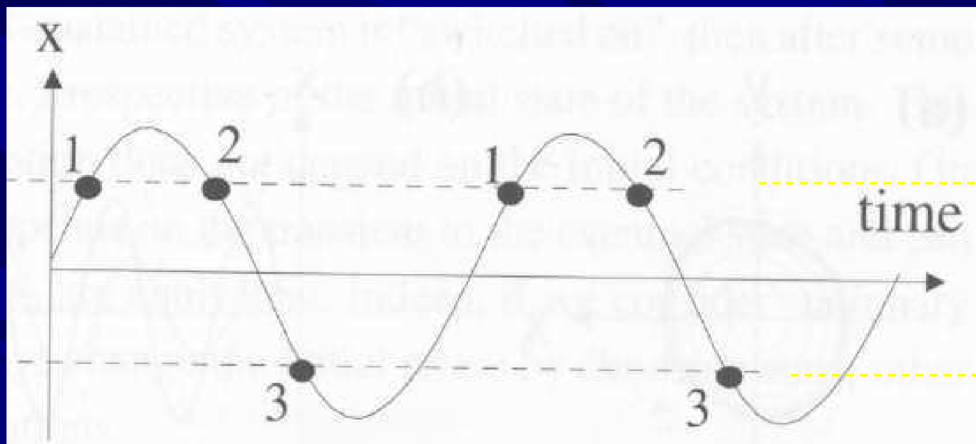
Synchronization of two self-sustained oscillators due to coupling

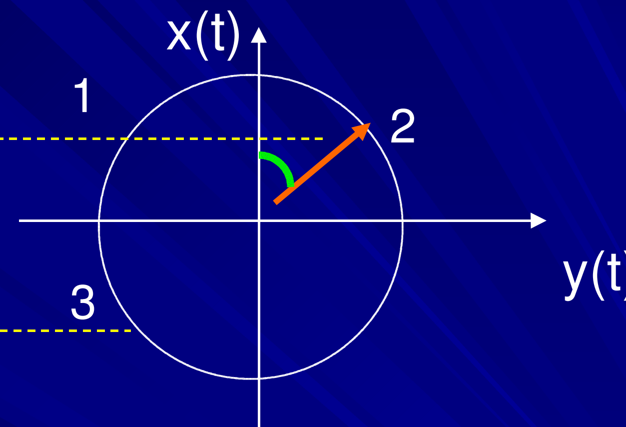
Self-sustained oscillators exhibit "regular" rhythm
driven by internal source

How to describe the motion of self-sustained oscillator?

Linear self-sustained oscillators

x(t) : time series of a self-sustained oscillator





Description of the motion: One variable is not enough!

$x(t) + i\, y(t)$ ⟺ $Ae^{i(\ \ t)}$

Phase: tan (t) =y/x (angular speed ˙ (t)= constant)

Amplitude: $A^2 = x^2 + y^2$ (constant)

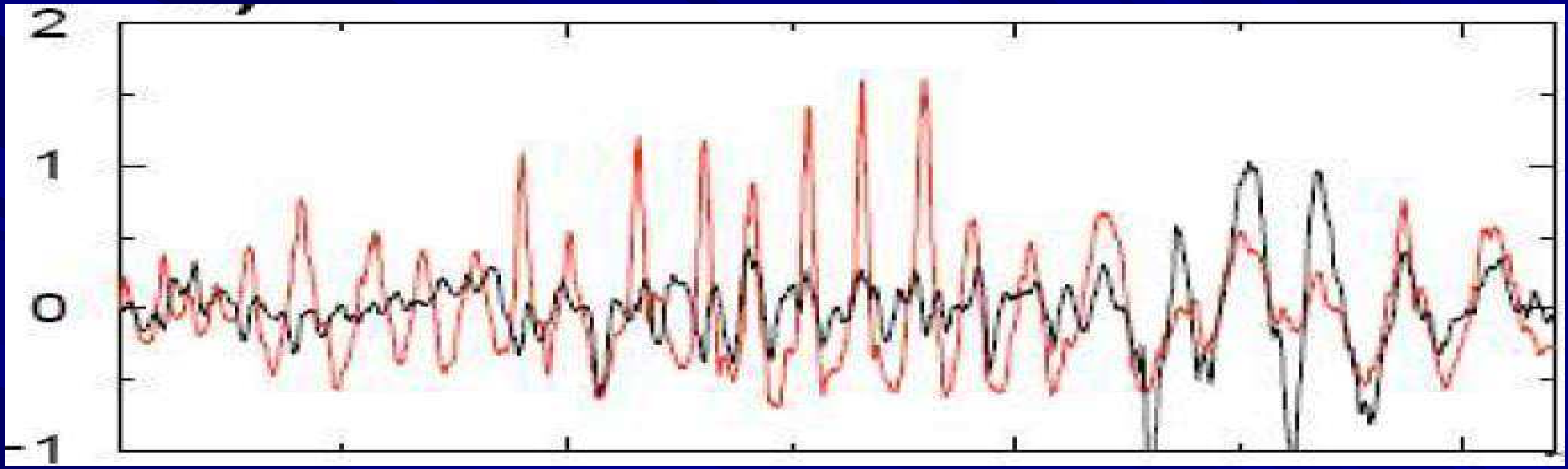
Output signals of biological systems are more complex=>nonlinear oscillators

Nonlinear Oscillators: signal of human postural control

X_1(Red): back-forth sway X_2(Black): left-right sway

$X_1(t)$

$X_2(t)$



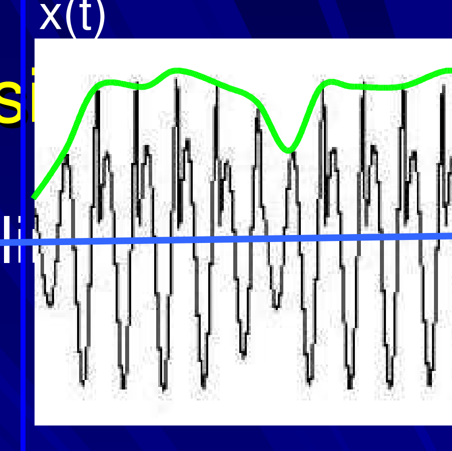
time

Characteristics:

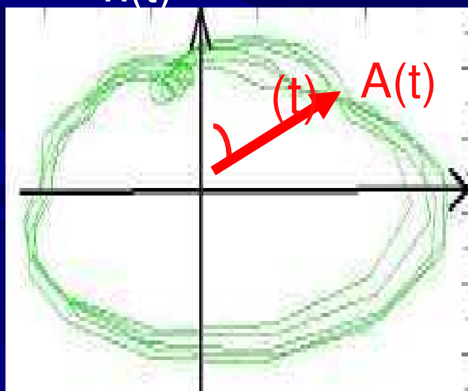
both amplitude and rhythm are time dependent

extraction of amplitude and phase is nontrivial

Extract phase and amplitude from si



for arbitrary time series x(t), analog to the case of li



- time dependent amplitude of the original signal is preserved

for time series of postural control: $x_1(t)$, $x_2(t)$

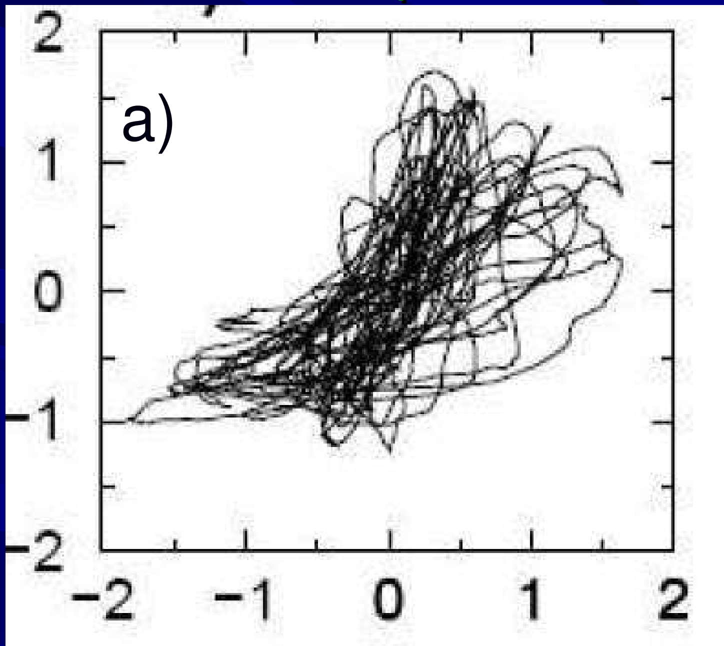
$x_1(t) + i\, y_1(t)$ ⟺ $A_1(t)e^{i\,{}_1(t)}$

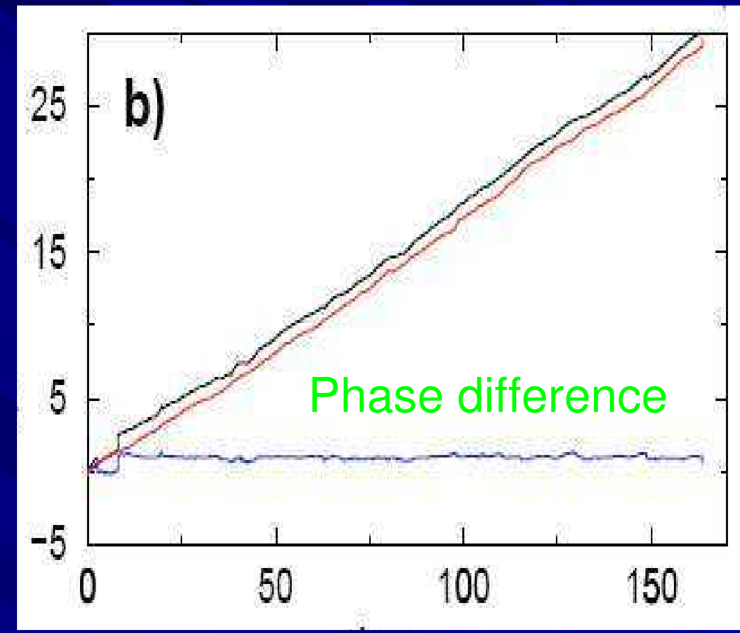
$x_2(t) + i\, y_2(t)$ ⟺ $A_2(t)e^{i\,{}_2(t)}$

How to detect coupling between two time series?

Coupled nonlinear oscillators

time series of human postural control





Amplitude: not synchronized

synchronized: $A_2/A_1 \equiv$ constant

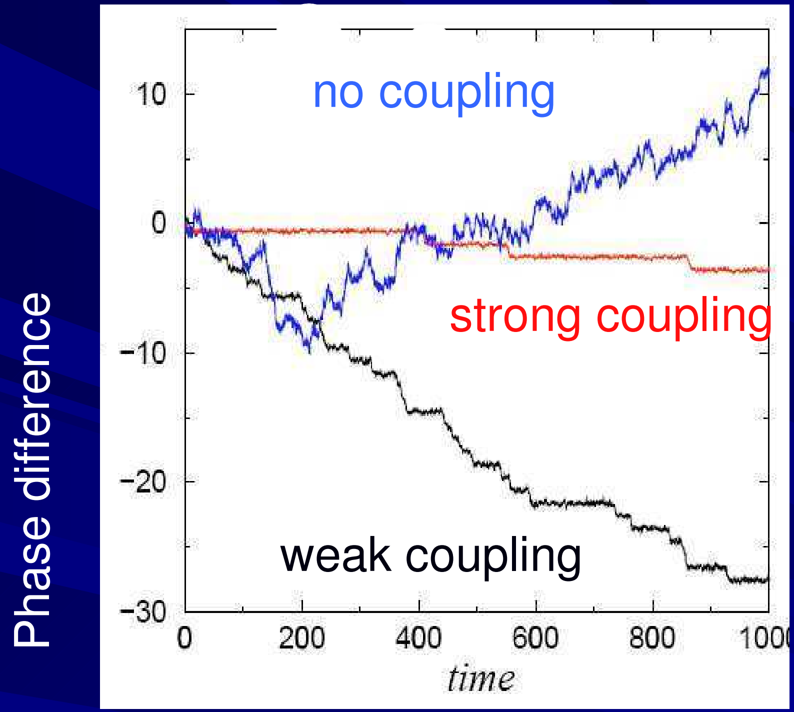
Phase: synchronized

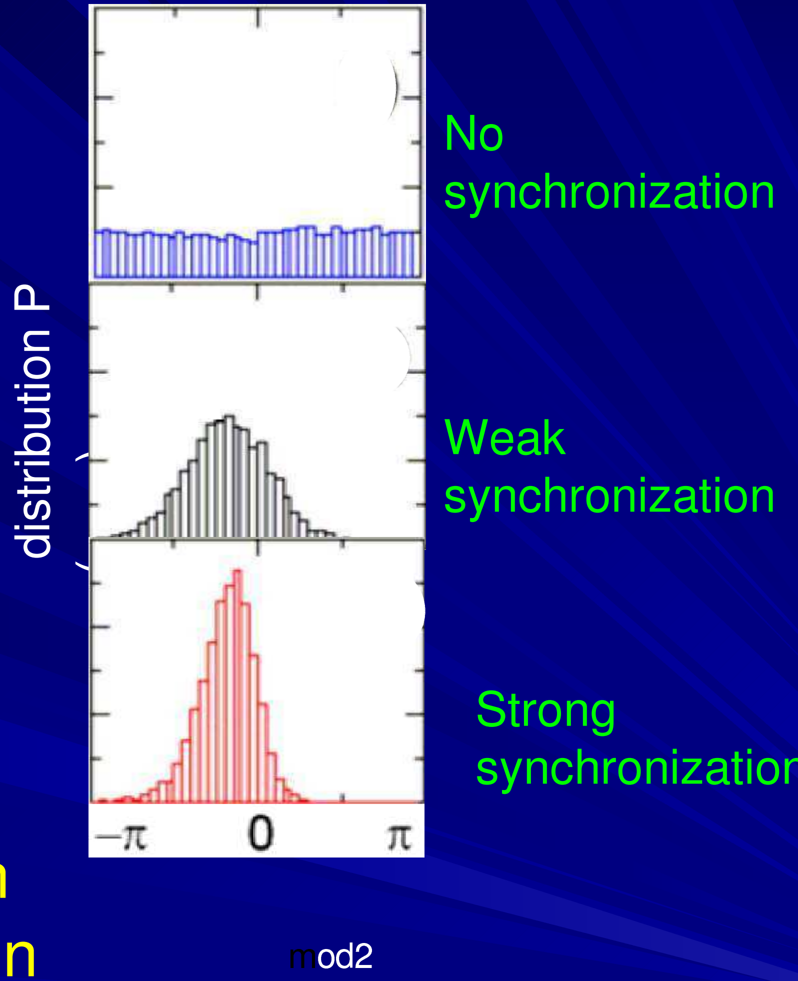
$$|\phi_1 - \phi_2 - Const| < \delta$$

Phase information is important to detect coupling between systems!

A. Piously et al., Intl. J. Bifurcation and Chaos, Vol. 10, 2291 (2000)

Characterization of Phase Synchronization



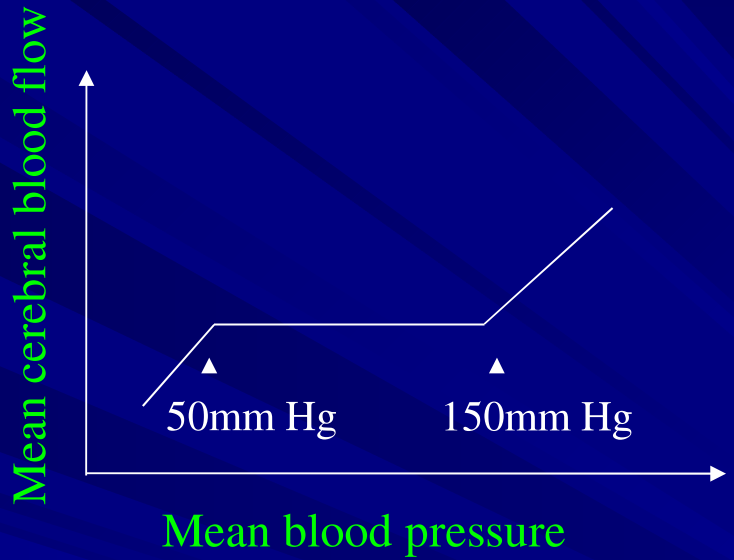


no coupling →uniform distribution
weak coupling →broad distribution
strong coupling→sharp distribution

Synchronization is a measure of the strength of coupling

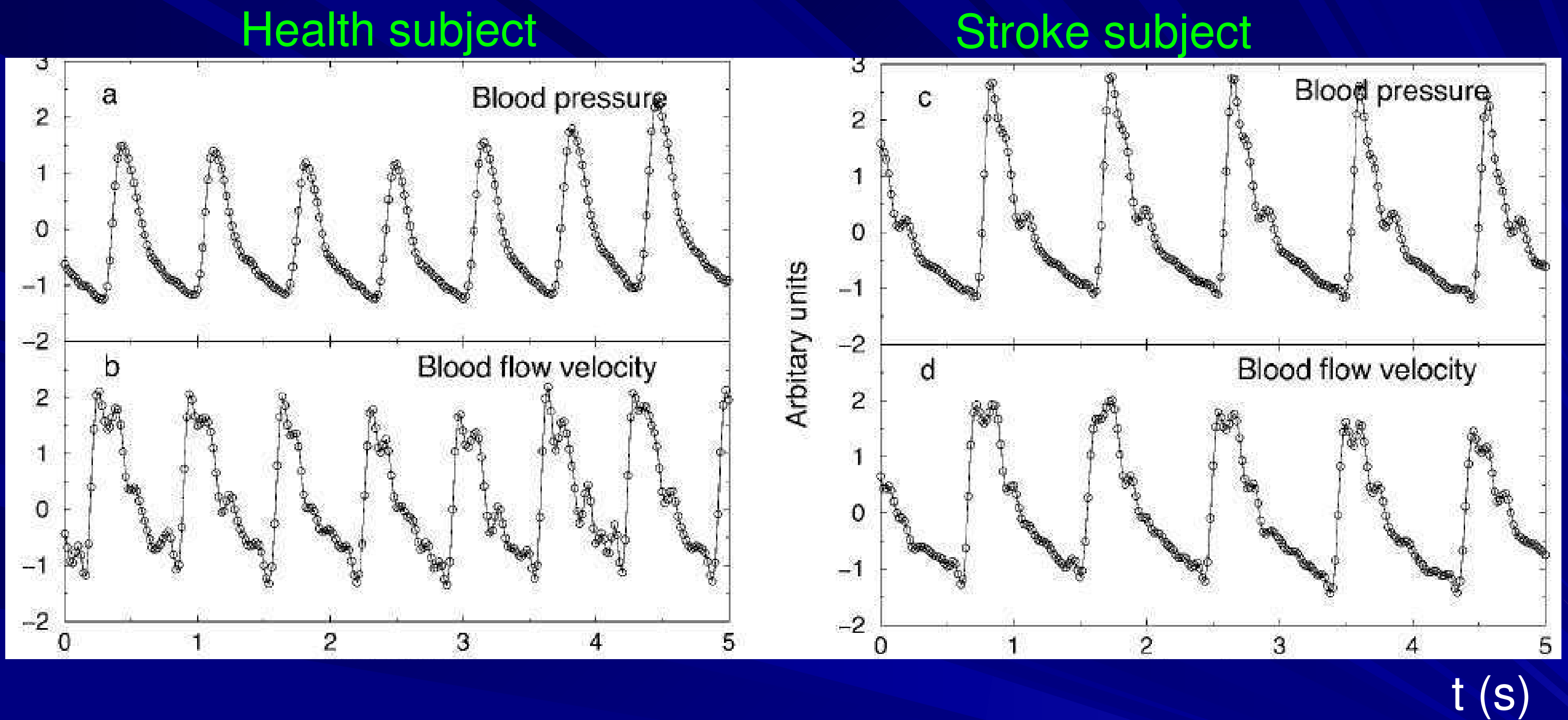
Detect of correlation between blood flow velocity in the brain and blood pressure in the limbs for healthy and stroke subjects

Background: cerebral autoregulation



Cerebral autoregulation is an autonomic function to keep constant blood flow velocity (BFV) in the brain even when blood pressure (BP) changes.

Application to Cerebral Autoregulation



Characteristics: (1) regular rhythm 1Hz (driven by heart beat)

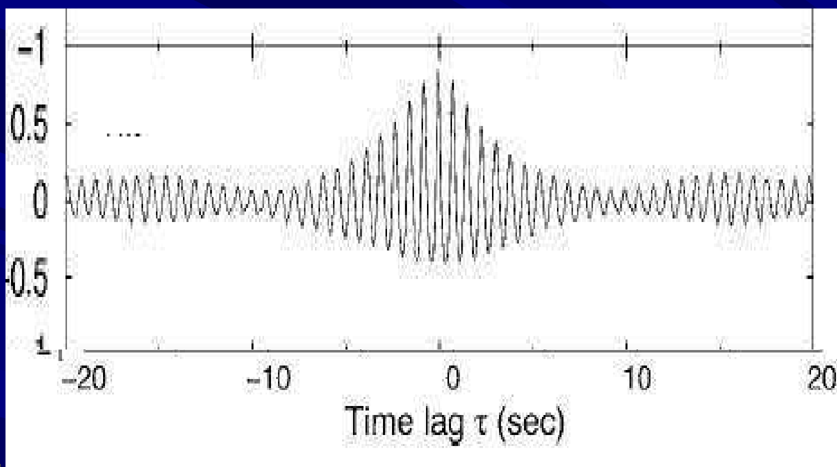
(2) nonstationarity

Question: Is there any difference in the correlations between two signals for healthy subject and unhealthy subjects?

Amplitude cross-correlation between BP and BFV

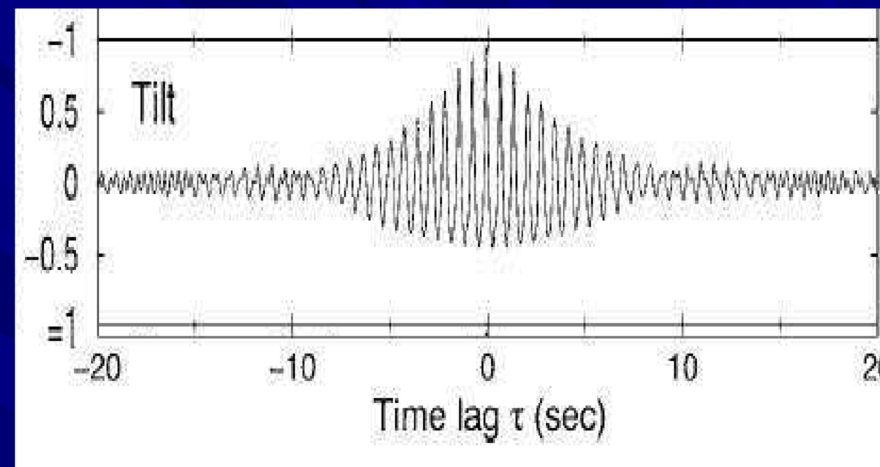
Amplitude cross-correlation

Healthy subject



Time lag

Stroke subject



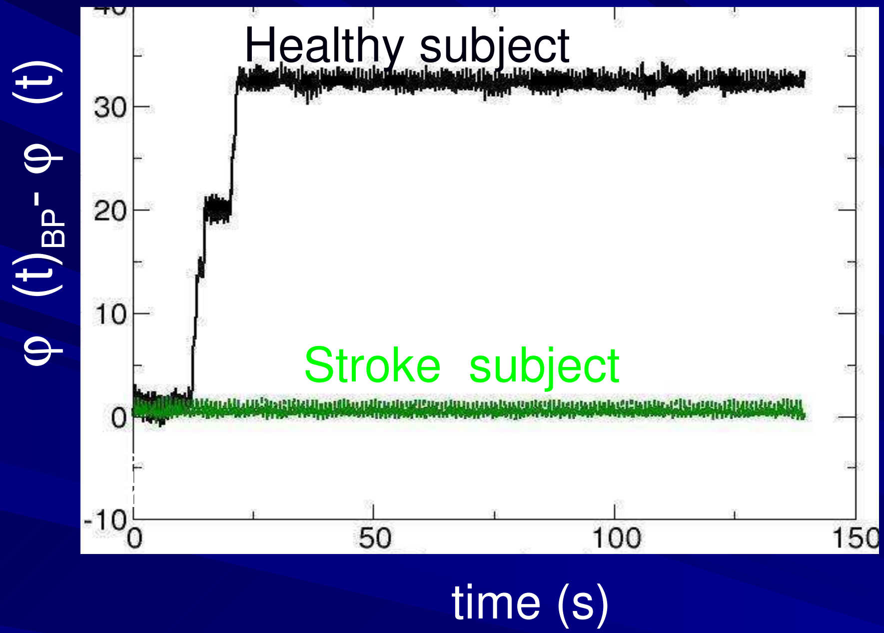
Time lag

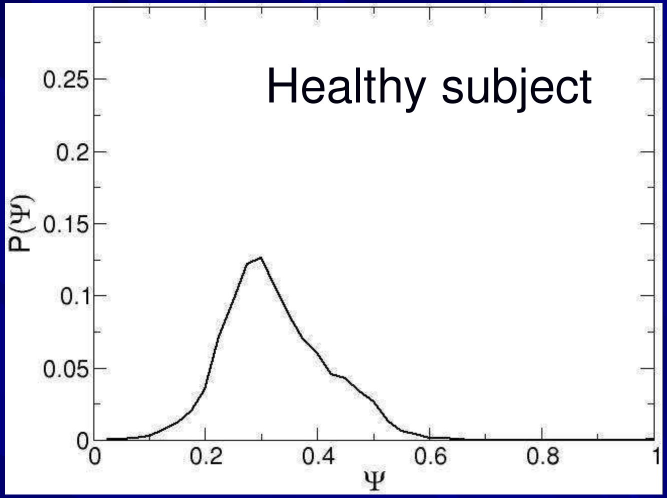
Amplitude cross correlation:

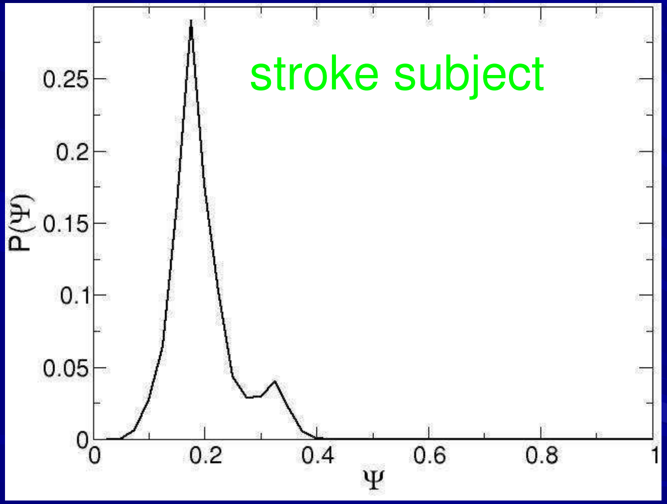
$$C(\tau)=\langle A(t)_{BP}\, A(t+\tau)_{BFV}\rangle / \langle A(0)_{BP}\, A(0)_{BFV}\rangle$$

Simple approach does not work well ☹

Phase synchronization between BFV and BP







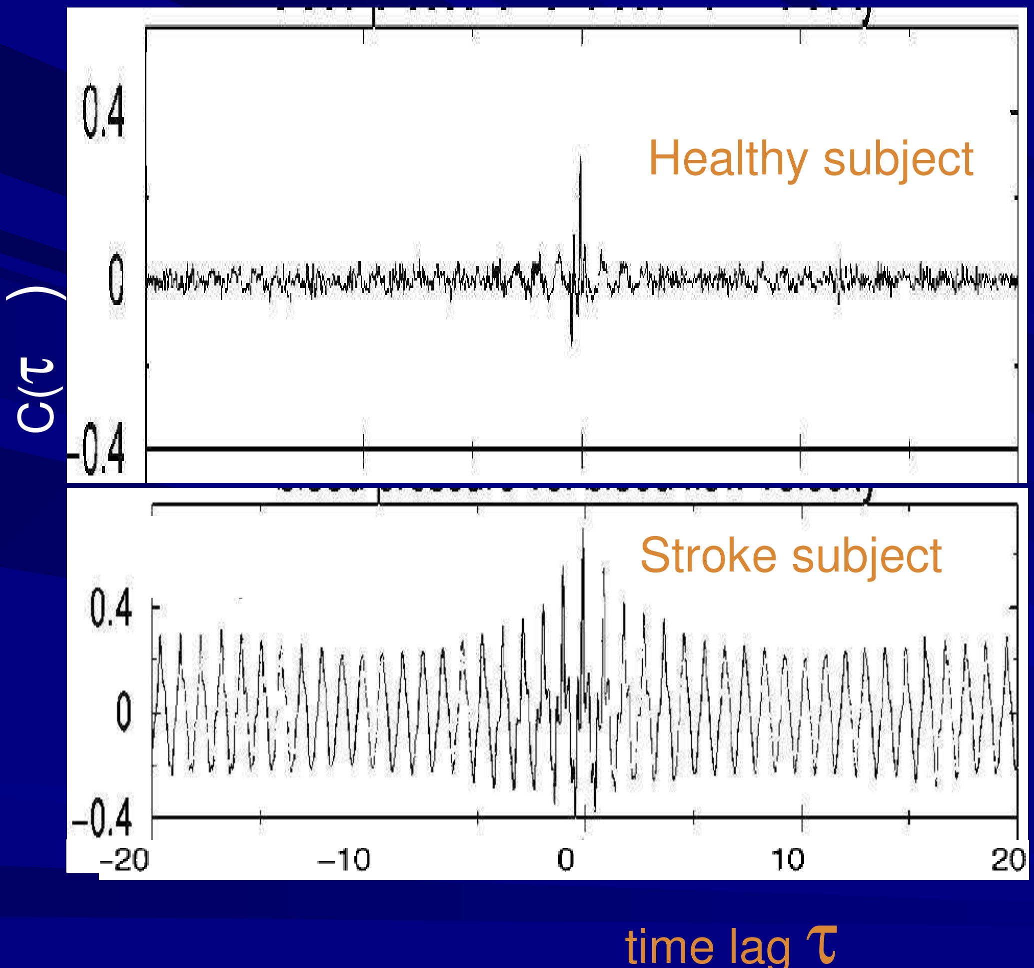
Healthy subject: weak synchronization

Stroke subject: strong synchronization

A change in blood pressure, how long its effects on BFV will last?

Cross-correlation of phases between BFV and BP

$C(\tau) = \langle \varphi(t)_{BP}\, \varphi(t-\tau)_{BFV} \rangle / \langle \varphi(0)_{BP}\, \varphi(0)_{BFV} \rangle$



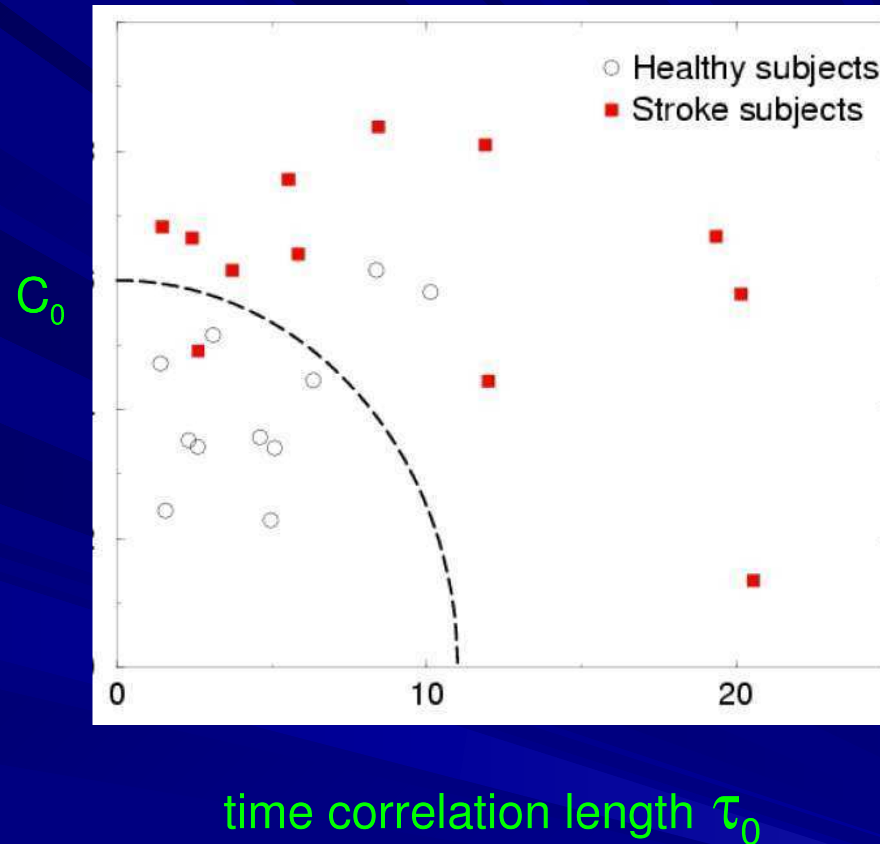
correlation strength at time 0: C_0

time correlation length τ_0: time lag at which correlation drops by 80%

Health subject: short ranged→weak synchronization → Cerebral autoregulation works
Stroke subject: long ranged →strong synchronization→ Cerebral autoregulation impaired

Diagnosis for stroke?

11 healthy
12 stroke subjects



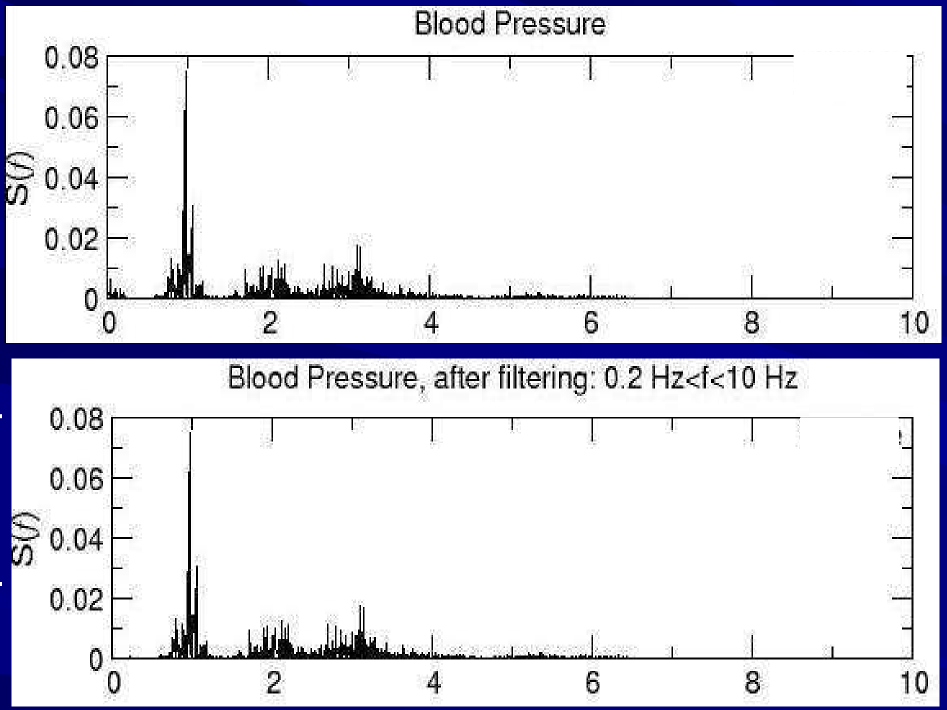
stroke subjects: the instantaneous response of BFV to the change of BP is more pronounced and lasts longer compared to healthy subjects

Effect of band-pass filtering on phase synchronization

phase synchronization approach:

- sensitive to the choice of band-pass filtering
- excessive band-pass filtering → spurious detection

power spectrum



f (Hz)

filtering:

Low frequency range 0-0.2Hz
nonstationarity (trend)

High frequency:
f>10Hz, same results

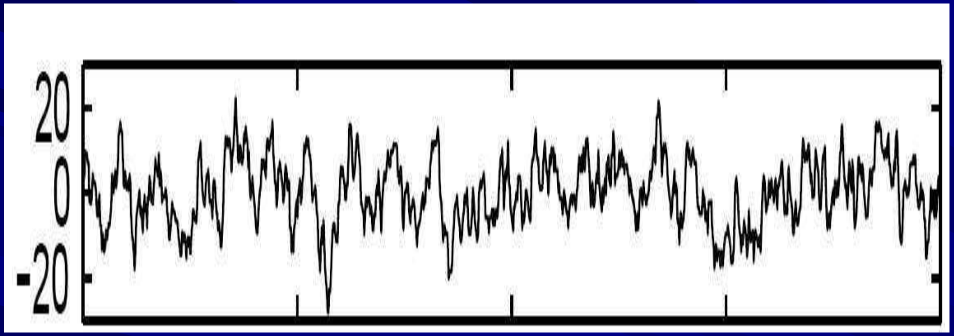
Our detection of the synchronization between BP and BFV is valid

Conclusion

- Phase synchronization is a useful concept to quantify complex behavior in coupled nonlinear systems.
- Useful approach to detect the coupling between systems
- Phase synchronization has practical applications to physiological data and is useful to understand mechanisms of physiologic regulation.

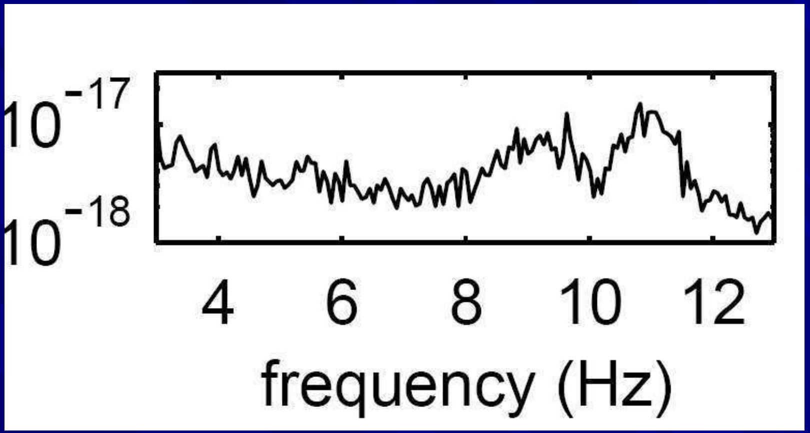
Effect of band-pass filtering for the phase synchronization approach

MEG Signal for Parkinson's patient



time

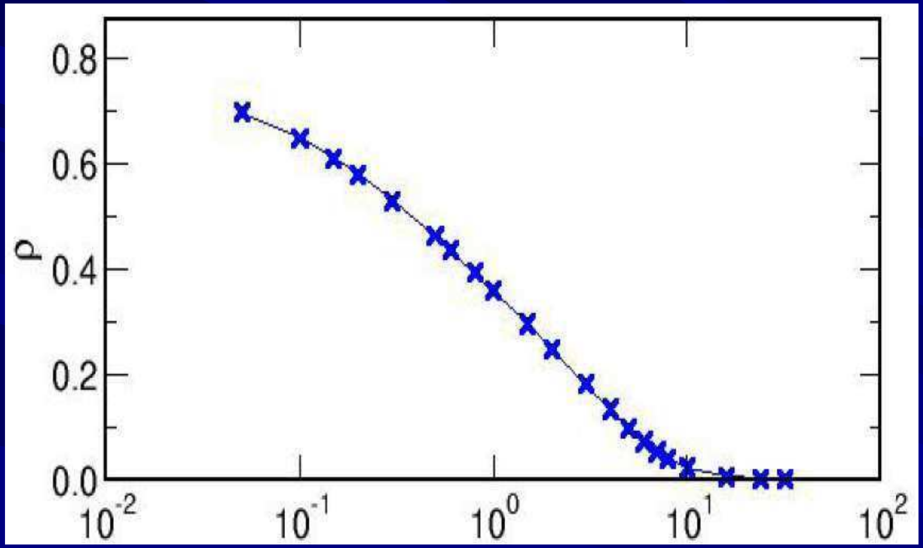
Power spectrum for Parkinson' patient



More than one peak
broad power spectrum

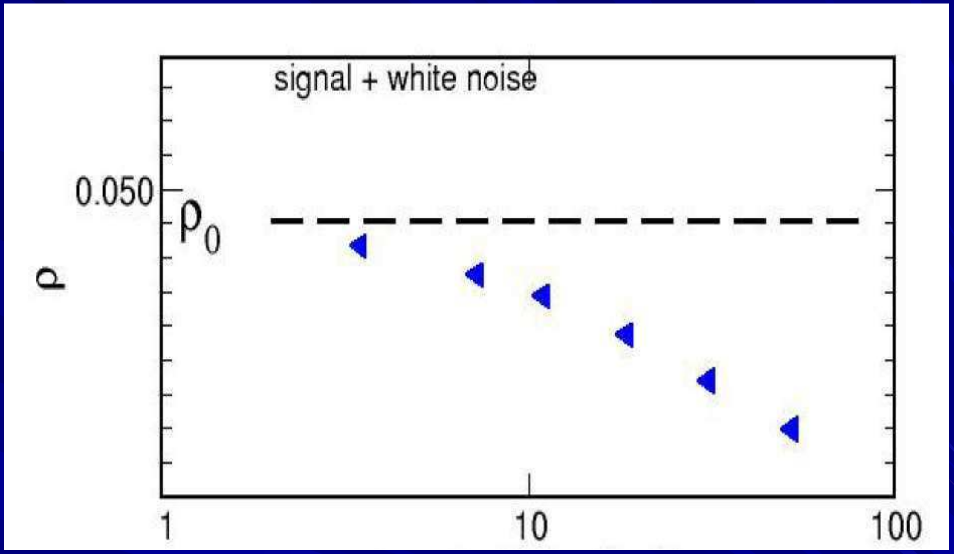
Effect of band-pass filtering on external noise on coupled output of oscillators

High noise hides synchronization



Increasing Noise strength

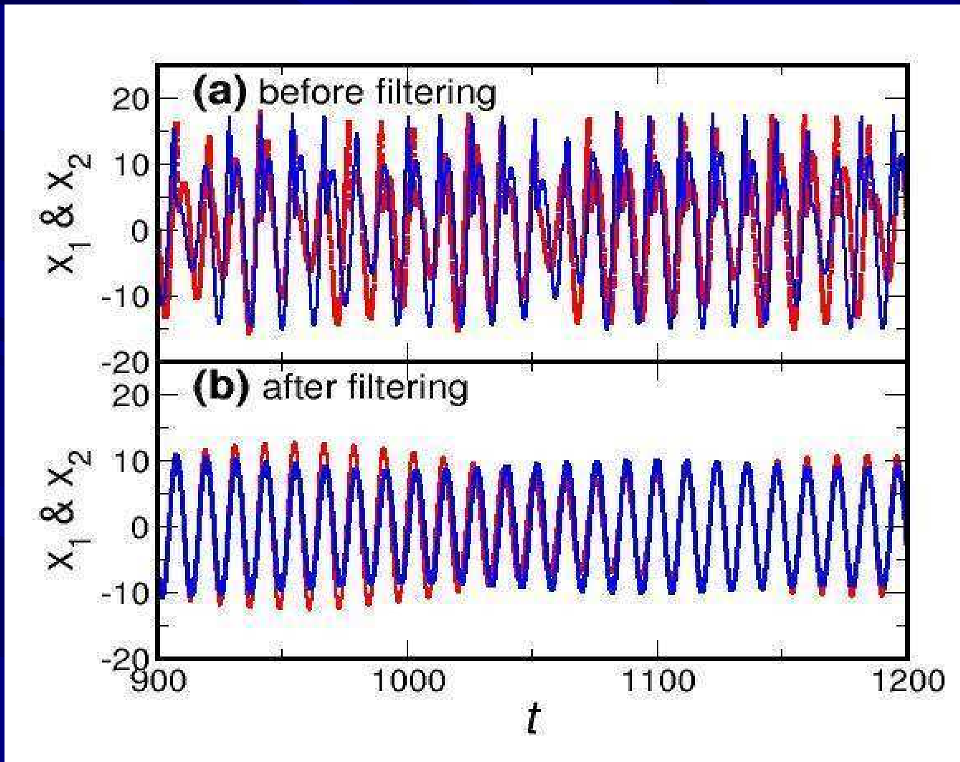
filtering recovers true behavior

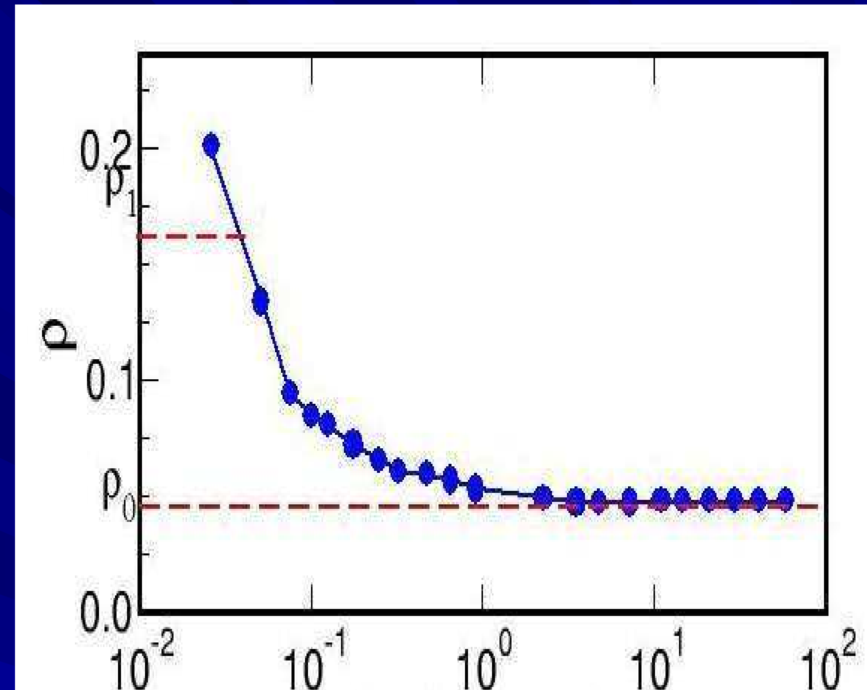


narrowing down band-width

Band-pass filtering is effective to reduce external noise in coupled systems!

Effect of band-pass filtering on phase synchronization





Remaining band-width

Band-pass filtering generate spurious synchronization

Recipes for band-pass filtering

- Phase synchronization is very sensitive to the choice of band-width,
- it is effective to reduce external noise
- excess of band-pass generating spurious detection of phase synchronization

Therefore, many trials has to be done before getting consistent results.