Spiral wave annihilation by low-frequency planar fronts in a model of excitable media

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Abstract – We perform numerical lattice simulations of an excitable medium. We show that the interaction of a spiral wave with a periodic train of planar fronts leads to annihilation of the spiral wave even when i) the period of the fronts is longer than the period of the spiral and ii) the annihilating fronts are released at a significant distance from the spiral. The observed annihilation is not due to spiral drift, and occurs well inside the lattice.

The dynamics of wave propagation in excitable media has been extensively studied in the last years [1–15] and the existence of spiral waves in this kind of system has been reported in many cases, including aggregating slime-mould cells [16], retinae [17], the Belousov-Zhabotinsky chemical oscillator [18], CO oxidation [19], and heart muscle [20,21]. Of special interest, in areas like cardiology, is the study of wavefront stability, as wave breaks generate spatio-temporal patterns that are associated with potentially fatal arrhythmias [1–3,22–32]. Therefore, it is of great interest to find out how to attenuate and annihilate spiral waves.

Here, we investigate the possibility of annihilating spiral waves by a train of periodic plain waves, which is of relevance to the dynamics in the myocardium, where fronts descending from the heart pacemaker (the sinoatrial node) interact with spiral waves. It is widely assumed that stable spiral waves cannot be eliminated by wavefronts of similar or lower frequency, since the domains of faster spiral waves grow at the expense of the slower wavefronts [31,33–37]. In this study, we consider the interaction of a stable spiral wave and planar wavefronts with frequency lower than the frequency of the spiral rotation and we show that this interaction leads to annihilation of the spiral wave over a range of physiologically meaningful parameter values.

It has been reported recently [38,39] that the interaction of a single spiral wave with a train of periodic planar fronts can lead, under certain circumstances, *i.e.*, when the fronts have long excitation duration, and are delivered at a specific phase relative to the rotational phase of the spiral, to the formation of complex spatio-temporal patterns characterized by the presence of periodic attenuation of the spiral wave. Since the results reported in [38,39] are based on a cellular automaton model, it is natural to raise the question of the validity of those findings for continuous models based on partial differential equations.

To perform our analysis we choose the Aliev-Panfilov model of the cardiac tissue [40]. This is a modification of the classical Fitzhugh-Nagumo model aimed at introducing the experimentally observed restitution curve, which the original model lacks [41–45]. As we will explain later, this feature is crucial for the annihilation of the spiral wave. The model equations are

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x_i} d_{ij} \frac{\partial u}{\partial x_j} - ku(u-a)(u-1) - uv, \qquad (1)$$

$$\frac{\partial v}{\partial t} = \epsilon(u,v)\left[-v - ku(u-a-1)\right], \qquad (2)$$

where u is the transmembrane potential, v is the recovery variable (related to the conductance of the cell membrane), $\epsilon(u,v) = \epsilon_0 + \mu_1 v/(u+\mu_2)$ and the typical parameter
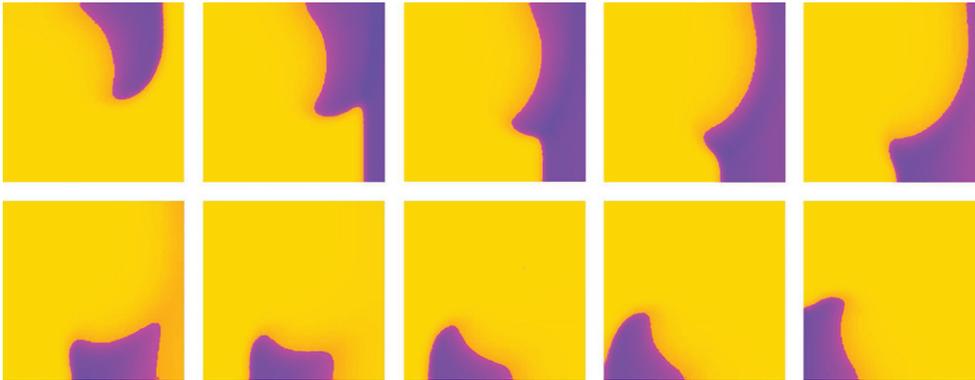
Fig. 1: (Color online) Evolution of the system during the first spiral rotation. The color-coding shows the gradient of variable u in eq. (1) (the transmembrane potential) from high values, in violet-blue, corresponding to excited areas, to low values, in yellow-orange, corresponding to resting areas. From left to right and top to bottom: initial condition; collision with first front, at $t = 3$; and eight other snapshots, each one after every three time units. The parameter values used are: $k = 8$, $a = 0.02$, $\epsilon_0 = 0.4$, $\mu_1 = 0.35$, $\mu_2 = 0.5$. For these parameter values, the model simulations generate a stable spiral with a period $T_{\mathrm{sp}} = 55$. The value chosen for the period of the fronts is $T = 59$.

values are $k = 8$, $a = 0.02$, $\epsilon_0 = 0.4$, $\mu_1 = 0.35$, and $\mu_2 = 0.5$. Here, k is related to the upstroke of u when the excitation occurs, a represents the excitation threshold, and $\epsilon(u, v)$ relates to the difference between the characteristic time scales of u and v. We consider a homogeneous isotropic medium with a diffusion coefficient $d_{ij} = 1$.

We use a 128×128 two-dimensional square lattice with the usual no-flux boundary conditions, by means of a standard multigrid scheme. The space and time integration steps we have used are $\delta x = 0.3$ and $\delta t = 0.1$, respectively. The model involves dimensionless variables, u, v, and t, related to the actual transmembrane potential and time by means of the formulae: $E\,(\mathrm{mV}) = 100u - 80$ (the equilibrium potential is set at $-80\,\mathrm{mV}$ and the pulse jump is $100\,\mathrm{mV}$ since the natural upper bound for u is $u = 1$) and $t(\mathrm{ms}) = 12.9t(\mathrm{t.u.})$ [40]. Assuming a propagation speed of $75\,\mathrm{cm/s}$, in agreement with the experimental results in myocardial cells of guinea pigs [13], the lattice size in our simulations corresponds to $0.6\,\mathrm{cm}$.

We generate the spiral according to a standard procedure, by propagating a planar front with one end close to the center of the lattice and the other end on the lattice boundary [46]. We wait for ≈ 15 spiral rotations until the spiral reaches a stable rotation. We next introduce planar fronts with a period T, starting from the boundary of the lattice. The fronts are introduced by setting $u = 1.0$ (the highest transmembrane potential) and $v = 0.0$ (no refractoriness) for all elements on the right boundary of the lattice; this initial condition for the fronts gives rise to long excitation durations, which plays an instrumental role to achieve spiral annihilation. We have explored periods T in the range 45–65, while the period of the spiral wave is $T_{\mathrm{sp}} = 55$ for the parameter values given above.

Once a single, stable spiral wave is generated (see fig. 1, snapshot 1) and the rotational period of the spiral

stabilizes, we introduce a periodic train of planar fronts that propagate from right to left. The parameter values used for the simulation shown in fig. 1 are: $k = 8$, $a = 0.02$, $\epsilon_0 = 0.4$, $\mu_1 = 0.35$, and $\mu_2 = 0.5$. Figure 1 shows the interaction of the spiral wave and the first front of the train during the first spiral rotation. The first snapshot shows the initial condition, at $t = 0$. Then, the first front is introduced at the right-hand-side boundary of the lattice at $t = 1$ (fig. 1, snapshot 2). Every subsequent snapshot in fig. 1 is taken every 3 time units. Figure 2 shows the interaction of the spiral wave with the second and third fronts, and partially, in the last snapshot, with the fourth one. The first snapshot in fig. 2 corresponds to $t = 67$, and every following snapshot is taken after 13 time units. In agreement with the results found in [38,39], where a cellular automaton was used, we find that subsequent incoming fronts progressively invade the spiral wave domain successfully both along the advancing contour of the spiral and further away from the spiral tip, near the upper right corner of the lattice. However, in contrast with our previous analysis, it is the invasion near the lattice boundary and away from the spiral tip, and not the invasion along the spiral edge, which is successful in altering the spiral dynamics. While the invasion process is not self-evident in the figure, every subsequent front advances a little more than the previous one, in turn creating the conditions for the advance of the next one. The invasion process can also be tracked by the movement of the spiral tip which is progressively kept further and further away from the fronts. Finally, during the interaction with the fourth front (fig. 3), a second counter-rotating spiral develops and both spiral waves collide and annihilate each other. The first snapshot in fig. 3 corresponds to $t = 193$, and every following snapshot is taken after 13 time units. The remaining excited area in the upper left corner of the last but one snapshot
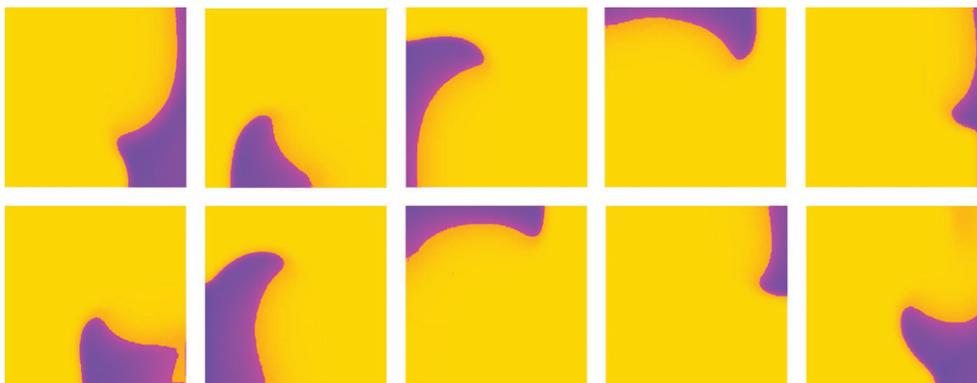
Fig. 2: (Color online) Invasion process. Color-coding and parameter values are as in fig. 1. These snapshots show how the same simulation as in fig. 1 continues. The three first fronts reach the spiral domain and progressively invade the region. The first snapshot was taken at $t = 67$ and every subsequent snapshot was taken after $\Delta t = 13$.
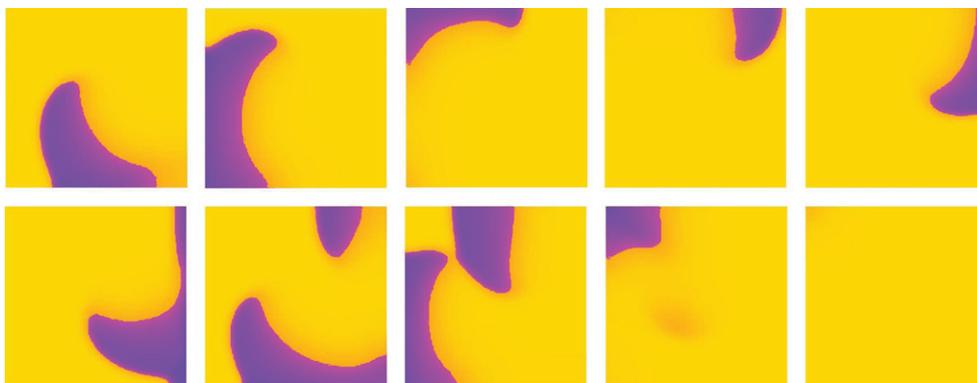
Fig. 3: (Color online) Spiral annihilation. Color-coding and parameter values are as in fig. 1. The fourth subsequent front invades the spiral domain deep enough to achieve the annihilation of the spiral waves. Again, the figure shows the same simulation as in figs. 1 and 2. The three first fronts reach the spiral domain and progressively invade the region. The first snapshot was taken at $t = 193$ and every subsequent snapshot was taken after $\Delta t = 9$. The last snapshot corresponds to $t = 274$. Our simulations show that, within the range of model parameters where spiral annihilation is found, releasing a single planar front at the same specific phase relative to the rotational phase of the spiral as in snapshot 6 does not lead to spiral attenuation. Thus, a train of several fronts (4–6 fronts) is needed to create the necessary preconditions leading to the spiral annihilation observed in snapshot 10. We note that the number of fronts in the train needed to annihilate the spiral does not depend on the timing of the release of the first front relative to the rotational phase of the spiral.

develops into a continuous front that does not form new spiral waves. After the last snapshot, the activity entirely disappears, and the resting state of the whole lattice is only altered by new fronts, if they are introduced. The last snapshot in fig. 3 corresponds to $t = 274$. We note that the process of spiral annihilation (shown in snapshots 6–10 in fig. 3) occurs while the spiral tip is close to the center of the lattice, and that the spiral annihilation is not due to spiral drift to the lattice boundary. Our simulations show that within the whole range of model parameters where spiral annihilation is observed, the distance between the last released front, which leads to spiral annihilation, and the spiral tip is of the order of one spiral wavelength (as shown in snapshot 6 in fig. 3 for a given choice of model parameters). Our observations of a relatively large distance between the annihilating front and the spiral are complementary to earlier studies where spiral annihilation

was achieved by point stimuli released at a distance to the spiral much shorter than a wavelength [47].

In order to further survey the annihilation process, in fig. 4 we plot the total number of excited cells in the lattice as a function of time for three different values of the period of the planar fronts, two of them higher and the other one lower than the spiral period. For this purpose, we have defined a lattice element as excited when $u > 0.1$ and, simultaneously, $v < 1.0$. The curves follow a quasiperiodic behavior until the annihilation of the spiral wave occurs for the two cases of higher period than the spiral. After the annihilation the number of excited cells drops to zero, as expected.

To check the robustness of the annihilation process we have considered different parameter sets: $k = 8$, $a = 0.01$, $\epsilon_0 = 0.5$, $\mu_1 = 0.35$, $\mu_2 = 0.5$; $k = 8$, $a = 0.02$, $\epsilon_0 = 0.4$, $\mu_1 = 0.35$, $\mu_2 = 0.5$; $k = 8$, $a = 0.04$, $\epsilon_0 = 0.1$, $\mu_1 = 0.3$,
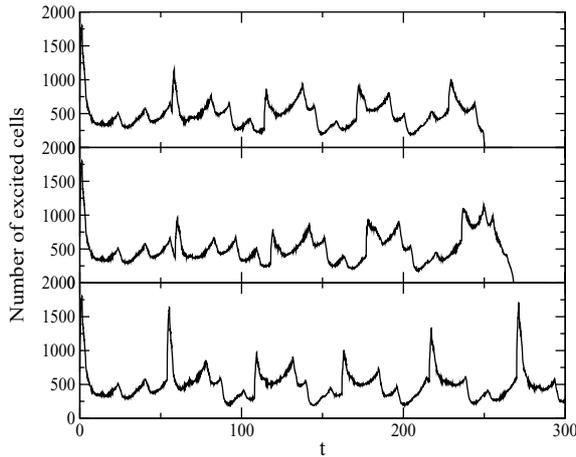
Fig. 4: Total number of excited cells in the lattice as a function of time (in dimensionless units) for three different simulation runs with, from top to bottom, $T = 57$, $T = 59$ and $T = 54$. The parameter values used are the same as in fig. 1, which implies $T_{sp} = 55$. The plots clearly show that only when $T > T_{sp}$ the annihilation process takes place as the number of excited cells goes to zero.

$\mu_2 = 0.4$; $k = 8$, $a = 0.08$, $\epsilon_0 = 0.07$, $\mu_1 = 0.25$, $\mu_2 = 0.35$; $k = 8$, $a = 0.10$, $\epsilon_0 = 0.05$, $\mu_1 = 0.2$, $\mu_2 = 0.3$; and $k = 8$, $a = 0.15$, $\epsilon_0 = 0.002$, $\mu_1 = 0.2$, $\mu_2 = 0.3$. The annihilation process occurs only for values of the excitation threshold parameter $a < 0.04$, and for decreasing values of a (low excitation threshold) the conditions for spiral annihilation become more favorable. Therefore, within the considered low-excitability regime (where spirals are stable), the more excitable the medium is, the easier it is to achieve spiral annihilation. We emphasize that an advantage of our model approach is that, for a broad range of parameter values, spiral annihilation is achieved even when the period of the planar fronts is longer (by 1–4 dimensionless time units or approximately 10–50 ms than the period of the spiral. We note that traditionally spiral attenuation has been achieved with periods shorter than the period of the spiral.

The long excitation duration in the fronts allows every front to leave a lasting mark of refractory area (an area that cannot be excited until the refractory time has elapsed) that outlasts the rotational period of the spiral and, therefore, affects the spiral during the next spiral rotation. The effect of this refractory area is to prevent to some extent the propagation of the spiral wave, but it is very small in the case of the first incoming front. The longer period of the fronts, compared to the rotational period of the spiral, implies that this effect is stronger with every subsequent front. The Fitzhugh-Nagumo model is unable to take into account this kind of interaction between the spiral and the fronts, since it features no significant restitution curve (all excitations have essentially the same duration); therefore, it is critical to introduce a more complex model, as in eqs. (1)

and (2), where an empirically realistic restitution curve is introduced in a natural way.

We have also investigated how the front-spiral interaction depends on the relative phase between the spiral and the fronts, since it has been reported that i) the rotational phase of the spiral wave, when the first front is released, may play a critical role in the dynamics as ii) certain spatio-temporal patterns appear only for specific values of this phase [38,39]. To answer this question, we performed several tests by releasing the first front at a time T_0 after the stabilization of the period of the spiral, followed by a train of fronts with period T. We repeated the simulations for different values of T_0 and for different parameter sets. We find that the relative phase between the first released front and the spiral does not appear to play a significant role in achieving spiral annihilation in these continuous model simulations at least for the set of parameter values used in this work.

In summary, we have found that the interaction of a single, stable spiral wave with a periodic train of planar fronts, with period longer than the rotational period of the spiral and of sufficiently long excitation duration can lead, not only to spiral attenuation (as reported in [38,39]), but also to the complete annihilation of the spiral wave. After collision with as few as 4–6 fronts, the spiral domain is invaded by the fronts and the spiral is annihilated. Thus, the domain of the lower frequency planar waves expands at the expense of the spiral wave until it completely vanishes. This occurs only when the period of the fronts is slightly longer, by 1–4 time units, than the period of the spiral waves. Notably, the annihilation process is not due to spiral drift outside of the lattice boundary, and results from the front-spiral interaction which occurs well within the spatial domain of the lattice.

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